CHAPTER - 7

ANALYSIS OF CHANNEL CAPACITY OF GENERALIZED-K FADING BASED ON MARGINAL MOMENT GENERATING FUNCTION

7.1 INTRODUCTION

The received signal over a wireless channel is usually characterized by the joint effect of two independent random processes such as small scale fading due to the arrival of multiple, randomly delayed, reflected and scattered signal components at the receiver side and large scale fading due to shadowing from various obstacles in the propagation path. Therefore, it is useful for various wireless system designers to have a general statistical model that encompasses both of these random processes. To model the small-scale fading channel, various fading models such as Rayleigh, Rician and Nakagami have been proposed in [2]. In addition to the multipath fading in the wireless environment, the quality of signal is also affected due to the shadowing from various obstacles in the propagation path. The Nakagami-$m$ and Rayleigh-lognormal (R-L) are well known composite statistical distribution to model the multipath and shadowing [160-166]. As these distributions don’t have closed-form mathematical solution, so it is difficult to use it. However, they have been approximated by the Generalized-K distribution [163] and K-distribution [160]. The diversity combining [167-173] is an effective technique for mitigating the detrimental effects of the multipath fading and shadowing in the wireless mobile channels.

In general, the channel capacity in fading environment is a complex expression in terms of the channel variation in time and/or frequency depending upon the transmitter’s and/or receiver’s knowledge of the channel side information. For the various channel side information assumptions that have been proposed, several definitions of the channel capacity have been provided. These definitions depend on the different employed power and rate adaptation policies and the existence, or not, an outage probability [22]. Earlier, the channel capacity has been studied by various researchers for different fading environment [104-110]. In [104], Goldsmith and Varaiya have examined the channel capacity of the Rayleigh fading channels under
different adaptive transmission techniques. Lee [105] has derived an expression for the channel capacity of a Rayleigh fading channel. In [106], Gunther has extended the results presented in [105] by deriving the channel capacity of Rayleigh fading channels under diversity scheme. Alouini and Goldsmith in [107] have derived the channel capacity of Rayleigh fading channels under different diversity schemes and different rate adaptation and transmit power schemes. Other fading channels like K-fading, Nakagami, Weibull, Rician, and Hoyt fading channels were studied in [108-112]. As per best of author’s knowledge, the ref. [113, 114] is the main article, which deals with the channel capacity on generalized-K fading. In [113], the channel capacity under different diversity schemes and different rate adaptation and transmit power schemes for K-fading channel has been derived. However, in [113] has limitation that the channel capacity has been analyzed only for special value of the shaping parameter that is $m = 2$ and moreover in [113], for calculation of channel capacity under the optimal rate constant (Cora) policy, author shows two methods for calculation of (Cora), first method involves Lommel function and second method shows that, when $k$ is an integer plus one half, the capacity can be expressed in the term of the more familiar sine and cosine integrals. These two methods as discussed above having complex expression for the channel capacity. In [114], the channel capacity under different diversity schemes and different rate adaptation and transmit power schemes for K-fading channel has been derived. For computation of the Cora, in [114] Ethymoglou et al have been taken the approximate value by calculating limit at ($a → 1$) and formula for Cora is valid for non-integer value of shaping parameters $k$ and $m$. If $k$ and $m$ are integers then formula for Cora fails. In [111, 112], the characteristics function (CF) is developed for computing the ergodic channel capacity. In [112, 115], the moment generating function based (MGF) approach is proposed for computation of the channel capacity only for Cora scheme by using numerical techniques. In [116], a novel MGF based approach is developed for evaluation of the channel capacity for various rate adaptations and transmit power. In [116], the integral is evaluated by using mainly two type of numerical technique and both the numerical techniques are lengthy and much more complex.

In this chapter, we have presented MGF based channel capacity analysis over Generalized-K fading channel with M-branch maximal-ratio combining (MRC)
diversity. The main contribution of this chapter consists in the evaluation of the MGF function and the derived MGF function is used to evaluate a closed-form expression for the channel capacity under optimal rate adaptation (ORA) and channel inversion with fixed rate (CIFR). The derived results are obtained in the terms of well known Meijer G function, which can be easily implemented using Maple or Mathematica software.

7.2 GENERALIZED-K FADEING CHANNEL MODEL

The channel model for M-branch MRC diversity receiver operating over a generalized-K fading channel is similar with Section 6.4 in the previous chapter and the probability density function (PDF) is given by Equation (6.8) in same chapter.

7.2.1 MARGINAL MOMENT GENERATING FUNCTION

In this Section, marginal MGF is evaluated of SNR of M-branch MRC diversity as Equation (5.2) and further it is used to obtain the channel capacity. The marginal MGF is defined as [155]:

\[
\hat{M}(s, \gamma_0) = \int_{\gamma_0}^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma
\]  

(7.1)

By substituting the value of \( f_\gamma(\gamma) \) from Equation (6.8) in Equation (7.1), we get:

\[
\hat{M}(s, \gamma_0) = \int_{\gamma_0}^\infty \frac{2}{\Gamma(mL)\Gamma(k)} (\Xi)^{(\alpha+1)/2} (\gamma)^{(\alpha-1)/2} K_\beta \left[ 2\sqrt{\Xi \gamma} \right] d\gamma
\]  

(7.2)

By putting \( 2\sqrt{\Xi \gamma} = t \) and after some mathematical manipulation, the Equation (7.2) can be written as:

\[
\hat{M}(s, \gamma_0) = \frac{1}{(2)^{\alpha-1}\Gamma(mL)\Gamma(k)} \Gamma \int_{2\sqrt{\Xi \gamma_0}}^\infty t^{\alpha-1} e^{-t} e^{s/4t} K_\beta(t) dt
\]  

(7.3)

By expressing \( K_\beta(t) = \frac{I_\beta(t) - I_{\beta}(t)}{\sin \pi \beta} \) form [149] and putting it in Equation (7.3), we get:
\[ \hat{M}(s, \gamma_0) = \frac{1}{(2)^{\alpha-1} \Gamma(mL) \Gamma(k) 2\sqrt{\pi \gamma_0}} \int_0^\infty t^{\alpha \beta} e^{-r^2 s / 4 \pi} \left[ \frac{I_{-\beta}(t) - I_{\beta}(t)}{\sin \pi \beta} \right] dt \]  

(7.4)

From [149], by expressing \( I_{-\beta}(t) \) and \( I_{\beta}(t) \) as:

\[ I_{-\beta}(t) = \sum_{k=0}^\infty \frac{1}{p! \Gamma(k - \beta + 1)} \left( \frac{t}{2} \right)^{-\beta + 2k} \]  

and

\[ I_{\beta}(t) = \sum_{k=0}^\infty \frac{1}{p! \Gamma(k + \beta + 1)} \left( \frac{t}{2} \right)^{\beta + 2k} \]  

(7.5)

(7.6)

By putting the value of \( I_{-\beta}(t) \) and \( I_{\beta}(t) \) from the Equation (7.5) and (7.6), respectively in the Equation (7.4), we get:

\[ \hat{M}(s, \gamma_0) = \frac{1}{(2)^{\alpha-1} \Gamma(mL) \Gamma(k) \sin \pi \beta 2\sqrt{\pi \gamma_0}} \int_0^\infty t^{\alpha+\beta+2p} e^{-r^2 s / 4 \pi} \left( \sum_{p=0}^\infty \frac{1}{p! \Gamma(p - \beta + 1)} \left( \frac{t}{2} \right)^{-\beta + 2p} - \sum_{k=0}^\infty \frac{1}{p! \Gamma(p + \beta + 1)} \left( \frac{t}{2} \right)^{\beta + 2p} \right) dt \]  

(7.7)

By changing the order of integration in Equation (7.7), we get:

\[ \hat{M}(s, \gamma_0) = \frac{1}{(2)^{\alpha-1} \Gamma(mL) \Gamma(k) \sin \pi \beta} \left( \sum_{p=0}^\infty \frac{1}{p! \Gamma(p - \beta + 1)} \left( \frac{t}{2} \right)^{-\beta + 2p} \int_2^{\sqrt{2}/\gamma_0} t^{\alpha + \beta + 2p} e^{-r^2 s / 4 \pi} dt \right) \]

\[ \sum_{p=0}^\infty \frac{1}{p! \Gamma(p + \beta + 1)} \left( \frac{t}{2} \right)^{\beta + 2p} \int_2^{\sqrt{2}/\gamma_0} t^{\alpha + \beta + 2p} e^{-r^2 s / 4 \pi} dt \]  

(7.8)

Say \( I_1 = \int_2^{\sqrt{2}/\gamma_0} t^{\alpha - \beta + 2p} e^{-r^2 s / 4 \pi} dt \)  

(7.9)

\[ I_2 = \int_2^{\sqrt{2}/\gamma_0} t^{\alpha + \beta - 2p} e^{-r^2 s / 4 \pi} dt \]  

(7.10)

Now evaluation of \( I_1 \) as given by Equation (7.9), is obtained as given below:

\[ I_1 = \int_2^{\sqrt{2}/\gamma_0} t^{\alpha - \beta + 2p} e^{-r^2 s / 4 \pi} dt \]
Using [149 Equation (3.381.3)] and after some mathematical manipulation, the Equation (7.9) can be expressed as:

\[ I_1 = (2)^{(\alpha - \beta + 2p + 1)} (s)^{(\alpha - \beta + 2p + 1)} \Gamma \left( \frac{\alpha - \beta + 2p + 1}{2}, \gamma_0 s \right) \]  

(7.11)

Similarly, \( I_2 \) can be expressed as given below:

\[ I_2 = (2)^{(\alpha + \beta + 2p + 1)} (s)^{(\alpha + \beta + 2p + 1)} \Gamma \left( \frac{\alpha + \beta + 2p + 1}{2}, \gamma_0 s \right) \]  

(7.12)

By substituting the value of \( I_1 \) and \( I_2 \) in Equation (7.8), marginal MGF can be expressed as:

\[
\hat{M}(s, \gamma_0) = \frac{1}{\Gamma(mL) \Gamma(k) \sin \pi \beta} \sum_{p=0}^{\infty} \left( \frac{2}{\Gamma(p - \beta + 1)} (s)^{\frac{\alpha - \beta + 2p + 1}{2}} \Gamma \left( \frac{\alpha - \beta + 2p + 1}{2}, \gamma_0 s \right) \right) - \\
\sum_{p=0}^{\infty} \left( \frac{1}{p! \Gamma(p + \beta + 1)} (s)^{\frac{\alpha + \beta + 2p + 1}{2}} \Gamma \left( \frac{\alpha + \beta + 2p + 1}{2}, \gamma_0 s \right) \right) 
\]  

(7.13)

If we put lower limit \( \gamma_0 = 0 \) in Equation (7.13), then marginal MGF changes to MGF.

MGF for generalized-K fading channel is similar with Equation (6.10) as discussed in the previous chapter.

### 7.3 MARGINAL MGF-BASED CHANNEL CAPACITY ANALYSIS

The channel capacity has been used as the fundamental information theoretic performance measure to predict the maximum information rate of a communication system. It is extensively used as the basic tool for the analysis and design of new and more efficient techniques to improve the spectral efficiency of modern wireless communication systems and to gain insight into how to counteract the detrimental effects of the multipath fading propagation via opportunistic and adaptive communication methods. The main reason for the analysis of the spectral efficiency over fading channels is represented by the fact that most framework described in various literature make use of the so-called PDF based approach of the received SNR to be applied, which is a task that might be very cumbersome for most system setups and often require to manage expression including series. It is also well known that a prior knowledge of channel state information at the transmitter may be exploited to...
improve the channel capacity such that in the low SNR regime, the maximum achievable data rate of a fading channel might be much larger than when there is no fading. The MGF and CF based approaches have extensively been used for analyzing average bit error rate probability and outage probability. Alouini et al [170] have also pointed out the complexity of using and generalizing MGF and CF based approaches for channel capacity computation. Moreover, the application of the PDF based approach for channel capacity computation turns out to be in evident counter tendency with recent advances on performance analysis of digital communication over fading channels. Several researchers [171-174] have clearly shown the potential of using either an MGF or CF-based approach for simplifying the analysis in most situation of interest with the computation of important performance parameters where the application of PDF based approach seems impractical. Recent advances on the performance analysis of digital communication systems in fading channels has recognized the potential importance of the MGF or Laplace transforms as a powerful tools for simplifying the analysis of diversity communication systems. This led to simple expressions to average bit and symbol-error-rate for wide variety of digital communication scheme on fading channels including multipath reception with correlated diversity [173-174]. Key to these developments was the transformation of the conditional error-rate expressions into different equivalent forms in which the conditional variable appears only as an exponent. In this section, we have proposed some alternative expressions for the channel capacity computation relying on the knowledge of the MGF, \( M_\gamma() \) of \( \gamma \). We have obtained novel expression for \( C_{\text{ORA}}, C_{\text{CIFR}}, C_{\text{OPRA}} \) and \( C_{\text{TCIFR}} \) schemes using a novel marginal MGF based channel capacity analysis approach.

### 7.3.1 OPTIMAL RATE ADAPTATION

When transmitter power remains constant, usually as a result of channel state information being available at receiver side, the channel capacity with optimal rate adaptation (\( C_{\text{ORA}} \)) in the terms of MGF based approach can be expressed as [112]:

\[
C_{\text{ORA}} = \frac{1}{\ln(2)} \int_0^\infty \frac{e^{-s} \left(1 - M(s)\right)}{s} ds \tag{7.14}
\]
\[ I_3 = \frac{1}{\ln(2)} \int_0^\infty \frac{e^{-s}}{s} ds \]  \(7.15\)

and

\[ I_4 = -\frac{1}{\ln(2)} \int_0^\infty \frac{e^{-s} M(s)}{s} ds \]  \(7.16\)

From (7.15) by putting the values \(M(s)\) in \(I_4\) that is Equation (7.16), we get:

\[ I_4 = -\frac{1}{\ln(2)} \int_0^\infty e^{-s} \left[ \frac{\Xi}{s} \right]^{\alpha+1} \frac{1}{\Gamma(mL) \Gamma(k)} \frac{G^2}{s} \left[ 1 \left( \frac{(1-\alpha)/2}{\beta/2} - \frac{\beta/2}{\beta/2} \right) \right] ds \]  \(7.17\)

By putting \(s = \Xi t\) and after some mathematical manipulation, the Equation (7.17) can be expressed as:

\[ I_4 = -\frac{\Xi^{\alpha+1}}{\ln(2) \Gamma(mL) \Gamma(k)} \int_0^\infty e^{-\Xi t} \left[ \frac{1}{2} \left( \frac{(1-\alpha)/2}{\beta/2} - \frac{\beta/2}{\beta/2} \right) \right] dt \]  \(7.18\)

In order to make integral more simpler by using [157 Equation (8.2.2.14)], we get:

\[ G^2 \left[ 1 + \left( \frac{1}{2} \left( \frac{(1-\alpha)/2}{\beta/2} - \frac{\beta/2}{\beta/2} \right) \right) \right] = G^2 \left[ 1 - \left( \frac{1-\beta/2}{1+\beta/2} \right) \right] \]  \(7.19\)

The Equation (7.18) can be expressed as:

\[ I_4 = -\frac{\Xi^{\alpha+1}}{\ln(2) \Gamma(mL) \Gamma(k)} \int_0^\infty e^{-\Xi t} \left[ \frac{1}{2} \left( \frac{1-\beta/2}{1+\beta/2} \right) \right] dt \]  \(7.20\)

From [25 Equation (2.24.3.1)], the Equation (7.20) can be expressed as:

\[ I_4 = -\frac{\Xi^{\alpha+1}}{\ln(2) \Gamma(mL) \Gamma(k)} G^2 \left[ 1 + \left( \frac{1}{3} \left( \frac{1+(\alpha+1)/2}{\beta/2} - \frac{1-\beta/2}{1+\beta/2} \right) \right) \right] \]  \(7.21\)

Expression \(I_3\) shown in the Equation (7.15), is singular at zero and diverging between 0 to 1. So after approximation, Equation (7.15) can be expressed as:

\[ I_3 = \frac{1}{\ln(2)} \int_1^\infty \frac{e^{-s}}{s} ds \]  \(7.22\)
From [23] the above integral can be written as:

\[ I_3 = \frac{0.21938}{\ln(2)} \quad (7.23) \]

By substituting results of eq. (7.21) and (7.23) in (7.14), \( C_{ORA} \) can be expressed as:

\[ C_{ORA} = \frac{1}{\ln(2)} \left[ \frac{0.21938}{\Gamma(mL) \Gamma(k)} \right] G^2 \left[ \frac{1}{3} \left\{ \frac{1 + (\alpha + 1)/2}{(\alpha + 1)/2} \right\} \right] \tag{7.28} \]

\[ C_{ORA} \approx \frac{1}{\ln(2)} \left[ \frac{0.21938}{\Gamma(mL) \Gamma(k)} \right] G^2 \left[ \frac{1}{3} \left\{ \frac{1 + (\alpha + 1)/2}{(\alpha + 1)/2} \right\} \right] \tag{7.29} \]

The above expression for the capacity with ORA policy evaluates correctly for arbitrary non-integer values of shaping parameters \( k \) and \( m \) whereas the limit for these special values can be obtained numerically, an approach for evaluation of \( C_{ORA} \) through this is much simpler than [113] and [114]. The Equation (7.29) is also valid for non-integer value of \( k \) and \( m \). The channel capacity with optimal rate adaptation (ORA) can be calculated in closed-form, by using another method in the terms of MGF based approach as [116]:

\[ C_{ORA} = \frac{1}{\ln(2)} \int_0^\infty E_s(-s) M_j^Y(s) ds \tag{7.30} \]

where \( E_s(\cdot) \) denotes the exponential integral function defined in [157] and \( M_j^Y(s) \) is the first derivative of MGF. By expressing \( E_s(-s) \) as [157 Equation (8.4.11.1)] and by putting value Equation (6.10) in the Equation (7.30), we get:

\[ I_s = -\left[ \frac{0.21938}{\Gamma(mL) \Gamma(k)} \right] G^2 \left[ \frac{1}{3} \left\{ \frac{1 + (\alpha + 1)/2}{(\alpha + 1)/2} \right\} \right] \tag{7.31} \]

By using [157 Equation (8.2.1.35)] along with [157 Equation (8.2.1.14)] and by putting \( s = \Xi t \), the integral given in Equation (7.31), can be expressed as:

\[ I_s = \frac{1}{\Gamma(mL) \Gamma(k)} \left[ (t)^{-(\alpha + \beta)/2} \right] G^2 \left[ \frac{1}{2} \left\{ \frac{1 - \beta/2}{1 + (\alpha + 1)/2} \right\} \right] dt \tag{7.32} \]
From [157 Equation (2.24.1)], the integral \( I_5 \) in Equation (7.32), can be expressed as:

\[
I_5 = \frac{(\Xi)^{\alpha+1}}{\Gamma(mL)\Gamma(k)} \cdot \frac{1}{4} \cdot \left[ \frac{1}{\Xi} \left( \frac{1}{1+\alpha+1/2} \right) \frac{1}{1+(\alpha+1)/2} \right] \]

(7.33)

By putting \( I_5 \) in (7.30), \( C_{OR4} \) can be expressed as:

\[
C_{OR4} = \frac{1}{\ln(2)} \frac{(\Xi)^{\alpha+1}}{(mL)\Gamma(k)} \cdot \frac{1}{4} \cdot \left[ \frac{1}{\Xi} \left( \frac{1}{1+\alpha+1/2} \right) \frac{1}{1+(\alpha+1)/2} \right] \]

(7.34)

### 7.3.2 OPTIMAL SIMULTANEOUS POWER AND RATE ADAPTATION

The channel capacity in case of optimal simultaneous power and rate adaptation policy is evaluated numerically by using standard software like Maple and Mathematica similarly as discussed in Chapter 5. To obtain the optimal cut-off SNR, \( \gamma_c \), we need to solve MMGF based Equation (5.34) as given below.

\[
\frac{\hat{M}(0, \gamma_c)}{\gamma_c} - \int_0^\infty \hat{M}(s, \gamma_c) \, ds = 1
\]

(7.35)

\[
I_6 = \int_0^\infty \hat{M}(s, \gamma_c) \, ds
\]

(7.36)

\[
I_7 = \frac{\hat{M}(0, \gamma_c)}{\gamma_c} = \frac{1 - I_{opt}(\gamma_c)}{\gamma_c}
\]

(7.37)

By substituting value of \( \hat{M}(s, \gamma_c) \) from Equation (7.13) in Equation (7.36), we get:

\[
I_6 = \int_0^\infty \frac{1}{\Gamma(nM)\Gamma(k)\sin\pi\beta} \left( \sum_{p=0}^{\infty} \frac{2}{p!\Gamma(p+\beta+1)} \left( \frac{\alpha - \beta + p + 1}{2} \right) \frac{(\alpha - \beta + p + 1)}{2} \right) ds
\]

(7.38)

where

\[
I_8 = \int_0^\infty \frac{(s)^{(\alpha - \beta + p + 1)}}{2} \Gamma\left( \frac{\alpha - \beta + p + 1}{2}, \gamma_s \right) \, ds
\]

and

(7.39)
\[ I_0 = \int_0^\infty (s)^{-(\alpha+\beta+2p+1)/2} \Gamma\left(\frac{\alpha + \beta + 2p + 1}{2}, \gamma_0 s\right) ds \]  

(7.40)

By putting the value from [157 Equation (8.4.16.2)] along with [157 Equation 2.24.2.1] in Equation (7.39), \( I_8 \) can be written as:

\[ I_8 = (\gamma_0)^{-(\alpha+\beta+2p-1)/2} \frac{\Gamma\left(1 - \left(\frac{\alpha - \beta + 2p}{2}\right)\right)}{\Gamma\left(1 + 1 - \left(\frac{\alpha - \beta + 2p}{2}\right)\right)} \]  

(7.41)

Similarly,

\[ I_9 = (\gamma_0)^{-(\alpha+\beta+2p-1)/2} \frac{\Gamma\left(1 + \left(\frac{\alpha + \beta + 2p}{2}\right)\right)}{\Gamma\left(1 + 1 - \left(\frac{\alpha + \beta + 2p}{2}\right)\right)} \]  

(7.42)

By substituting value of \( I_8 \) and \( I_9 \) from Equation (7.41) and (7.42), respectively to Equation (7.38), we get:

\[ I_6 = \frac{1}{\Gamma(mM)\Gamma(k)} \mathcal{S} \left( \sum_{p=0}^{\infty} \frac{2^{\alpha+\beta+2p+1}}{\Gamma(\beta+1)} \frac{\Gamma\left(\frac{\alpha+\beta+2p}{2}\right)}{\Gamma\left(1 + 1 - \left(\frac{\alpha+\beta+2p}{2}\right)\right)} \right) \]  

(7.43)

For the evaluation of integral \( I_7 \), \( P_{out}(\gamma_s) \) evaluated is evaluated first. From Equation (5.51) along with (6.8), \( P_{out}(\gamma_s) \) can be expressed as:

\[ P_{out}(\gamma_s) = \int_0^{\gamma_s} \frac{2^{(\gamma_s)^2/2}(\Xi)^{(\alpha+1)/2}}{\Gamma(mM)\Gamma(k)} K_\beta \left[ \sqrt{\Xi} \gamma \right] d\gamma \]

\[ P_{out}(\gamma_s) = 1 - \int_{\gamma_s}^{\infty} \frac{2^{(\gamma)^2/2}(\Xi)^{(\alpha+1)/2}}{\Gamma(mM)\Gamma(k)} K_\beta \left[ \sqrt{\Xi} \gamma \right] d\gamma \]  

(7.44)
By replacing $K_{\beta}$ as [157 Equation (8.4.23.1)] and applying [157 Equation 2.24.2.3] in the Equation (7.44) can be expressed as:

$$P_{ou}(\gamma_o) = 1 - \left(\frac{\Xi/\gamma_o}{\Gamma(mM)\Gamma(k)}\right)^{\frac{(\alpha+1)/2}{\Gamma(mM)\Gamma(k)}} G^{3} \left[\begin{array}{cc} 1-(\alpha+1)/2 \\ \beta/2 \\ -\beta/2 \end{array}\right]$$  \hspace{1cm} (7.45)

From Equation (7.37), integral $I_{\gamma}$ can be expressed as:

$$I_{\gamma} = \frac{(\Xi/\gamma_o)^{\frac{(\alpha+1)/2}{\Gamma(mM)\Gamma(k)}} G^{3}}{\gamma_o} \left[\begin{array}{cc} 1-(\alpha+1)/2 \\ \beta/2 \\ -\beta/2 \end{array}\right]$$  \hspace{1cm} (7.46)

By putting result $I_6$ and $I_{\gamma}$ in Equation (7.35), we get:

$$\left(\frac{\Xi/\gamma_o}{\Gamma(mM)\Gamma(k)}\right)^{\frac{(\alpha+1)/2}{\Gamma(mM)\Gamma(k)}} G^{3} \left[\begin{array}{cc} 1-(\alpha+1)/2 \\ \beta/2 \\ -\beta/2 \end{array}\right] \frac{1}{\gamma_o} = \frac{1}{\Gamma(mM)\Gamma(k)} \sin \pi \frac{\beta}{2} \left(\Xi\right)^{\frac{\alpha+1}{2}} \left\{ \sum_{p=0}^{\infty} \frac{2}{p! \Gamma(p+1)} \left(\Xi\right)^{\frac{-\beta+2p}{2}} \left(\frac{1}{\Gamma\left(1+\frac{1-(\alpha-\beta+2p)}{2}\right)} - \frac{\Gamma\left(1+\frac{1-(\alpha+\beta+2p)}{2}\right)}{\Gamma\left(1+\frac{1-(\alpha-\beta+2p)}{2}\right)} \right) \right\} = 1$$  \hspace{1cm} (7.47)

In order to get optimal cut-off SNR, $\gamma_o$, Equation (7.47) is evaluated numerically.

### 7.3.3 CHANNEL INVERSION WITH FIXED RATE

The channel capacity for channel inversion with fixed rate ($C_{\text{CIFR}}$) scheme by using MGF based approach is used to obtain Equation (5.47), which is:

$$C_{\text{CIFR}} = \log_2 \left(1 + \frac{1}{\int_0^\infty M(s) \, ds} \right)$$  \hspace{1cm} (7.48)

For evaluation of $I_{10}$, by substituting the value $M(s)$ from Equation (6.10) to the Equation (7.48), we get:

$$I_{10} = \int_0^\infty M(s) \, ds = \int_0^\infty \left(\frac{\Xi}{\gamma_o}\right)^{\frac{\alpha+1}{2}} G^{3} \left[\begin{array}{cc} 1-(\alpha+1)/2 \\ \beta/2 \\ -\beta/2 \end{array}\right] ds$$  \hspace{1cm} (7.49)

By putting $t = 1/s$ in Equation (7.49) and by using [157 Equation 2.24.2.1], Equation (7.49), can be expressed as:
\[ I_{10} = \frac{\Xi \Gamma ((\alpha + \beta - 1)/2) \Gamma ((\alpha - \beta - 1)/2)}{\Gamma (mM) \Gamma (k)} \]

\[ C_{\text{CIFR}} = \log_2 \left( 1 + \frac{\Gamma (mM) \Gamma (k)}{\Xi \Gamma ((\alpha + \beta - 1)/2) \Gamma ((\alpha - \beta - 1)/2)} \right) \] (7.50)

So now the channel capacity for channel inversion with fixed rate scheme is:

For \( M = 1 \) and if \( m \) is an integer in the Equation (7.50), we get:

\[ C_{\text{CIFR}} = \log_2 \left( 1 + \frac{\bar{F} (m-1)(k-1)}{km} \right) \] (7.51)

The above Equation is similar with [113 Equation (29)] and [114 Equation (27)]. The approach for the derivation of this Equation has been discussed in [113] and [114] is quite complicated whereas the approach presented in chapter is very simple.

### 7.3.4 TRUNCATED CHANNEL INVERSION

The channel capacity for \( C_{\text{TTCF}} \) scheme is evaluated by using Equation (5.54) as:

\[ C_{\text{TTCF}} = \log_2 \left( 1 + \frac{1}{\int_0^\infty \frac{1}{\lambda} \left( t, \gamma_0 \right) \, dt} \right) \left\{ \bar{M} \left( 0, \gamma_0 \right) \right\} \]

From Equation (7.43) and from (7.45), \( C_{\text{TTCF}} \) can be expressed as

\[ C_{\text{TTCF}} = \log_2 \left( 1 + \frac{1}{\Gamma (mM) \Gamma (k) \sin \phi} \right) \left( \frac{\bar{F} \left[ \frac{1 - (\alpha + \beta + 2 \rho)}{2} \right]}{\Gamma \left[ \frac{1 - (\alpha + \beta + 2 \rho)}{2} \right]} \right) \]

\[ \left[ \frac{\bar{F} \left[ \frac{1 - (\alpha - \beta + 2 \rho)}{2} \right]}{\Gamma \left[ \frac{1 - (\alpha - \beta + 2 \rho)}{2} \right]} \right] \]

\[ \left( \frac{\bar{F} \left[ \frac{1 - (\alpha + \beta + 2 \rho)}{2} \right]}{\Gamma \left[ \frac{1 - (\alpha + \beta + 2 \rho)}{2} \right]} \right) \]

\[ \left( \frac{\bar{F} \left[ \frac{1 - (\alpha - \beta + 2 \rho)}{2} \right]}{\Gamma \left[ \frac{1 - (\alpha - \beta + 2 \rho)}{2} \right]} \right) \]

(7.64)

### 7.4 RESULT AND DISCUSSION

In this section, we have presented some numerical results for the channel capacity with MRC diversity over generalized-K fading channel. The proceeding performed
under infrequent light and frequent heavy shadowing. The corresponding slandered deviations ($\sigma$) of lognormal shadowing is 0.115 to 0.806, respectively. The parameter $k$ for generalized-K fading can be computed by using moment matching approach [114, 169], that lies from $k = 1.093$ to $k = 75.11$. Figure 7.1 shows the channel capacity for optimal rate adaptation ($C_{ORA}$) versus SNR ($\bar{\gamma}$) in case of heavy shadowing ($k = 1.0931, m = 2$) and light shadowing ($k = 75.11, m = 2$). As the $M$ increases from $M = 1$ to $M = 3$ channel capacity improves significantly. Figure 7.2 shows the plot of channel capacity for optimal rate adaptation ($C_{ORA}$) versus fading parameter for various values of average SNR per branch such as $(\bar{\gamma}) = 5$ dB, 10 dB and 15 dB. From the figure it is clear that as fading parameter increases, the channel capacity improves slightly and it becomes constant after certain value. Figure 7.3 depicts the channel capacity for optimal rate and power adaptation ($C_{OPRA}$) versus SNR ($\bar{\gamma}$) plot in light shadowing ($k = 75.11, m = 2$) and heavy shadowing ($k = 1.0931, m = 2$) for $M = 1, 3$. By focusing on the effect shadowing, particularly in case of heavy shadowing ($k = 1.0931$) the channel capacity degrades significantly as shown in Figure 7.3 and it improves with the increase of $M$. Figure 7.4 shows the channel capacity for channel inversion with fixed rate ($C_{CIFR}$) as a function SNR ($\bar{\gamma}$) for light shadowing ($k = 75.11, m = 2$) and heavy shadowing ($k = 1.0931, m = 2$).

For heavy shadowing, the channel capacity ($C_{CIFR}$) improves less with increase of SNR ($\bar{\gamma}$) and for light shadowing ($C_{CIFR}$) increases rapidly. Figure 7.5 shows the plot of channel capacity for channel inversion with fixed rate ($C_{CIFR}$) versus fading parameter, m, for various values of the average SNR per branch such as $(\bar{\gamma}) = 5$ dB, 10 dB and 15 dB. The channel capacity improves slightly with increase of fading parameters and it becomes constant after certain values of $m$. Figure 7.6 shows dependence of Channel capacity with truncated channel inversion ($C_{TCIFR}$) on cutoff SNR ($\gamma_0$), for various values of SNR ($\bar{\gamma}$) and shadowing, $k$. All curves shows that ($C_{TCIFR}$) is maximized for an optimal value of cutoff SNR and optimal cut-off rate increases with the increase of SNR ($\bar{\gamma}$). Figure 7.7 shows the dependency of channel capacity with truncated channel inversion ($C_{TCIFR}$) on MRC diversity receivers at various values of shadowing, as the number of diversity receivers increases the
channel capacity improves significantly. In the Figure 7.8, comparison of the channel
capacity of proposed MG F based method and PD F based method [11.3] for various
value of shadowing parameters and diversity receivers is shown.

![Graph showing channel capacity versus SNR for different values of k and M.]

**Figure 7.1** The channel capacity for optimal rate adaptation ($C_{ora}$) versus SNR for heavy shadowing ($k = 1.0931$) and light shadowing ($k = 75.11$).

![Graph showing channel capacity versus fading parameter for different values of k.]

**Figure 7.2** The channel capacity for optimal rate adaptation ($C_{ora}$) versus fading parameter.
The PDF based approach which is discussed in detail in [113] is only valid for fixed value of the shaping parameter (i.e. $m = 2$) but the MGF based proposed method discussed in this chapter is valid for any arbitrary chosen values of the shaping parameter $m$.

**Figure 7.3** The channel capacity for optimal rate and power adaptation ($C_{OPR}$) versus SNR plot for heavy shadowing ($k = 1.0931$) and light shadowing ($k = 75.11$).

**Figure 7.4** The channel capacity for channel inversion with fixed rate ($C_{CIFR}$) versus SNR for light shadowing ($k = 75.11$) and heavy shadowing ($k = 1.0931$).
Figure 7.5 The channel capacity for channel inversion with fixed rate ($C_{CIFR}$) versus fading parameter.

Figure 7.6 The channel capacity with truncated channel inversion ($C_{TCIFR}$) versus cut-off SNR ($\gamma_0$), MRC diversity ($M = 1$).
Figure 7.7 The channel capacity with truncated channel inversion ($C_{TCII/RI}$) versus MRC diversity, cutoff ($\gamma_0$) = 10 dB and average SNR ($\bar{\gamma}$) = 15 dB for various values of shadowing parameters.

Figure 7.8 Comparison of channel capacity for optimal rate adaptation ($C_{cm}$) with proposed method and PDF based method [113].
Figure 7.9 Comparison of the channel capacity for optimal rate adaptation ($C_{ora}$) with proposed method and PDF based method [114].

The Figure 7.9 shows the comparison of channel capacity of the proposed MGF based method and PDF based method as reported in [114]. The PDF based method in approximates ($C_{ora}$) at $a = 1$ and also approach is valid only for non-integer value of $k$ and $m$.

7.5 CONCLUSION
In this Chapter, we have obtained the mathematical expression for the marginal MGF for generalized-K fading channel with M-branch MRC diversity. Also, the obtained marginal MGF function is used to evaluate channel capacity under different adaptation policies. We derived the expression of channel capacity with optimal rate adaptation ($C_{ora}$) which valid for arbitrary value of the shaping parameters $k$ and $m$. We also derived expression for capacity for channel inversion with fixed rate ($C_{clf}$). The $C_{ora}$ and $C_{clf}$ is very easily calculated using MGF based approach and for $M = 1$, $C_{clf}$ is calculated by using Equation (25) is similar with [113 eq. 29] and [114 eq. 27]. In this work, we also derived marginal MGF based channel capacity for truncated channel inversion($C_{tcfr}$) and for optimal rate and power adaptation ($C_{ora}$) schemes for Generalized-K fading channel, which is novel and can be applied to other fading channels also.