CHAPTER -1

INTRODUCTION
1.1 BACKGROUND

The single notch / multi notch filters are often used in instrumentation, biomedical signal processing and in a host of other practical situations to eliminate very narrow bands of undesirable frequencies or disturbances. Such notch filters can be either Infinite Impulse Response (IIR) filters or Finite Impulse Response (FIR) filters. A number of design approaches are available in the literature for IIR as well as for FIR filters [13-24, 26-46, 50-52]. These FIR and IIR digital notch filters have their own specific features. Based upon the requirements for the specific application one of the above digital filter is chosen.

The main advantage of an IIR filter is the low order in its realization. It can be designed for lower orders to approximate a given set of specifications as compared to the FIR designs. This implies that the signal processing by IIR filters is faster than that by the FIR ones. However, the implementation complexity of an FIR filter is not of much concern as tremendous advancement in DSP has taken place due to FFT techniques and field programmable gate array (FPGA) technology. The major problems with IIR filters are that they are essentially unstable and can not provide linear phase response. The decisive advantages of FIR filters are their linear phase response and finite length of the impulse response. They can be easily designed to give linear phase characteristics. FIR filters, therefore, find extensive use where frequency dispersion due to non-linear phase is undesirable. These are the primary reasons because of which a large
number of commercial chips carry out signal processing with FIR filters. This is the precise reason for selecting FIR designs for our thesis.

1.2 PROBLEM SPECIFICATION AND APPROACH

In this thesis, we propose new designs for application specific FIR digital notch filters and analyze their performance. Both linear phase and non linear phase FIR notch filters are considered. The research work carried out in this thesis is with reference to the following problems.

1. Design of maximally flat linear phase FIR notch filters with controlled null width.
2. Design of special bi-notch filters.
3. Design of FIR notch filters for highly narrow rejection band width.
4. Design of FIR comb filters with extremely narrow rejection band width.

*Design of maximally flat linear phase FIR notch filter (NF) with controlled null width* is an innovative approach for eliminating the interference frequency which varies within certain fixed known range. Proposed FIR notch filter can be an alternative to data dependent filter and finds applications in many communication systems where interference frequency is found to vary within certain specified range. Analytical solutions have been provided for computations of design weights of maximally flat linear phase FIR notch filter with controlled null width. Also, empirical formulas are suggested for calculating the length of the filter and
degree of flatness for the specified null width around specified notch frequency.

Design procedure has been suggested and a few design examples are worked out.

The design approach used is now explained briefly.

The transfer function $H(z)$ of a causal FIR filter of length $N$ in terms of unit sample response $h(i)$ is given by

$$H ( z ) = \sum_{i=0}^{N-1} h (i) z^{-i}$$  \hspace{1cm} (1.1)

For the linear phase requirement, $h(i)$ must satisfy the constraint [1,3,5]

$$h(i) = \pm h(N - 1 - i), \quad i = 0, 1, 2, 3, \ldots, N-1$$ \hspace{1cm} (1.2)

The filter length $N$ can be odd or an even integer. However, $N$ is chosen to be an odd integer so as to avoid problems due to fractional delays. By using (1.1) and (1.2) the transfer function of a symmetric linear phase FIR filter can be written as [5]

$$H ( z ) = \sum_{i=0}^{n} z^{-\binom{i}{0}} h (n - i) \left[ z^{-(n-i)} + z^{-(n+i)} \right], \quad n = (N-1)/2$$ \hspace{1cm} (1.3a)

$$= z^{-n} \sum_{i=0}^{n} D_i (z^i + z^{-i})/2$$ \hspace{1cm} (1.3b)

where,

$$\binom{0}{i} = \begin{cases} 0, & i = 0 \\ i, & \text{otherwise} \end{cases}$$ \hspace{1cm} (1.3c)
The weights $D_i$ are related to the unit sample response $h(i)$ [5] as

$$D_i = \begin{cases} h(n), & i = 0 \\ 2h(n), & i = 0, 1, 2, \ldots n \end{cases} \quad (1.4)$$

The frequency response $H(e^{j\omega})$ of the causal filter may be written as

$$H(z)|_{z = e^{j\omega}} = e^{-j\omega n}H_0(\omega) \quad (1.5)$$

where $H_0(\omega)$ is the amplitude function given by

$$H_0(\omega) = \sum_{i=0}^{n} D_i \cos i\omega \quad (1.6)$$

and $\omega$ is the normalized frequency in radians. The term $e^{-j\omega n}$ contributes only to linear phase (equaling $-n\omega$ radians) and has no effect on the magnitude of $H(e^{j\omega})$.

Also, an algorithm has been proposed to compute the coefficients $D_i$’s to achieve design of notch filter NF2 as explained in Chapter 2. For this design the following optimality criteria has been imposed.

$$H_d(\omega)|_{\omega=0} = 1 \quad (1.7a)$$

$$H_d(\omega)|_{\omega=\pi} = -1 \quad (1.7b)$$

$$\ddots$$

$$\frac{d^u H_d(\omega)}{d\omega^u}|_{\omega=\pi} = 0, \quad u = 1, 2, 3, \ldots, (2m-1) \quad (1.7c)$$

$$\frac{d^v H_d(\omega)}{d\omega^v}|_{\omega=0} = 0, \quad v = 1, 2, 3, \ldots, (L-2m) \quad (1.7d)$$
Here, ‘m’ is an integer specifying the degree of flatness at $\omega = \pi$, which can have values within the range $1 \leq m \leq n$. For (1.7f) two cases have been considered. In one case ‘k’ can take values 1 or 3 or 5 etc. (selective addition of odd derivatives) and in the other case ‘k’ can take values 1, 3, 5 etc. (i.e. successive addition of odd derivatives). In either case (1.7a-1.7f) gives (n+1) non-trivial equations which can be solved to compute the design weights $D_i$’s.

Next, the problem of designing special bi notch filters is addressed. In this case, a methodology for designing FIR bi notch filters derived from second order prototype IIR notch filters is suggested. The transfer functions of two prototype IIR notch filters at notch frequencies $\omega_1$ and $\omega_2$ [1,5] are

$$G_1(z) = K_1 \frac{1 - 2 \cos \omega_1 z^{-1} + z^{-2}}{1 - 2r \cos \omega_1 z^{-1} + r^2 z^{-2}}$$ (1.8a)

$$G_2(z) = K_2 \frac{1 - 2 \cos \omega_2 z^{-1} + z^{-2}}{1 - 2r \cos \omega_2 z^{-1} + r^2 z^{-2}}$$ (1.8b)

Now, obtain the following two FIR notch filters both of order $L$ from the two prototype IIR notch filters given in (1.8a) and (1.8b)

$$N_i(z) = \sum_{i=0}^{L} D_i(i) z^{-i} \quad \text{(for notch at } \omega_1)$$ (1.9a)
The computation methodology for calculating the weights \( D_1(i) \) and \( D_2(i) \) is proposed. Then, \( N_1(z) \) and \( N_2(z) \) are cascaded to obtain the composite bi notch filter \( N_3(z) \).

\[
N_3(z) = N_1(z)N_2(z) \tag{1.10a}
\]

\[
N_3(z) = \sum_{i=0}^{2L} D_3(i)z^{-i} \quad \text{(with notches at } \omega_1 \text{ and } \omega_2) \tag{1.11b}
\]

The procedure to compute composite weights \( D_3(i) \) for the bi notch filter is suggested. Rejection bandwidth for the designed bi notch filter can be controlled by suitable choice of \( 'r' \), the pole length of the IIR prototype notch filter. The suggested bi notch filter can also be adapted to eliminate second, third and fourth order harmonics of periodic noise besides the fundamental noise frequency component. A special case when two notch frequencies \( \omega_1 \) and \( \omega_2 \) are such that \((\cos \omega_1 \cdot \cos \omega_2) = -1/2\) has also been discussed.

The third problem deals with the design of FIR notch filters (NF) with highly narrow rejection bandwidth (RBW). Reduction in the RBW is achieved progressively in three stages. In the first stage an FIR notch filter is designed from prototype IIR notch filter with high value of \( 'r' \) (pole length). In the second and third stage decrease in the rejection bandwidth is ensured with amplitude change function ACF: \( H(z)(2 - H(z)) \). This filter is most appropriate for video and audio signal processing where elimination of extremely narrow band of interference
frequencies must be ensured without disturbing pass band frequencies so that reproduced signals are the exact replica of the transmitted signals.

Finally, a methodology for designing an FIR comb filter (CF) with very narrow rejection bandwidth (RBW) is suggested. This comb filter can be used in the separation of luminance and chrominance components in video signals and in processing of music signals.

1.3 Organization of the thesis

There are seven chapters in this thesis including the introductory and the concluding ones.

In this introductory Chapter we have explained about our thesis problems and the approaches that have been adopted. Notch filter applications and the various digital notch filter design techniques available in the literature have been discussed in Chapter 2.

In the third Chapter, a technique for designing a maximally flat, linear phase FIR notch filter with controlled null width is presented. Design analyses is carried out with first, third and fifth order zero derivative constraints of the amplitude response of the FIR filter at notch frequency. An algorithm is presented for designing maximally flat, linear phase FIR notch filter with specified null
width around specified notch frequency with third order zero derivative constraint at the notch frequency. Analysis is also carried out for controlling the null width with successive addition of odd zero derivatives.

In Chapter 4, a methodology for designing FIR bi-notch filters derived from second order prototype IIR notch filters is suggested. The rejection bandwidth for the designed filter can be controlled by suitable choice of ‘r’, the pole radius of the IIR prototype notch filters. The suggested bi-notch filter can also be adapted to eliminate second, third and fourth order harmonics of periodic noise besides the fundamental noise frequency component. A special case when two notch frequencies $\omega_1$ and $\omega_2$ are such that $(\cos \omega_1)(\cos \omega_2) = -1/2$ has also been discussed.

Chapter 5 deals with the design of FIR notch filters (NF) with highly narrow rejection bandwidth (RBW). Reduction in the RBW is achieved progressively in three stages. In the first stage, an FIR notch filter is designed from a second order IIR prototype filter. For a given length $L$ of the NF, the maximum permissible value of ‘r’ (the pole length of IIR prototype filter) is chosen to achieve very narrow RBW of the FIR filter. In the next stage by using an Amplitude Change Function (ACF): $H(z)(2 - H(z))$, the designed filter is sharpened. Consequently, the RBW of the resulting NF is reduced to almost half of the earlier value. This reduction of bandwidth makes the resulting notch filter
of length 2L. In the next stage, RBW can be further reduced by the repeated sharpening of the filter by the same ACF.

Results of Chapters 4 and 5 are used in Chapter 6 for designing FIR comb filter with very narrow rejection bandwidth (RBW). The proposed design using Amplitude Change Function can give an extremely sharp rejection bandwidth which is not possible in conventional designs.

Chapter 7 is the concluding chapter in which critical review of the research work presented in the thesis has been discussed. Scope for further work in the area of notch filters has been identified.