Chapter - 1

Development and Basics of Production Scheduling Problems

1.1 Introduction:

The subject matter of this dissertation deals with some special classes of flow-shop sequencing problems which is an important area of Operation Research. The term Operation Research was first coined in 1940 by McClosky and Trefthel in a small town, Bowdsey of the United Kingdom. The main origin of Operation Research was during the second world-war. At that time, the military management in England called upon a team of scientists to study the strategic and tactical problems related to air and land defence of the country. Since they were having very limited military resources, it was necessary to decide upon the most effective utilization of them, e.g. the efficient ocean transport, effective bombing etc.

Thus, this new science came into existence in military context. During Second World War II, the Military Commands of U.K. and U.S.A. engaged several inter-disciplinary teams of scientists to undertake scientific research into strategic and tactical from various disciplines and organized them into teams to assists in solving strategic and tactical military operations. Their mission was to formulate specific proposals and plans for aiding the Military Commands to arrive at the decision on optimal utilization of scarce military resources and efforts and also to implement the decisions effectively. The OR teams were not actually engaged in military operations and in fighting the war. But, they were
only advisors and significantly instrumental in winning the war to extent that the scientific and systematic approaches involved in OR provided a good intellectual support to the strategic initiatives of the military commands. Hence OR can be associated with “an art of winning the war without actually fighting it”.

With the development of industrial organization, the managerial functions in the field of defense, business, industry, civil government etc., have become increasingly intricate and complex. In fact, the rapid growth of industrial decisions regarding purchasing, planning and supervising the production, selling, hiring and so on. These increased activities have led to the problem of utilizing the limited resources like man, machine and material to the best possible advantages. The science of O.R. attempts to provide techniques for such an objective approach to decision making problems. In fact O.R is the quantitative decision making approach suggest an optimal solution to the problems which is in the best interest of organization. Operation Research problems fall into any of the following categories; sequencing, routing, allocation, replacement, inventory, queuing, competitiveness and search.

Allocation problem may be linear programming problem or an assignment problem.

The problems in which the facilities are fixed and the sequence of servicing the waiting customers in subject to control; the investigation to deal with such problems have given rise to an elegant theory of sequencing. Scheduling/sequencing problems are of common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, waiting aircraft for landing clearances, Programmes to be run in a
sequence at a computer center etc. such problems exit whenever there is an alternative choice as to the order in which a number of jobs can be done. The selection of an appropriate order or sequence in which to receive waiting customer (or jobs) is called sequencing. As with other Operational Research problems the objective is to optimize the use of available facilities to effectively process the items or the jobs.

It is also essential to select from the situation and its environment a criterion or objective function. If it is assumed that the time required to prepare the processing of a particular item at a specific machine is given. The question of optimization becomes the question to ordering the terms at each facility so that the objective of minimization of total elapsed time or minimization of total waiting time or cost associated with Operations is achieved.

In industries man, machine, material and money are involved for production of items. The manager of an industry is interested to use man, machine, material and money in an economic manner so that the cost associated with the production of an item is not increased or it is minimum in the competitive market.

In the last three and half decades deterministic sequencing and scheduling has received considerable attention in the literature. The scheduling field can be divided in to several major areas, namely single machine scheduling, parallel machine scheduling. Flow-shop, Job-shop and Open-shop scheduling etc.

In the last decades stochastic sequencing has begun to receive attention in the literature too. In an industry or business activity scheduling techniques are more useful and give scientific systems approach to utilize
machines idle times or operators time to cut down or reduce the cost of production. The scheduling methods can be utilized for alternate purpose of industry or in business unit in an economical manner.

1.2 Difference Between Scheduling and Sequencing:
Sequencing simply refers to the determination of order in which the jobs are to be processed on various machines while scheduling refers to the time table that include the state time and completion time of jobs on machines. Scheduling contains time tabling as well as sequencing information. The two terms viz. sequencing and scheduling though distinct, to some extent, are used as synonymous terms in the project.

This chapter, INTRODUCTION is divided into following sections.
1.3 Classification of scheduling models
1.4 Performance measure
1.5 Brief survey of scheduling problems

1.3 Classification of Scheduling Models

The different types of scheduling problems are:

(i) Flow-Shop Scheduling:

In these types of problems we have n jobs and m machines. Any given job requires and execution on each machine. The ordering of processing on various machines is same for all the jobs; also the sequence in which the jobs go through any of the sub-sequence machine, i.e. a job may not pass another job while waiting on processing on a machine. A Flow-shop with this restriction is often called a permutation Flow-shop. In the permutation Flow-shop not only the machine order is same for the all
jobs, but the search to schedule is completely specified by a single permutation of a number 1,2,3,…n, giving the order in which the jobs processed on each and every machines.

(ii) **Job-Shop Scheduling Problems:**
In these types of problems, we have n jobs and m machines, and each job has its own machines order specified.

(iii) **Open-Shop Scheduling Problem:**
In these types of problems, any given job requires and execution on each of the m machines. The order in which a job passes through the machines is immaterial.

(iv) **Project Scheduling:**
It is type of project where all the resources are brought to the job. In this project we emphasis Flow-shop scheduling problems.

1.4 **Basic Assumption in Flow-Shop**
- No machine processes more than one operation at a time.
- Each operation on a machine, one started, must be performed to its completions.
- Each operation takes finite time and it must be completed before any other operation begins. The given operation time includes setup time.
- There is only one machine of each type.
- Each machine operates independently of the order.

1.4 (a) **Assumption Regarding Jobs**
- All jobs are available for processing at time zero.
- All jobs allow the same sequence of operations.
• Jobs are independent of each other.
• No job is processed more than once of any machine.
• Each job consists of a specified number of operations and each operation is performed by only one machine.
• The processing times of jobs are independent of the order in which jobs are performed.
• Each job once started, must be processed to completion.

1.4 (b) Assumption Regarding Operating:
• Each job is processed as early as possible.
• Each job is considered as indivisible even though it may consist of a number of individual units.
• Each machine is provided with sufficient waiting space for allowing jobs to wait before starting their processing.
• Each machine processes jobs in the same sequence i.e. on passing or overtaking of jobs is permitted.
• Transportation time of a job between machines is negligible.
• Set-up times are sequence-independent.
• The given operation times include setup times.

1.5 Various Performance measure

(i) Release time \(r_i\):

This is the time at which job is released to the shop by some external job- generation process.
It is the earliest time at which processing of first operation of the job could be begin. It is also known as ready time or arrival time of job i.

(ii) Due date\( (d_i)\):

It is the promised delivery date of the job \( i \). It is the time when the processing of the last operation should be completed i.e., the time when the job \( i \) should be completed.

(iii) Completion time\( (c_i)\):

It is the time at which processing time of the last operation of the job \( i \) is completed.

(iv) Flow-time\( (F_i)\):

It is the total time that the job \( i \) spends in the shop.

\[
F_i = C_i - r_i
\]

The flow time \( F_i \) is also called the manufacturing interval and the shop time.

**Note:** when \( r_i=0 \) then \( F_i=C_i \) i.e., completion time of job \( i \) is equal to flow time of job \( i \).

(v) Lateness \( (L_i)\):

It is simply difference between completion time of job \( i \) and its due date.

(vi) Tardiness\( (T_i)\):

The tardiness of job \( i \) is equal to \( \text{Max} \{0,L_i\} \).
(vii) **Earliness** \((E_i)\):

The earliness of job \(i = \text{Max} \{0,-L_i\}\)

**Note:** lateness, tardiness and earliness are three different ways of comparing the actual completion time with the desired completion time. Lateness considers the algebraic difference of each job regardless of the sign of difference. Tardiness considers only positive difference i.e. jobs which are completed after their due dated and earliness considers only negative difference i.e. jobs completed ahead of their due date, \(L_i\) is negative and when a job is completed after its due dater, \(L_i\) is positive and non zero and the job is tardy.

(viii) **Total Elapsed Time** \((C_{\text{max}})\):

It defined as total completion time in which the set of all the jobs finish processing on the machines. It is also known as make span.

\[ C_{\text{max}} = \text{Max} \{C_i\}; \text{ i= 1,2,3,4,} \ldots \ldots \ldots \ldots \ldots ,\text{n.} \]

(ix) **Idle Time** \((I_k)\):

It is idle time on machine \(M_k\). \(L_k = C_{\text{max}} - P_{i,k}\), where \(P_{i,k}\) is the processing time of job I on machine \(M_k\).

(x) **Mean Completion Time** \((C)\):

It is the average completion time of any job \(C = (C_i)/n\).

(xi) **Mean Flow Time** \((F)\):

It is the average time a job spends in the shop. \(C = (F_i)/n\)
(xii) **Penalty Cost**: 
It is defined as the total penalty paid by virtue of jobs being late in completion by their due dates.

(xiii) **Total Production Cost**: 
It is defined as the total production cost for the production of a set of products on machines.

Two performance measures are equivalent if a schedule which is optimal with respect to one performance is also optimal with respect to other performance measure and vice-versa. The performance measures are $C$ and $F$ are equivalent while the measures $C_{\text{max}}$ and $F_{\text{max}}$ are not equivalent.

In this project, we have considered only total elapsed time; mean flow time (or mean completion time); total rental cost; and combination of mean flow time and total elapsed time as the performance of measures for a class of sequence problem.

**1.5.1 Parameters in Scheduling Problem**

There are four parameter $n/m$ $A/B$ in a scheduling problem

$n$: the number of jobs

$m$: the number of machine

$A$: describes the flow patterns or discipline within the machine Shop.
F: for the flow shop case i.e., the machine order for all the jobs is the same.

P: for the permutation flow shop case.

G: for the general job-shop case when there are no restrictions on the form of the technological constraints.

B: describes the performance measure by which the schedule is to be evaluated. As an example, \( n/2/f/C_{\text{max}} \) is the n-jobs, 2 machines flow-shop problem where the objective is to minimize total elapsed time.

In this project, we have considered only \( n/m/F/B \) type of the sequencing problems.

### 1.5.2 Terminologies

#### 1.5.2 (a) No-idle constraint:

Under no idle constraint, machine work continuously without idle interval i.e., each machine after starting the processing of first job will work continuously without break till the last job completed on it. This means a machine works for the time equal to the sum of processing times of all the jobs on this machine.

#### 1.5.2 (b) No wait Constraint:

No wait constraint prescribes for the jobs to work continuously without waiting times between consecutive machines i.e. a job needs to go through a sequence of machines without any delay between successive operations of the job. This means the difference between the completion
time of a job last operation and the starting time of its first operation is equal to the sum of its operation times on all machines.

1.5.2 (c) Non-Availability Constraint:

Most literature in scheduling assumes that machines are available at all times. However, this availability may not be true in real industry settings. For example,

Machines may not be available during the scheduling period during break down (stochastic) or preventive maintenance (deterministic). The machines may not be available at beginning of the planning period of machines continue to process unfinished jobs that were scheduling in the previous planning period.

1.5.2 (d) Pre-emption:

It is a complex situation in which processing is interrupted and a job is removed from a machine before an operation is completed. Pre-emption is said to occur when the processing of a job that has been started is stopped before completion. If pre-emption is considered, then a sequence of n integers is no longer sufficient to describe a schedule of n jobs, since individual jobs may appear two or more times in the schedule and one must satisfy how much time is to be allowed at each appearance. Different kinds of pre-emption are possible, depending on the treatment accorded to the interrupted job when it returns to the machine for further work. At one extreme, the processing may resume where it let off without extra work and time being occasioned by the interruption that it may suffer. This is called a pre-empt – resume
discipline. At the other extreme, called pre-empt – repeat, the benefit of any processing that has been done is forfeited with the interruption, so that processing must be repeated when the job returns to the machine. They stated processing time for the job is then a minimum and is achieved only if schedule permits the jobs to be processed without interruption. One very reasonable and important discipline between these extremes involves pre-empt resume operation for the processing time and pre-empt – repeat for the setup time.

1.5.2 (e) Inserted Idle Time:

It is another scheduling tactics in which a machine is held idle although there is work ready and waiting to be done. In flow-shop sequencing problems in which there is only one machine available of each time, the location of the machine is completely fixed by the sequencing decisions of the job only. In case more than one machine of the same type (ie multiple machines) are available for processing a set of jobs the scheduling in journal requires sequencing of the jobs as well as the allocation of the machine for processing the jobs. This journal area of theory is known as parallel machine sequencing.

There are three classes of problems depending on whether the machines (i) identical (ii) uniform (iii) unrelated.

When the machines are identical, the processing time of job I is the same on all machines.

When the machines are uniform, the processing time of job I on one of the parallel machine $M_k$ is $P_i/S_k$, where $P_i$ is constant for job I and $S_k$ is
a speed factor associated with machine $M_i$. When the machines are unrelated, the processing time of the job $I$ varies between parallel machines but in an arbitrary manner.

### 1.5.2 (f) Shortest Processing Time (SPT):

Sequencing the jobs in non-decreasing order of processing time is known as shortest processing time (SPT) sequencing or by shortest operation time and the proof of optimality illustrate a useful technical called the method of adjacent pair-wise interchange.

- Mean Flow-Time ($F$) is minimized by SPT sequencing ($p_1 < p_2 < \ldots < p_n$), where $p_i$ is the processing time of job $i$.

- Weighted Mean Flow-Time ($F_n$) is minimized by WSPT sequencing ($p_1/w_1 < p_2/w_2 < \ldots < p_n/w_n$); where $w_i$ is waiting factor to each job.

### 1.6 Brief Survey of Scheduling Problems

Early industrial engineers and management consultants were busy trying to sort out many aspects of modern manufacturing and production-control. Knoeppel [1915] discussed pulling work through a factory; Gantt [1919] explained how to use visual charts. Coestt [1928] described a mechanical scheduling technique that shared many characteristic of today’s kan ban system and there were, many others, tackling the problem as well. For the past four decades, scheduling has been researched from a mathematical point of view, embodied by the Operation-Research, Operations management and artificial intelligence communities. In these communities, the scheduling is taken as
sequencing i.e. the production system scheduling translates to allocating a set of machines to perform a set of work-orders within a certain time period or in other words scheduling deals with sequencing of operations on several machines.

Johnson [1954] made a successful attempt to find optimal solution using heuristic algorithm for n jobs 2 and 3 machines flow shop problem for given processing time on each machine under several restrictions. Mitten [1959] treated some-what less restrictive cases which was applicable to a wider variety of problems. Wagner [1959], Conway etal [1960] formulated the integer programming models for scheduling problems. The branch and bound technique has been used by Ignall & Scharge [1965]. Gupta [1969] studied economic aspect of scheduling theory.

A number of heuristic approaches are used in general job shop/flow shop scheduling. Moreover through the use of computer program various scheduling procedures and their impact on job-performance have been investigated under relatively homogeneous conditions.
