INTRODUCTION

Plates are the main parts in various structures as bridges, hydraulic, water tanks…. etc; in these structures the plate is an important component in carrying directly the applied loads. The advancements made in material science and manufacturing capabilities has lead to rapid increase in the application of composite materials in modern engineering industries. Composite materials, in the form of single or multiple layer laminated plates, are increasingly being used in various structural and machine elements needed in modern engineering systems like aircrafts, space vehicles, satellites, ships, submarines, turbines, bridges, automobiles etc. The study of the behavior of laminated composite plates in terms of bending, vibration and buckling is necessary to meet the design challenges.

Enormous amount of literature exists dealing various aspects of elastic plates like shape (rectangular, polygonal, circular or sector), material (isotropic, anisotropic), thickness (thin, thick plates), deflections (small, large), loadings (static or dynamic, mechanical, thermal, hydrostatic etc.), boundary conditions (Clamped, Simply supported or Free) etc.

The word “Vibration” is vigorously used in engineering applications, machines, nuclear field etc. Any motion that repeats itself after an interval of time or can be as simply the cyclic or oscillating motion of a machine or machine component from its position of rest is called vibration in scientific language. Vibration is the most important
modes of failure in plates with sprung mass and it plays a crucial role in engineering. Vibration analysis of plate shaped structures has been of interest to structural, aeronautical and mechanical designers for several decades. Vibration is an indicator of Machinery conditions. 

How many times have you touched a machine to see if it was "running right"? With experience, you have developed a "feel" for what is normal and what is abnormal in terms of machinery vibration. Even the most inexperienced driver knows that something is wrong when the steering wheel vibrates or the engine shakes. In other words, it's natural to associate the condition of a machine with its level of vibration. Of course, it's natural for machines to vibrate. Even machines in the best of operating condition will have some vibration because of small, minor defects. Therefore, each machine will have a level of vibration that may be regarded as normal or inherent. However, when machinery vibration increases or becomes excessive, some mechanical trouble is usually the reason. Vibration does not increase or become excessive for no reason at all. Something causes it - unbalance, misalignment, worn gears or bearings, looseness, etc.

Since not everyone has the long-term experience needed to judge a machine's condition based on how it feels, various instruments have been developed over the years to measure the actual level or amount of vibration. In addition, human perception of touch and feel is somewhat limited, and there are many common problems such as the early stages of bearing and gear failure that are generally out of the range of human perception. Thus, modern instrumentation for
measuring vibration on rotating and reciprocating machinery not only minimizes the need for extensive experience, but makes it possible to detect developing problems that are outside the range of human senses of touch and hearing. Further, human perception differs from individual to individual. What one person may consider as bad, another may consider as normal. Attempting to trend changes in machinery condition using human perception is nearly impossible, since it is nearly impossible to put a documented number on "how it feels". To overcome this problem, instrumentation has been developed to actually measure a machine's vibration level and assign it a numerical value. This tool not only overcomes the limitations of inexperience, but it addresses the limitations of human perception as well. Dynamic behavior of these structures strongly depends on boundary conditions, geometrical shapes, material properties etc. Closed-form solutions are possible only for a limited set of simple boundary conditions and geometries. For analysis of plate shaped structures, several numerical methods such as finite element method, finite difference method, boundary method etc. are usually applied.

The subject of vibration of plates is an extremely important area owing to wide variety of its applications. The research in the field of vibration is quite mesmerizing and continuously acquiring a great attention of scientists and design engineers because of its unbounded effect on human life. In the engineering we can not move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. Study of effect of
vibration can’t be restricted only in the field of science but, our day to
day life is also affected by it. Whether it be a constructive aspect e.g.
aircraft, space shuttle, satellite or design engineering to the destructive
aspect, e.g. tsunami, earthquake etc., none of these are remained
untouched with the effect of vibrations.
Vibration of plates of various shapes, homogeneous or non-
homogeneous, orthotropic or isotropic, with or without variation in
thickness, have been studied by various authors, with or without
considering the effect of temperature. Since beams, plates and shells
form integral parts of structures, a prior knowledge of a first few
natural frequencies and the associated modes is quite essential for an
engineer before finalizing the design of a given structure. In
particular, plates with different shapes and boundary conditions at the
edges are often encountered in several engineering applications such
as aeronautical engineering, telephone industry, machine design,
nuclear reactor technology, naval structures and earthquake resistant
structures etc. The study of vibration of plates is an old area in which
a lot of work has been done. In the earlier periods, results were
available for some simple cases only where the analytical solution
could be found. To get the reasonable accurate results even in these
cases were impossible due to lack of good computational facilities.
With the advent of fast computers, there was a tremendous amount of
rise in the research work using approximate and numerical methods.
Most of the structural components are generally subjected to dynamic
loadings in their working life. Very often these components may have
to perform in severe dynamic environment where in the maximum damage results from the resonant vibrations. Susceptibility to fracture of materials due to vibration is determined from stress and frequency. Maximum amplitude of the vibration must be in the limited for the safety of the structure. Hence vibration analysis has become very important in designing a structure to know in advance its response and to take necessary steps to control the structural vibrations and its amplitudes. Actually, machines without vibration can’t be imagining. But vibration directly affects the life and work power of the machine. Since vibrations affect the efficiency and strength of machine as well as durability of the system, it is necessary for the engineers to develop more and more accurate design of the structure to give better performance and efficiency. Since, vibrations affect the efficiency and strength of machine as well as durability of the system. Knowledge about the first few modes of vibration is essential to a mechanical engineer, before finalizing a design. In our work, first two modes of vibrations are calculated and try to minimize the first two mode of the vibration.

**Visco- Elastic Material**

Visco-elasticity, as its name implies, is a generalization of elasticity and viscosity. These are those materials, when the applied stress is removed the materials have the ability to reach its original position slowly over a period of time. Some materials are viscous rather than elastic. When a force is applied to a viscous material the material does not stretch, it flows like a liquid. Most materials used in electronics
don't flow as easily as water but at high temperatures can be a bit like extremely ‘thick’ (or viscous to give it a scientific name) syrup. The important thing about a viscous material is that when the force is removed it does not return to its original shape because the force has been ‘used up’ in the fluid flow.

The property of these types of materials is lies between elastic & plastic.

- **Elasticity** is the physical property of a material that returns to its original shape after the stress (e.g. external forces) is removed. For example, a stress on a spring, when we remove the stress, it comes to its original position.

- **Plasticity** is the physical property of a material that doesn’t return to its original position when the applied force is removed. For example; a piece of metal being bent to give it’s a new shape. It shows the property of plasticity i.e. when we removed force it doesn’t return to its original position.

Recent development in technology, the study of vibrations of visco-elastic plates (materials) has acquired great importance. But the consideration of visco-elastic behavior of the plate material together with variation in thickness not only ensure the reduction in the rate and size but also meets the desirability of high strength. The visco-elastic behaviors of some materials invigorated scientists for modern designs and analysis techniques and their application to many practical problems. As technology develops new discoveries have
intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications. Visco-elasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. Visco-elastic materials have elements of both of these properties and, as such, exhibit time dependent strain. Whereas elasticity is usually the result of bond stretching along planes in an ordered solid, viscosity is the result of the diffusion of atoms or molecules inside an amorphous material. Visco-elastic material i.e. Duralium It is the composition of Aluminum: 95%, Copper: 4%, Magnesium: 0.5%, Manganese: 0.5%. Duralium alloy are relatively soft, duelite and work ship in the normal state, it may be rolled, forged, extruded or drawn into variety of shapes and products. Also, its light weight and consequent high strength per unit weight as compared to steel suit it for aircraft construction. Because, it loses strength in welding at a special sheet called ALCLAD. It is used for aircraft construction.

Plates of Variable Thickness
Plates of variable thickness have been used in aircraft structures for many years and are frequently used in order to economize on the plate materials or to lighten the plates, especially when used in wings for high-speed, high-performance aircrafts. By carefully designing the thickness distribution, a substantial increase in stiffness, buckling and vibration capacities of the plate may be obtained over its uniform thickness counterpart. In the aeronautical field, analysis of non-homogeneous plates with thermal gradient and variable thickness has been of great interest due to their utility in aircraft wings. The research in the field of vibration is quite mesmerizing and continuously acquiring a great attention of scientists and design engineers because of its unbounded effect on human life. In the engineering we cannot move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications.

The freedom to taper the plate thickness has allowed the designer to achieve greater structural efficiency. However, if a further increase in efficiency is to be achieved, then such plates must be allowed to
operate in the large-deflection. In order to increase the durability and reliability of a structure, the plates of variable thickness are used extensively in engineering applications. Plates of variable thickness is commonly used in modern technology such as in aerospace, nuclear plants, power plants, marine engineering, mechanical, civil, chemicals, ship hulls, building etc. It may also used for construction of wings, tails and fins of the airplane, rockets and missiles.

**Effect of Temperature**

With the advancement of technology, the requirement to know the effect of temperature on visco-elastic plates of variable thickness has become vital due to their applications in various engineering branches such as nuclear, power plants, aeronautical, chemical etc. Further in mechanical system where certain parts of machine have to operate under elevated temperature, its effect is far from negligible and obviously cause non-homogeneity in the plate material i.e. elastic constants (young modulus etc.) of the materials becomes functions of space variables. The plate type structural components in aircraft and rockets have to operate under elevated temperatures that cause non-homogeneity in the plate material i.e. elastic constants of the materials becomes functions of space variables. In an up-to-date survey of literature, authors have come across various models to account for non-homogeneity of plate materials proposed by researchers dealing with vibration but none of them consider non-homogeneity with thermal effect on visco-elastic plates.
Actually, most of the machines temperature produced due to vibration and machine efficiency decreases. So, it is very necessary to study the temperature variations on plates. Due to variation in temperature, non homogeneity occurs in material, which cannot be neglected.

In an up-date survey of literature, authors have come across various models to account for non-homogeneity of plate materials proposed by researchers dealing with vibration but none of them consider non-homogeneity with thermal effect on visco-elastic plates. It also indicates that sufficient work on one dimensional temperature variation has been done but negligible work has been done in the field of two dimensional temperature variation. The proposed studies aim to carry out studies in the area of two-dimensional variation in temperature.

When the elastic constant (young’s modulus, poison’s ratio, modulus of rigidity) of a material does not vary from point to point in the material, it is said to be elastically Homogeneous.

In a Kelvin model; the following assumption are made

- The deflection is considered from the middle plane of the plate.
- The upward force which is acting on the plate is directly proportional to the deflection.

A study of the literature on vibration problems shows that the visco-elastic plates with thickness variation in two directions has received rather less attention than that of in one direction.
Recently, A.K Gupta and Lalit Kumar [1] studied Thermal effects on the vibration of non-homogeneous visco-elastic rectangular plate of linearly varying thickness.


The effect of thermal gradient on the frequencies of an orthotropic plate of linearly varying thickness has been discussed by Tomar and A.K Gupta [5].

D, Young [6] discussed the study on vibration of rectangular plates by the Ritz method.


Tomar and Gupta [8] discussed the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions.

An analysis is presented on the vibration of clamped visco-elastic rectangular plate with parabolic thickness variations by Gupta and Anupam Khanna [9].


Gupta and Kaur [12] studied the effect of thermal gradient on free vibration of visco-elastic rectangular plates with linearly thickness variation in both directions.

Vibration behavior and simplified design of thick rectangular plates with variable thickness considered by Sasajima, Kakudate and Narita [13].


An interesting analysis of the free vibration of rectangular plate is given by A.W. Leissa [15].

Tomar and Tewari [16] presented an analysis on effect of thermal gradient on frequencies of a circular plate of linearly varying thickness.

Gupta and Khanna [17-18] discussed the free vibration of clamped visco-elastic rectangular plate having bi-direction {exponentially in 17 & linearly in 18} thickness variations.

Larrondo, Avalos, Laura and Rossi [19] studied vibrations of simply supported rectangular plates with varying thickness and same aspect ratio cutouts.

Free vibrations of rectangular plates of parabolically varying thickness have been investigated by Jain and Soni [20].

Bhatnagar and Gupta [21] discussed thermal effect on vibration of visco-elastic elliptic plate of variable thickness.


Laura, Grossi, and Carneiro [22] studied transverse vibrations of rectangular plates with thickness varying in two directions and with edges elastically restrained against rotation.

T. Sakiyama and M. Huang [23] studied the free vibration analysis of right triangular plate with variable thickness.


Rossi, R.E. [26] studied the 'Transverse vibrations of thin, orthotropic rectangular plates with rectangular cutouts with fixed boundaries.

Cox, H.L. and Boxer, J. [27] studied the 'Vibration of rectangular plates point supported at the corners.

The fundamental frequency of transverse vibration of a clamped rectangular plate of cylindrical orthotropy has been investigated by Bombill and Laura [28].

Filipich, Laura and Santos [29] considered the rectangular plates with two opposite edges simply supported and the other two with very general boundary conditions and studied the vibrations of such rectangular plates of variable thickness.


Nagaya [32] discussed the vibrations and dynamic response of visco-elastic plates on non-periodic elastic supports.

Fung, Y. C. [33], discussed the ‘Bending of thin elastic plates of variable thickness.


Turvey, G. J. and Wittrick, W. H. [40], studied ’The large deflection and post buckling of some laminated plates’, The Aeronautical Quarterly.

The aim of present investigation is to study the two dimensional thermal effect on the vibration of visco-elastic square plate whose thickness varies linearly in both directions. A frequency equation has
been obtained by Rayleigh-Ritz technique with two terms of
deflection function for visco-elastic square plate.

**H.-N. Chu and G. Herrmann** [41] studied ‘Influence of large
amplitudes on free flexural vibrations of rectangular elastic plates’.

**P. C. Dumir and A. Bhaskar** [42], discussed the ‘Nonlinear forced
vibration of orthotropic thin rectangular plates’.

**M. Ganapathi, T. K. Varadan, and B. S. Sharma** [43], discussed
the ‘Nonlinear flexural vibrations of laminated orthotropic plates’.

**K. Kanaka Raju and E. Hinton** [44], discussed the ‘Nonlinear
vibrations of thick plates using Mindlin plate elements’.

**L. V. Kurpa, V. L. Rvachev, and E. Ventsel** [45], studied ‘The R-
function method for the free vibration analysis of thin orthotropic
plates of arbitrary shape’.

**T. Manoj, M. Ayyappan, K. S. Krishnan, and B. Nageswara Rao**
[46], ’Nonlinear vibration analysis of thin laminated rectangular plates
on elastic foundations’.

**C. Mei and K. Decha-Umphai** [47], studied, ’A finite element
method for nonlinear forced vibrations of rectangular plates’.

**Laura P.A.A., Grossi R.O., Carneiro G.I** [48] studied the
‘Transverse vibrations of rectangular plates with thickness varying in
two directions and with edges elastically restrained against rotation’.

**Leissa A.W.** [49] studied the, ’Recent studies in plate vibration 1981’.

Since the area of present research is very vast, therefore there is
needed to make some assumption which is as follows:
It is assumed that the plate is clamped on all the four edges.
Temperature varies linearly in both the directions.
It is also assumed that deflection is small and linear.
Visco-elastic properties are of ‘Kelvin’ type.
The Poisson ratio is assumed to remain constant.
Young Modulus becomes the function of space variable due to thermal effect.

For various values of thermal gradient and taper constants; frequency of vibration for the first two modes of vibration are evaluated with the help of MATLAB. All the numerical calculations will be carried out using the material constants of alloy 'Duralium'. All results are shown in Graphs and Tables.