2.1 Digital Images

An image defined in the real world is a function of two real variables. For instance, \( a(x,y) \) with \( a \) as the amplitude of the image at the real coordinate position \((x, y)\). Any perceived image is characterized by two components namely luminance (the quantum of incident light) and reflectance (the quantum of the reflected light). These two functions combine as a product to form \( a(x, y) \) as [Gonzalez and Woods, 2004]:

\[
a(x,y) = i(x,y)r(x,y),
\]

(2.1)

where \( 0 < i(x, y) < \infty \),

and \( 0 < r(x,y) < 1 \)

That reflectance \( r(x,y) \) is bounded by 0 (total absorption) and 1 (total reflectance).

Irrespective of the ways by which images are acquired, they are represented as continuous voltage waveforms whose amplitude and spatial behaviour are basically related to the physical phenomenon being sensed. Such sensed data is digitized to generate a digital image involving two processes viz., sampling and quantization. Digitization is a process of converting an image which is continuous in space and in its values, into a discrete numerical form.
2.1.1 Sampling

Image sampling is a process of selection of discrete locations in a continuous two dimensional space whose intensity is only considered during the process of digitization of image. For an image to be processed through computers, an image function $a(x,y)$ must be digitized both spatially and in amplitude. Digitization of spatial coordinates $(x,y)$ is called image sampling.

2.1.2 Quantization

Quantization is a process in which a continuous value is converted into discrete value. The values obtained by sampling usually comprise of infinite set of real numbers ranging from minimum to maximum, depending on the sensor calibration. The sampled values are represented by the finite set of integer numbers. The mapping of real numbers to a finite range of integers is called quantization. In other words, quantization is the process of amplitude digitization. The result of sampling and quantization of an image results in a matrix of real / integer numbers of two dimensional sizes of $M$ rows and $N$ columns as:

$$f^*(x,y) = \begin{bmatrix}
  f(0,0) & f(0,1) & \cdots & f(0,N-1) \\
  f(1,0) & f(1,1) & \cdots & f(1,N-1) \\
  \vdots & \vdots & \ddots & \vdots \\
  f(M,0) & f(M,1) & \cdots & f(M-1,N-1)
\end{bmatrix}$$ (2.2)

Thus a digital image $f(x, y)$ described in a two dimensional discrete space is derived from an analog image $a(x,y)$ in a two dimensional continuous space. The intersection of a row and a column is called a picture element, generally referred to as pixel. A pixel is thus the smallest unit of a digital image which is organized in rows called raster. The two dimensional representation of image is given in Figure 2.1. The traditional matrix notation often used to denote digital image and its elements is:
Hence, \( a_{ij} = f(x = i, y = j) = f(i, j) \).

The standard values used for rows, columns and gray levels to represent an image are given in Table 2.1 [Young et al, 1998]. Normally, \( M = N = 2^K \) where, \( K = 8, 9, 10 \). The number of distinct gray levels is usually a power of 2, i.e., \( L = 2^B \), where, \( B \) is the number of bits in the binary representation of the brightness levels. For a gray scale image \( B > 1 \), while for a binary image \( B = 1 \). For instance, if \( B = 8 \) then black is zero and white is 256 and if \( B = 1 \), black is 0 and white is 1. The intensity of monochrome image \( f \) at coordinates \((x, y)\) is known as gray level \((i)\) image of that point.

\[
L_{\text{min}} \leq i \leq L_{\text{max}}
\]
where $L_{\text{min}}$ is the minimum gray level and $L_{\text{max}}$ is the maximum gray level. The interval $[L_{\text{min}}, L_{\text{max}}]$ is called the gray scale of the image. An image is shifted to the interval $[0, L]$ where $I = 0$ is black and $I = L$ is white.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows</td>
<td>M</td>
<td>256, 512, 525, 625, 1024, 1035</td>
</tr>
<tr>
<td>Columns</td>
<td>N</td>
<td>256, 512, 768, 1024, 1320</td>
</tr>
<tr>
<td>Gray Levels</td>
<td>L</td>
<td>2, 64, 256, 1024, 4096, 16384</td>
</tr>
</tbody>
</table>

### 2.2 Image Representation in Spatial Domain

Any digital image may be represented in two major domains namely spatial domain and frequency domain. Any digital image may be represented in terms of spatial domain as the aggregate of pixels of digitized image whereas the spatial domain methods are techniques / procedures that manipulate the pixels of an image directly. Conventionally the spatial domain techniques are denoted as [Gonzalez and Woods, 2004]:

$$g(x, y) = T[f(x, y)]$$  \hspace{1cm} (2.5)

where $f(x, y)$ is the input image, $g(x, y)$ is the processed / restored image and $T$ is the operator applied on $F$ which is defined with respect to one or more neighbourhood of $(x, y)$. The principal approach in defining / formulating the neighbourhood is called mask, filter, kernel, template or window that is centered at $(x, y)$ can be rectangle, square, hexagonal, circle or any other desired geometrical shape. For practical convenience a window that represents sub image is chosen to be a square or rectangle.
When the neighbourhood size is restricted to 1 x 1 then $g$ depends only on the pixels of $f$ at $(x,y)$ and this transformation $T$ is called gray level intensity or mapping transformation function which is expressed as:

$$s = T(r) \tag{2.6}$$

where $r$ and $s$ are the variables that represent the gray levels of $f$ and $g$ respectively at a given point $(x,y)$. An example of this kind of transformation is contrast stretching that produces an image of higher contrast than the original image by darkening the gray levels below $m$ and by brightening the gray levels above $m$ in the original image. Such a kind of mapping is known as threshold. It is the process of making all pixels above a certain threshold level as white and the rest as black. The key parameter in thresholding is obviously the choice of the threshold. The process in which, the new value of pixel is determined by the original value of that single pixel alone, is referred to as point processing.

Alternatively, selection of large sized neighborhoods permits us to have greater flexibility to determine the value of $g$ at $(x, y)$. The approach involves the use of function of the values of $f$ in a predefined neighbourhood of $(x,y)$ so as to determine the value of $g$ at $(x,y)$. This is carried out by the use of filter or mask in which the values of mask coefficients determine the nature of the process. Image restoration techniques involve this technique of filtering or masking.

### 2.3 Noise Models

Based on the properties of noise introduced at various stages of acquisition, noise may be categorized [Gonzalez and Woods, 2004] as:
2.3.1 Additive Noise

For an image of \( f(x,y) \) and for a noise function of \( r(x,y) \) the additive noise, the degraded image \( f'(x,y) \) is defined as:

\[
f'(x,y) = f(x,y) + r(x,y)
\]

(2.7)

where input image / and noise \( r \) are the independent variables. This type of noise is signal independent and hence independent of the pixel values of the original image. Typically, this additive noise is symmetric with zero. Thermal noise, photographic noise and quantization noise are typical examples of additive noise model. This type of noise does not alter the average brightness of the image as shown in figure 2.2(b).

2.3.2 Multiplicative Noise

Multiplicative noise is otherwise known as speckle noise is a signal dependent noise whose magnitude is related to the value of the original pixels. Television raster degradation that depends on TV lines is a classic example of this type of noise. The noise is maximal in the area of line and is minimal between the lines. Another example of this kind of noise is degradation of film material caused by the finite size of silver grains used in photosensitive emulsion. This type of noise is illustrated in figure 2.2.(c). The multiplicative noise is represented as:

\[
f'(x,y) = f(x,y) + v(x,y)f(x,y) = f(x,y) + [1 + rt(x,y)]
\]

(2.8)
Mostly the magnitude of the multiplicative noise depends on the magnitude of the signal. It is appropriate to state that additive noise can be converted into multiplicative noise by exponentiation and the vice-versa conversion is done by logarithmization.

### 2.3.3 Gaussian Noise

Gaussian noise is evenly distributed over the input signal. This means that each pixel in the noisy image is the sum of the true pixel value and a random gaussian distributed noise value. As the name mentions this type of noise has a gaussian distribution having a bell shaped probability distribution function given [Scott, 1998, Pierre et al, 2004] by

\[
F(g) = \frac{1}{\sqrt{2\pi \sigma}} e^{-(g-\mu)^2/2\sigma^2}
\]  

(2.9)

where \(g\) represents the gray level, \(\mu\) represents the mean of the function and \(\sigma\) the standard deviation of the noise. Gaussian noise arises in an image due to the factors such as electronic circuit noise and sensor noise, due to poor illumination and/or high temperature. An image corrupted by gaussian noise of mean = 0 and variance = 0.1 is illustrated in figure 2.2(d).

### 2.3.4 Impulsive Noise

The impulse noise can be due to a noisy transmission channel or due to imperfections of the sensors with which the images are obtained [Ding and Venetsanapoulos, 1987]. This noise corrupts the image with individual noisy pixels whose brightness differs significantly from that of the neighbourhood. It has the property of either leaving the pixels uncorrupted with the probability of \(1-p\) or replacing it altogether with probability \(p\). This can be expressed as [Gonzalez and Woods, 2004]:

\[
f'(x, y) = \eta(x, y) \text{ with probability } p
\]

\[= f(x, y) \text{ with probability } 1-p \]  

(2.10)
Salt and pepper noise is the most popular impulsive noise. The corrupted pixels are set alternatively to the minimum or the maximum gray value giving the image a salt and pepper like appearance. This type of noise is also known as shot and spike noise. The uncorrupted pixels remain as such. In a gray scale image, if $a$ and $b$ are the minimum and the maximum density of the gray level, then the probability of occurrence of $a$ and $b$ are given as [Bertozzi and Greer, 2004]:

$$
f'(x, y) = \begin{cases} 
\eta(x, y) = a & \text{with probability } p_a \\
\eta(x, y) = b & \text{with probability } p_b \\
f(x, y) & \text{with probability } 1 - p_a - p_b 
\end{cases} \quad (2.11)
$$

Noise impulses can either be positive or negative. Impulse noise is generally digitized as extreme values (black/white), in an image because the impulse corruption is comparatively larger than the strength of the image signal. Thus the usual assumption is that ‘a’ and ‘b’ are the saturated values i.e., they are equal to the minimum and the maximum allowed values in the digitized image. Consequently, negative impulses appear as black dots (pepper noise) in an image whereas the positive impulses appear as white dots (salt noise). For an 8-bit image, this means that $a = 0$ (black) and $b = 255$ (white). When either $p_a$ or $p_b$ is zero, the impulse noise is referred to as unipolar. If neither probability is 0 and if both the probabilities are approximately equal this bipolar impulse noise is called salt and pepper noise. This type of impulse noise is found to be randomly distributed over an image. The salt and pepper appearance in an image is only one that is usually indicative of type of noise causing the degradation. Impulse noise is found in situations where quick transients such as faulty switching take place during imaging [Bertozzi and Greer, 2004]. An impulse noise corrupted image is given in figure 2.2.(e).
Figure 2.2: Types of noises: (a) Original image (b) Additive noise (c) Multiplicative noise (d) Gaussian noise (e) Impulse noise
2.4 Noise Types

Noise is categorized into different types [Young et al, 1998] depending upon the sources from / processes by which they emanate. The popular types of noise are:

i. quantization noise
ii. photon noise
iii. thermal noise
iv. on-chip electronic noise
v. amplifier noise

2.4.1 Quantization Noise

Quantization noise is due to the quantization of pixel values, during the process of analog-to-digital conversion and is inherent in the amplitude quantization process. This noise occurs when insufficient quantization levels are used. It is additive and independent of signal when the number of gray levels $L \geq 16$. This is equivalent to $B \geq 4$, where $L$ is the gray level and $B$ is the number of bits.

2.4.2 Photon Noise

The fundamental statistical nature of photon production results in photon noise. The photon production is governed by the laws of quantum physics. The probability distribution for $\rho$ photons in an observation window of length $T$ seconds is known as Poisson, which is given by the expression

$$P(\rho | \rho T) = \frac{(\rho T)^\rho e^{-\rho T}}{\rho!}$$  \hspace{1cm} (2.12)

where $\rho$ is the intensity parameter measured in photons/second. This type of noise is not altogether independent of the signal. It is neither gaussian nor additive.
2.4.3 Thermal Noise

Stochastic source of electron in a CCD well is thermal energy. Electrons that are freed from the CCD material through thermal vibration and trapped in the CCD well can not be distinguished from true photoelectrons. Often it is possible to reduce the number thermal electrons which are the cause for thermal noise or dark current by effectively cooling the CCD chip. As the integration time $T$ increases, the number of thermal electrons increases. The probability distribution of thermal electron is also a Poisson process, where the rate parameter is an increasing function of temperature. Dark current can also be suppressed by estimating the average dark current for the given integration time and then subtracting this value from the CCD pixel values before the A/D converter.

2.4.4 On-Chip Electronic Noise

This type of noise emerges in the process of reading the signal from the sensor in the case of Field Effect Transistor of a CCD chip. The general form of the power spectral density of readout noise is:

$$S_n(\omega) \propto \begin{cases} \omega^{-\beta} & \omega < \omega_{\text{min}} \\ k & \omega_{\text{min}} < \omega < \omega_{\text{max}} \\ \omega^{-\alpha} & \omega > \omega_{\text{max}} \end{cases}$$  \hspace{1cm} (2.13)$$

where $\alpha$ and $\beta$ are constants and $\omega$ is the frequency at which the signal is transferred from the CCD. Readout noise can be reduced to manageable level by appropriate readout and proper electronics.

2.4.5 Amplifier Noise

The standard model of this type of noise is additive, gaussian and independent of signal. This noise is generally negligible in well-designed
modern electronics. However, in colour camera, where more amplification is used, this type of noise assumes significant [Young et al, 1998].

2.5 Neighbourhood Operations of Images

Analysis of spatial relations of the gray value in a small neighbourhood provides the first clue for the recognition of the objects in images. If the gray value does not change in the small neighbourhood, the neighbourhood lies within an object. However, the gray value changes significantly, if an edge of an object crosses the neighbourhood. In this way areas of constant gray values and edges are recognized. A new class of operations is necessary, that combines the pixel of a small neighbourhood, in an appropriate manner and yields a result that forms a new image. Operations of this kind are referred to as neighbourhood operations which form the central tool for low-level processing.

The result of any neighbourhood operation is still an image with its contents being changed. A properly designed neighbourhood operation to detect edges should show bright values in pixels at an edge while all pixels independent of the gray value - should show low values. Thus by the application of neighbourhood operator the information is generally lost. The original gray values can no longer be inferred. Hence, neighbourhood operations are called filters as they extract certain features of interest from an image.

The neighbourhood operations enable us to perform various image processing tasks such as:

- Detection of local structures such as edges, corners, lines and areas of constant gray value.
- Texture analysis
- Motion determination
- Reconstruction of images such as tomography
- Restoration of images degraded by defocusing, motion blur or similar errors during image acquisition
- Correction of disturbances caused by errors in image acquisition, transmission and decoding

The neighbourhood operator $T$ takes values of neighbourhood around a point, performs some operations with them and writes the result back on the pixel. This operation is repeated for all points of the signal.

A continuous neighbourhood operator maps a multidimensional continuous signal $f(x)$ on to itself as [Jahne, 2003]:

$$g(x) = N\{f(x') \forall (x - x') \in W\} \quad (2.14)$$

where $W$, a compact area is called mask / window / region of neighbourhood / structure element of the neighbourhood operation. To compute $g(x)$, the size and shape of $W$ determines the neighbourhood operation by specifying the input values of $f$ in the area $W$ that is shifted with its origin to the point $x$.

A discrete neighbourhood operator maps a $M \times N$ matrix onto itself by the operation

$$G_{m,n} = N(F_{m',n'} \forall (m',n')^T \in W) \quad (2.15)$$

where $W$ is the window having discrete set of points [Jahne, 2003].

The size of the neighbourhood is the first characteristics of neighbourhood operation. The position of the pixel is specified relative to the window that receives the result of the operation. The window may be rectangular or any other form. But, a square window of size $n \times n$ where $n = 3, 5, ..., $ is the obvious choice to place the result of the operation at the central position. Even sized masks are not suitable for neighbourhood operation because no pixel lies at the centre of such mask [Jahne, 2003].
2.6 Fixed-Valued Impulsive Noise

Impulsive noise is categorized into fixed valued impulse noise and random valued impulse noise. The noise density of fixed valued impulse noise corresponds to 0 or 255 with equal probability. Salt and pepper noise is the best example for this type of noise. Random valued impulse noise is uniformly distributed in the range 0 and 255. This research work is aimed at the restoration of images degraded with fixed valued impulse noise. For this purpose, test images are superimposed with salt and pepper noise to study the performance of the proposed noise filters.

2.7 Simulation of Fixed Valued Impulsive Noise

In order to generate fixed valued impulsive noise, it is necessary to choose the probability of noise density. Accordingly, a percentage of pixels equal to the noise density is randomly chosen from the total number of pixels of the given image. From among such selected pixels, half of them are assigned the value 0 and 255.

MATLAB has an inbuilt facility namely \textit{imnoise} using which fixed valued impulsive noise is simulated whereas there is no such built-in function to generate random valued impulsive noise. We have made use of MATLAB tool for all our experiments reported in this thesis.

2.8 Image Restoration

One of the major application areas of image processing is to improve the quality of the captured images. Due to imperfections in imaging and capturing processes, the recorded image usually represents a degraded version of the original. Such images need to be restored for subsequent computer processing viz., segmentation, feature extraction, imager
representation, description, reconstruction and/or human viewing. There exists a wide range of different degradations that are to be taken into account like noise, geometric degradation, blur and illumination and colour imperfections. Degradation can also result due to the defect of optical lenses, non-linearity of electro-optical sensors, graininess of the film material, relative motion of object and the camera, wrong focus, atmospheric turbulence and scanning. The field of image restoration is concerned with reconstruction or estimation of the uncorrupted image from a blurred and/or a noisy image. In essence, image restoration tries to perform an operation on the image i.e., the inverse of imperfection in the image formation system. The use of image restoration methods, the characteristics of the degrading system and the noise are assumed to be known a priori. However in practice, it is difficult to obtain this information from the image formation process. The combination of image restoration and blur identification is often called as blind image deconvolution [Kundur and Hatzinakos, 1996]. Image restoration algorithms distinguish themselves from image enhancement methods, in that they are based on the models for the degrading process. A model of image degradation and restoration process is depicted in figure 2.3.

![Schematic Representation of Image Restoration](image)

**Figure 2.3 Schematic Representation of Image Restoration**

On an input image of \( f(x, y) \) the degradation function \( H \) and the additive noise term \( \eta(x, y) \) operates and produces the degraded image \( g(x, y) \). The objective of the restoration process is to generate \( f(x, y) \) which is the estimate of the original image. If the degradation function \( H \) is taken to
be linear and position-invariant then the degraded image in spatial domain is given as:

\[ g(x, y) = H(x, y) * f(x, y) + \eta(x, y) \]  \hspace{1cm} (2.16)

Studies on image restoration confirm the fact that spatial processing is an efficient approach when the source of degradation is additive noise. On the other hand if the degradation is due to blurring, then the frequency domain based techniques are more appropriate than using the spatial domain techniques. The restoration techniques on spatial domain may either be linear or non-linear. Both linear filter and non-linear filters are the most popular spatial domain restoration techniques and are more suitable for restoration of degraded images corrupted by impulsive noise.

2.9 Filters for Noise Removal

The term filter is commonly used to refer to any device or system that takes mixture of elements from its input and process them according to some specific rules to generate a corresponding set of elements as output. Noise filters generally attempt to restore the degraded image by neighbourhood operation. When the presence of noise is the only cause for degradation of image, then it may be represented in spatial domain as:

\[ g(x, y) = f(x, y) + \eta(x, y) \]  \hspace{1cm} (2.17)

where \( \eta(x, y) \) is an additive noise that corrupts the input image \( f(x, y) \) resulting in a degraded image \( g(x, y) \).

Noise elimination is a main concern in computer vision and image processing. In applications where operations are based on image derivatives, any noise in an image may result in serious errors. Many of the state-of-the-art impulse noise filters by and large employ median filter at some level of noise detection / suppression. This chapter concerns with the principles of
median based filters, which are proved to be triumphant in fixed valued impulse noise suppression. According to the recent studies, impulse noise intensive reduction demands for refined techniques which handle additional information about the image. Such additional information allows the use of statistics and threshold to estimate replacement values for the corrupted pixels and hence makes the filters adaptive with respect to the image regions [Windygo, 2001],

Linear and non-linear filters are the two major classes of the filters which are widely applied to restore the quality of degraded images. Linear filters are frequently used for removing additive and gaussian noise whereas the non-linear filters are effective in removing impulsive noise. Linear filters are found to blur the edges and degrade the quality of an image that may be more objectionable than the noise. Non-linear filters are known for their ability to preserve the edges better than that of linear filters [Jain, 2001; Jahne, 2003], Non-linear filters basically control the direction and/or degree of averaging with the local contents of neighbourhood pixels can recognize the objects in an image. Rank-value filter is a class of non-linear filter that sorts the gray values and selects a gray value based on the underlying principle of filters and replaces the pixels in the process [Justusson, 1981],

Impulsive noise removal may well be achieved when the corrupted image is represented in spatial domain. Normally, a corrupted image of size 256x256 or 512x512 in spatial domain representation is given as the input for the denoising filter. This filter, based on its principle, divides the image into small sub images, by using a sliding or a non-sliding window of odd number sized window \( W \) as 3x3, 5x5, 7x7 and so on. Then, each pixel that lies at the centre of the window, called inprocess pixel which is assumed to be corruptive is treated by the filter. The filters use the neighbouring pixels bounded by the window \( W \) and estimate the replacement value for the inprocess pixel. The nature of computation of replacement value for the inprocess pixels categorizes filters into linear, non-linear and adaptive. In figure 2.4, the shaded pixels constitute a window of size 3x3 with \( X(i,j) \) as its
inprocess pixel. The remaining pixels in the window are the neighbouring pixels of \(X(i,j)\).

<table>
<thead>
<tr>
<th></th>
<th>1,1</th>
<th>1,2</th>
<th>1,3</th>
<th>. . .</th>
<th>1,256</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,1</td>
<td></td>
<td>X(i,j)</td>
<td></td>
<td>2,3</td>
<td></td>
</tr>
<tr>
<td>3,1</td>
<td></td>
<td>3,2</td>
<td>3,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td></td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>256,1</td>
<td></td>
<td>256,2</td>
<td>256,3</td>
<td>. . .</td>
<td>256,256</td>
</tr>
</tbody>
</table>

Figure 2.4 Image in spatial domain representation with a \(3 \times 3\) window

### 2.9.1 Linear Filter

Linear spatial filters may either be done with simple averaging or with weighted averaging by convolving a suitable mask of size \(n \times n\). For a simple averaging filter the convolution mask of \(3 \times 3\) is given as in figure 2.5(a) and for weighted averaging as in figure 2.5(b).

**Figure 2.5: Convolution mask of (a) Simple averaging (b) Weighted averaging**

- The operation of simple averaging and the weighted averaging is given in equations (2.18) and (2.19) respectively.

\[
\bar{g} = g \ast W_k
\]  

(2.18)

where \(W_k\) is the mask.
\[ \bar{g}(x, y) = \sum_{m=-k}^{k} \sum_{n=-k}^{k} W_{mn} g_{mn} \]  

(2.19)

2.9.2 Non-Linear Filter

In linear filtering, pixels with the neighbourhood are taken to give the average value. Though the noise removal ability of this filter is remarkable, the principle of unselected averaging blur the fine image details and edges and destroys the image details like edges, thin lines and other sharp details. Increase in the size of filtering mask size in turn increases the degree of blur in edges as well as sharp details. So, linear filtering techniques have serious limitations in dealing with images exhibiting some degree of non-linearity. In view of these factors, non-linear filtering techniques are proved to be superior over linear filters. Hence the problem of blurred edges and smoothing is addressed by the non-linear filters. This is because in many image processing tasks, the image is just two-valued as black and white and the order is more or less arbitrary [Astola and Kuosmanen, 1997].

Non-linear filters are also known as order statistics filters / rank value filters. In this type of filtering, all the gray values of the pixels which lie within the mask are taken and are sorted by ascending gray values. Sorting is a common phenomenon to all non-linear filters. Median filter is the most popular filter of this kind. This filter selects the median of the sorted pixels of a given mask and uses it to replace the central pixel. It is important to note that for a given mask, the rank value filters do not perform any arithmetic operation as such.

Median filter is the most popular non-linear filter used for noise removal in image processing application, because of its good denoising power and computational efficiency. Median filter works well in removing the impulsive noise, which distorts the pixels randomly. It is evident that many of the state-
of-the-art filters designed for impulsive noise reduction use the principle of median filters at one point or the other [Lee, 1060]. Median based filter are also known as order statistics filters as a window of size nxn is made to slide over the digitized image and the median of the pixels bounded by that is used to replace the central pixels (ie. inprocess pixels) of the chosen window size [Bovik and Munson, 1983]. Median filter has been found to remove the signal structure when a large window size is chosen [Qiu, 1994] and has a disadvantage of removing thin lines and corners while reducing noise from an image. The weighted median filter, the center weighted median filter, mask median filter are a few variations of median filter. Recursive median and iterative median filters are also the improvements made over the median filters. There is another type of improved median filter known as decision-based median filter or switching median filter. This type of filter makes use of a threshold value to detect the presence of fixed valued impulsive noise [Sun and Neuvo, 1994; Chen et al, 1999], Filters like adaptive median filter [Hwang and Haddad, 1995] and median-based filters based on homogeneity information [Eng and Ma, 2001] are the classical examples of switching median filters.

2.10 Standard Median (SM) Filter

Median filtering is a simple and efficient point estimator, frequently used in signal, image and speech processing applications [Tukey, 1971], Tukey was the first person to suggest the use of median filtering for signal smoothing. The application of median filters in image processing was first reported by Pratt [1975] and Frieden [1976]. In one-dimensional domain, running medians track the local monotonic signal trends, including the edges. Two dimensional median filter has been widely used in image processing applications especially to restore the images corrupted by impulse noise. For this purpose, a running window of nxn is made to slide over the entire image, where the output associated with the center of the window is pixels of the square window of size nxn.
Median filtering is a discrete time process in which a $n \times n$ size window is taken from the input image. The pixels present in the window are ranked according to their intensity and the median of the ranked set is obtained as the output. The median filter is obtained as [Ko and Lee, 1991]:

$$y_{ij} = \text{Median} \{x(i-s, j-t) \mid (s, t) \in W \}$$

(2.20)

where $W$ is a square window of size $n \times n$ and $W = \{(s, t) \mid -N \leq s \leq N, -N \leq t \leq N \}$. For a window of size $3 \times 3$, $W = \{(s, t) \mid -1 \leq s \leq 1, -1 \leq t \leq 1 \}$. Hence it processes $n^2$ pixels and the complexity of the filter is $O(n)$. The median filter is computationally efficient [Huang and Yang, 1979], suppresses impulsive noise and preserves edges [Gabbouj et al, 1992, Astola and Kuosmanen, 1997]. The principle of median filter with a window size of $3 \times 3$ is illustrated in figure 2.6.

![Image](image-url)

Figure 2.6: Illustration of median filter (a) input pixels of $3 \times 3$ mask (b) sorted list of pixels of mask (c) output of median filtering

### 2.11 Recursive Median Filters

Generally, a median filter uses a sliding window and computes the running median. Filtering is done over the entire degraded image, by choosing a square window of size $n \times n$, where $n$ is chosen to be an odd number, for
the purpose of computational convenience though even number may be assigned, such a window is made to run over the image, starting from top left corner, in such a way that each assumed corrupted pixel appears at the center of the sliding window. The computed running medians replace the respective pixels lying at the center of the window [Huang et al., 1979]. During this process of filtering, from one output pixel to next, the window moves by one column, i.e. the number of pixels for a new window is chosen from the subset of preceding window by omitting $n$ pixels and by adding in new $n$ points. Hence the rest of the $n^2 - 2n$ pixels remain unchanged. Such a type of filtering is referred to as recursive filtering. This filtering technique makes use of the advantage of including the pixels which are already treated by the filter. In case of nonrecursive filtering the previously filtered pixels are not included while estimating the intensity value of an inprocess pixel of a window under processing.

2.12 Weighted Median (WM) Filter

Median filter has proved to be effective in removing impulse noise using small neighbourhood. Although the median is a robust estimator that possesses many optimality properties, the performance of median filter is limited by the fact that it is temporarily blind. To be specific, all the observation samples (i.e., the pixels of the chosen window) are treated equally regardless of its location within the observation window. This shortcoming can be addressed by choosing weighted median filter wherein each pixel within the window is assigned certain weight based on which the output is computed. The weighted median filter, an extended form of median filter, was first used by Brownrigg in 1984 for signal processing applications and it received considerable attention thereafter [Brownrigg, 1984]. The median of weighted median filter is given by:

$$y(n) = \text{Median} [W_1 0 x_1, W_2 0 x_2, \ldots -W_N x_N]$$  \quad (2.21)
where \( y(n) \) is the output of weighted median filter and \( W_i \) is the weight. For every \( W_i > 0 \) and \( \circ \) is the replication operator defined as:

\[
W_i \circ x_j = \underbrace{x_i \ldots x_i}_{W_i \text{ times}}.
\]

Here, the pixels of the given window are arranged as \( x_1, x_2, \ldots, x_N \) in such a way that \( x_1 < x_2 < \ldots < x_N \). Moreover, \( x_1, x_2, \ldots, x_N \) represent the pixels of a window size \( n \times n \). For instance, the pixels of window \( 3 \times 3 \) are arranged in one-dimensional representation as \( x_1, x_2, \ldots, x_9 \). Normally, the values of weighted mask are chosen to be positive integer, though negative integer / real-number weighted masks are also allowed. The weighing component allows us to have a greater flexibility either in emphasizing or deemphasizing the input pixels. In most of the applications, all the samples are not equally important. This filter can also be operated recursively. As in recursive median filter, recursive weighted median filters have an advantage over the nonrecursive weighted median filters. If \( W_i = 1 \), then WM filter is equivalent to median filter.

### 2.13 Center Weighted Median (CWM) Filter

Due to the symmetric nature of the odd sized filtering, the one which is the most correlated with desired output is the central pixel of the window. This principle has brought about a new filter called Center weighted Median filter (CWM). This is a subset of weighted median filter and is proved to be useful and efficient in many image processing applications. CWM filter is realized by assigning weight to the central pixel of the given window. The output of CWM is defined as:

\[
y(n) = \text{Median} \left[ x_1, \ldots, x_{c-1}, W_c \circ x_c, x_{c+1}, \ldots, x_N \right] \quad (2.22)
\]

where \( W_c \) is an odd positive integer, \( c \) is the index of the central pixel in a window of size \( n \times n \) and \( \circ \) is the replication operator. If \( W_c = 1 \), this filter behaves as that of median filter and if \( W_c \geq N \) it reduces to an identity filter.
operation, where $N$ is the total number of pixels in a window. This filter is easy to design and implement than the weighted median filter. CWM filter preserves image details at the expense of less impulsive noise suppression.

### 2.14 Rank Order Mean (ROM) Filter

Rank order mean (ROM) is another type of order statistics filter, proposed by Abreu et al [1996], as an alternative to median filter. This filter takes a window having the corrupted pixel in its center. The neighbourhood pixels of the window excluding the corrupted pixel are ordered by rank, which can be defined as a vector having eight elements as:

$$r(n) = [r_1(n), r_2(n), ..., r_8(n)]$$  \hspace{1cm} (2.23)

where $r_1(n) \leq r_2(n) \leq ... \leq r_8(n)$. Then, the ROM is defined as

$$m = \frac{r_4(n) + r_5(n)}{2}$$  \hspace{1cm} (2.24)

Unlike median filter, this filter does not include the corrupted pixel while estimating the replacement value for it. Hence this filter helps us to overcome the disadvantages of median filter.

### 2.15 Adaptive Median Based Filters

When sliding window filters are used in the restoration of nonstationary signals, the characteristics of the pixels present in each window varies as the window is moved across. In applications involving restoration, enhancement or smoothing of gray scale images corrupted by noise, obviously we encounter some discontinuities in the intensity level of the image in few regions due to the presence of edges or noise. Considering the uncertainty
regarding characteristics of pixels within each window, adaptive filtering techniques have been suggested wherein a set of parameters $I$ processes direct the selection of filter, independent of data. Simple adaptive filtering strategies make use of the local statistics of pixels to estimate the output. Hence such filters select an appropriate estimator from among the allowable estimators. Most of the adaptive order statistic filters allow necessary flexibility in selecting the most appropriate filter for processing different regions of nonstationary signal. A suitable filter for estimation is selected based on the test statistics got out of the local nature of the image. For instance, linear combination of the estimators may be taken, with a set of weights being adapted according to the local statistics prevails in each processing window. In addition, the selected estimators and test statistics may operate on the different subwindows of a given data window. However, exact statistical analysis of even individual order statistics filters is difficult to accomplish and simulation studies are often carried out for the performance characterization of such filters. A number of filters fit into this framework. Tri-state Median (TSM) Filter and Progressive Switching Median (PSM) Filter are few examples of adaptive median based filter for impulse noise removal. This type filtering is also referred to as switching median.

The purpose of adaptive filters is two fold. One is to avoid the undue distortion of restored image as a result of uncorrupted pixels getting modified during denoising. This problem is taken care of by devising an effective mechanism to identify the corrupted pixels prior to denoising, which is referred to as noise detection. The other purpose is to choose the best fitting replacement value from the available filtering mechanisms based on some decision using local statistics. Hence the adaptive filters have two phases, comprising of noise detection and noise correction. In general, adaptive median based filters use threshold as a criterion to detect the presence of impulse noise. The correction mechanism is done using one or more median based filters. These filters are found to exhibit higher degree of impulse rejection and detail preservation compared to SM, WM or CWM filters.
The working of an adaptive median based filter may be described using the principle of Tri-state median filter [Chen et al, 1999]. The block diagram of this filter is depicted in figure 2.7. The output of the filter is decided by computing the difference between the central pixel of the window and the standard median as $d_1$ and the central pixel of the window and the center weighted median as $d_2$. Based on the values of $d_1$ and $d_2$, impulse detection and correction is done adaptively using threshold.

Figure 2.7: Block diagram of Tri-state median filter

The replacement value for the impulse noise is given by

$$Y_{\tilde{d}}^{TSM} = \begin{cases} 
X_{\tilde{d}}^{W} & T > d_1 \\
Y_{\tilde{d}}^{CWM} & d_2 \leq T \leq d_1 \\
X_{\tilde{d}}^{SM} & T < d_2 
\end{cases} \quad (2.25)$$

where $d_1 = |Y_{\tilde{d}}^{SM} - X_{\tilde{d}}|$ and $d_2 = |Y_{\tilde{d}}^{CWM} - X_{\tilde{d}}|$. $T$ is the threshold that governs the switching of the filter between three options as given in equation (2.25).

Due to the advantages of adaptive filtering techniques, they are widely used in restoration techniques.
2.16 Image Quality Metrics

It is needless to emphasize the use of appropriate performance parameters to access the quality of restored images. Mean Square Error (MSE) and Peak-Signal-to-Noise Ratio (PSNR) are the widely adopted measures to ascertain the quality of any processed images. These metrics are extensively used in measuring the quality of restored imaged corrupted with impulsive noise too. The measure of success of restoration is usually an error measure between the original image $X$ and the restored image $Y$. No mathematical function is known that corresponds to human perceptual assessment of error. The most accepted measure of quality of restoration, MSE is commonly used because it is easy to compute and it corresponds to signal energy in the total error [Gonzaloz and Woods, 2004].

$$\text{MSE} = \frac{1}{M \times N} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (Y_{mn} - X_{mn})^2$$  \hspace{1cm} (2.26)

where $M$ and $N$ are the number of rows and columns of the original and the restored image. This metric is computed based on the luminance value of the images which varies between 0 and 255.

Similarly, for a gray scale image, the PSNR is computed using MSE as:

$$\text{PSNR} = 20 \log_{10} \left( \frac{b}{\text{MSE}} \right)$$  \hspace{1cm} (2.27)

In applications involving images, sequences and colour images, the order statistics filter and its variants have been the most prominent and successful in noise suppression. Weighted median filter, centre weighted median filter, max filter, min filter, midpoint filter, alpha-trimmed mean filter are a few variants of median filter. It is evident that non-linear filters based on order-statistics are applied to restore the images corrupted by impulsive noise.
and are proved to be the best substitute for linear filters. These filters have exhibited better detail preservation and impulse rejection capabilities [Astola and Kuosmanen, 1986]. Moreover these filters are conceptually simple and easy to implement [Pitas and Venetsanopolous, 1986]. But the main disadvantage of these approaches is the location-invariant nature, which results in altering even the pixels which are not disturbed by noise [Chen and Wu, 2001]. This called for a detection-based median filter with thresholding.