CHAPTER – 7

CONCLUSION
In this concluding chapter, critical reviews of the main results of the thesis are presented. The area for possible extensions of this research work has also been identified.

7.1 MAIN RESULTS OF THE THESIS

In the introductory Chapter 1, problem specifications, approach and thesis organization have been discussed. In Chapter 2, review of related topics such as notch filter types, characteristics and applications are carried out. Important FIR filter design techniques in the literature from the year 1978 to recent one are discussed.

The mathematical analysis for designing the maximally flat, linear phase FIR notch filter has been carried out in Chapter 3. It has been shown that it is possible to increase the null width at –120dB for a maximally flat, linear phase FIR notch filter by selecting suitable zero odd order derivatives of the filter response at notch frequency (ωd). Selection of zero odd order derivatives is done as per the requirement of null width. For example, for a notch frequency of 1.8 radians if the null width variation required is in the range 0.165 radians to 0.248 radians (i.e. approximately 9 to 14% of variation), the first order zero derivative is selected. If the requirement is in the range 0.034 radians to 0.079 radians (i.e. 1.8 to 4.5% of variation), the third order zero derivative is chosen.
A design algorithm for designing the MF linear phase FIR notch filter with specified –120dB null width and notch frequency has been proposed for the case when only third order zero derivative of the response contributes to the increase of null width. Empirical formulas for length of the filter ‘n’ (3.26) and degree of flatness ‘m’ (3.27) & (3.28) have also been proposed. The suggested algorithm is found to work satisfactorily up to ‘n’ equaling 35. Designed null width using this algorithm is found to be within an error range of 5%. One can increase the null width in steps by adding higher order odd zero derivatives of the response at the notch frequency. The proposed design approach for notch filter is simple because of the closed form equations for computing ‘n’ and ‘m’ parameters for the specified –120dB null width and notch frequency. The methodology proposed in this chapter, for controlling the null width is better than the design [30] in the following aspects.

1. The proposed design is **maximally flat** and is **linear phase** in the pass band.

2. Null width variation is achieved at -120dB compared to -50dB and -60dB in [30].

3. Null width variation for both quadratic constraint approach and derivative constraint approach of the design [30] is only from 2% to 10% where as our suggested design is more flexible. Higher null width variation from 9 to 14% is possible with first order zero derivative.
4. Smaller null width variation from 1.8 to 4.5% is possible with third order zero derivative and further refinement in the above range is possible with successive derivative approach proposed in Section 3.5

5. The proposed design is superior to the various designs found in the literature as it provides simple empirical formulas for filter length & degree of flatness, higher attenuation for the notch frequency at -120dB, besides higher control of null width over large range. Also, the proposed design produces almost linear null width variation with variation in ‘n’ where \( n = (N-1)/2 \), for the range \( n = 8 \) to \( 38 \). This can be seen from Figure 3.6.

In *Chapter 4*, a novel design technique for designing FIR bi notch filters from two prototype IIR notch filters has been proposed. The design weights for the proposed filters are computed from the suggested mathematical formulas obtained in the research work. Designed FIR bi-notch filter can be an effective eliminator of harmonic contents of periodic noise or unwanted signals. The rejection bandwidths of the bi notch FIR filter can be controlled from 17.19° to 9.16° by varying the pole length of prototype IIR filters used for the derivation of FIR filters. It is observed that ‘r’ cannot be increased beyond 0.91 and the filter length is limited to 100, if we aim at ripple free performance of the designed FIR filter. Design simplification for the special condition:

\[
\cos \omega_1 \cos \omega_2 = -1/2
\]
has also been brought out. A special IIR bi-notch filter designed with this condition requires less (almost half) number of multipliers. An FIR bi-notch filter designed with the special condition reduces half the number of weights to zero without compromising with the rejection band width for the desired response. Also, with increase in ‘r’ value, though the number of zero weights reduces, the quality of the frequency response improves due to elimination of ripples in the pass band.

It is possible to design an FIR notch filter from IIR prototype for very narrow rejection band width, by suitable selection of high value for ‘r’ (the pole length of the IIR prototype) as shown in Chapter 5. Rejection bandwidth of the FIR filter $H_1(z)$ can be reduced by about 50% by sharpening it using ACF : $H(z)(2 – H(z))$. The filter length of such sharpened FIR filter: $H_2(z)$ will be double that of the original filter $H_1(z)$. The frequency response of the sharpened FIR notch filter $H_2(z)$ has much narrower RBW than that of unsharpened FIR notch filter of the same order. By repeated sharpening the filter using the same ACF, one can obtain extremely narrow rejection bandwidth. However, every time the filter is sharpened by this ACF, the filter order increases to two fold. Thus, a compromise has to be made between the required minimum possible rejection bandwidth and the high filter order of the sharpened FIR filter.

While designing FIR notch filter for high ‘r’ value, low RBW is indeed obtained though high ripples in the pass bands also appear. However, by using
ACF: \( H(z)(2 - H(z)) \), the *ripples can be completely eliminated* and RBW is narrowed. From the performance analysis shown in Table 5.3, it is evident that every time an FIR filter is sharpened, RBW for the filter decreases to almost half of its original value. This observation of the analysis can be effectively used for designing FIR notch filters with “regulated” rejection bandwidth. With the designed FIR notch filters we can select low RBW as per the requirement for a particular application. In such regulated RBW FIR filters, one can achieve new RBW which is almost 1/2 of the designed RBW or 1/4\(^{th}\) of designed RBW or 1/8\(^{th}\) of designed RBW. A three stage FIR notch filter with regulated RBW is proposed in Figure 5.7. At every stage the RBW gets reduced to half and the order of the resulting filter gets doubled. Such arrangement may be extended for more stages to achieve very low RBW provided high order of the notch filter is acceptable. For such a proposed FIR notch filter with regulated RBW, FIR notch filter \( H_1(z) \), designed from IIR prototype, is the basic building block. Every stage of this arrangement consists of interconnected FIR filters \( H_1(z) \) through which the signal is passed repeatedly.

Finally in *Chapter 6*, the design of an FIR comb filter from IIR prototype for achieving very narrow rejection bandwidth, by suitable selection of high value for ‘r’ (the pole length of the IIR prototype) has been proposed. Rejection bandwidth of an FIR comb filter \( H_1(z) \) can be reduced by about 50% by sharpening it by using ACF: \( H(z)(2 - H(z)) \). A compromise has to be made between the required minimum possible rejection bandwidth and the high filter
order of the sharpened FIR comb filter. This analysis can be effectively used for
designing an FIR comb filter with “regulated” rejection bandwidth. The proposed
FIR comb filter using ACF is capable of achieving extremely sharp rejection
bandwidths which is not easily possible by conventional designs. The suggested
design methodology is simple and requires no computer optimization techniques.

7.2 SCOPE FOR FURTHER RESEARCH

The research work presented in this thesis may be extended as suggested
below.
1. A methodology for designing MF, linear phase FIR notch filter for the
specified null width and notch frequency with third order zero derivative of
the amplitude response has been proposed in Chapter 3. The, empirical
formulas suggested for the length of the filter ‘n’ and degree of flatness ‘m’
are working effectively within an error of 5%. There is a scope to extend the
design methodology and the empirical formulas for 5th or higher order zero
derivatives of the amplitude response.

2. In the present research work, suggested empirical formulas for the
computation of ‘m’ and ‘n’ (refer to (3.26), (3.27) and (3.28)) are found to
work satisfactorily up to a length of 35. There is need to derive exact
mathematical formulas for computing ‘n’ and ‘m’ which work effectively for
any length.
3. In Chapter 4, a special condition \( \cos \omega_1 \cos \omega_2 = -\frac{1}{2} \) has been suggested for design simplification to improve the computational efficiency by reducing the number of multipliers. It would be an interesting problem to identify some other special conditions by which even less multipliers are needed for the design.

4. In this research work, an amplitude change function \( H(z)(2-H(z)) \) for sharpening the non-linear phase FIR notch filter has been identified. It would be useful to find out whether the same ACF can be used for sharpening the MF, linear phase FIR notch filter. Then, the suggested methodology could be evolved for designing a MF, linear phase FIR notch filter with extremely narrow rejection bandwidth.

5. We have evolved the design of FIR notch filter in which rejection bandwidth (RBW) can be varied by varying pole length of the prototype IIR notch filter. Higher the pole length lower is the RBW. It should be investigated to obtain the exact relation between RBW and the pole length thereby one can develop tuned rejection bandwidth FIR notch filter.

6. The various concepts of filter design proposed in this thesis may be extended for the case of 2D and 3D as well.
7. It may be possible to propose the design of a notch filter which does not have exact null at $\omega_d$, but with the power level $|H(\omega_d)|^2$ less than some specified value $P_{in}$. This may render the design simpler and economical.
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