CHAPTER 1

INTRODUCTION

1.1 Introduction
1.2 Purpose of the thesis work.
1.3 Statement of the problem.
1.4 Stability theorems and stability testing.
1.5 Scope of the thesis work.
1.1 INTRODUCTION:

Stability is the most essential property possessed by all types of physical systems and investigation of it plays an important role in the fields such as computer tomography, image processing, biomedical engineering, geophysics, remote sensing and robotics; the mathematical representation of these types of systems may be represented either in one-dimensional (1-D) or two-dimensional (2-D) or multi-dimensional (N-D) digital or continuous forms [1-11].

Testing for the stability is an important task in the design of two-dimensional and multi-dimensional digital systems [6]. Stability itself has been defined in different ways. Of these various definitions, structural stability and bounded-input bounded-output (BIBO) stability are the most commonly used in design and implementation of 2-D digital systems [12]. Hence this thesis confines itself to ascertaining the BIBO stability of digital systems.

Problems related to the stability testing of polynomials arise in many engineering applications. Historically, such applications were first introduced by Ansell [13].

The stability property of two-dimensional system are many times more complicated than those of the one dimensional systems. This is mainly due to the fact that the fundamental theorem of Algebra does not extend to 2-D polynomials [4].

The stability of multi-dimensional systems arises in many applications (Foremost among is the stability of 2-D and N-D digital filters). These filters find widespread applications in many fields [8].

In classical stability theory of linear systems, the problem is to determine the location of zeros of a real or complex polynomial relative to the half-plane or the unit circle. A large number of solutions to the stability problem as a set of algebraic inequalities involving only polynomial coefficients
is available, as variations of the Hermite and Routh-Hurwitz criteria for the half-plane, and Schur-Cohn and Marden-Jury criteria for the unit circle [14,15].

But the problem of root distribution of polynomial with respect to the unit circle in the Z-domain has continued to receive much attention.

Similarly, the stability of a discrete system has been investigated quite thoroughly and numerous methods for testing the stability of such systems are well established.

Among the available methods, in the algebraic stability testings, actual root finding is avoided and only algebraic inequalities are deduced to ascertain the system stability.

1.2 PURPOSE OF THE THESIS WORK:

(1) To develop algebraic stability testing methods for 2-D Linear Time-invariant systems using 1-D Routh's test and Hurwitz determinants [1].

(2) To develop algebraic stability testing methods for 2-D and N-D Linear Time-invariant systems based on 1-D Mikhailov graphical stability criterion, developed by Mikhailov [16-18].

(3) To develop algebraic stability testing methods for 2-D and N-D Linear Time-invariant systems based on 2-D graphical method, due to Decarlo et. al. [19].

(4) To develop algebraic stability testing methods for 2-D and N-D Linear Time-invariant systems based on 1-D graphical method due to Pontryagin [20].

(5) To develop a scheme for stabilization of a class of 2-D digital system employing Pontryagin's results.
1.3 STATEMENT OF THE PROBLEM:

TWO-DIMENSIONAL SYSTEMS

2-D digital system can be represented by its rational transfer function

\[ H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} \]

where \( A(z_1, z_2), B(z_1, z_2) \) are co-prime polynomials.

Since the numerator polynomial does not affect the stability of the system, except the rare case of non-essential singularities of the second kind [6], the denominator polynomial \( B(z_1, z_2) \) or its characteristic equation is considered for stability analysis.

For the system to be stable, the denominator polynomial should not have any zeros inside the unit circle, i.e., the polynomial

\[ B(z_1, z_2) \neq 0, |z_1| \leq 1, |z_2| \leq 1 \]

To test a two-dimensional polynomial \( B(z_1, z_2) \neq 0, |z_1| \leq 1 \) and \( |z_2| \leq 1 \), the necessary and sufficient conditions for stability as proposed by Huang [4] and applied by Shanks et al [21] are,

(i) \( B(z_1, 0) \neq 0, |z_1| \leq 1 \)

and

(ii) \( B(z_1, z_2) \neq 0, |z_1| = 1 \) and \( |z_2| \leq 1 \)
Equivalently, the above conditions can also be written as under:

(i) \( B(0, 1/z_2) = B_1(z_2) \neq 0 \) for \( |z_2| \geq 1 \) and

(ii) \( B(z_1, 1/z_2) = B_1(z_1, z_2) \neq 0, |z_1| = 1 \) and \( |z_2| \geq 1 \)

where \( B_1(\ldots) \) is reciprocal polynomial.

**MULTI-DIMENSIONAL SYSTEMS**

Anderson and Jury [22] suggested a set of stability conditions to be checked for multi-dimensional digital systems. Those checking procedures imply the simultaneous satisfactions of (N-1) necessary conditions and one sufficiency condition as given below:

**(N-1) Necessary Conditions :**

(i) \( B(z_1, z_2, z_3, \ldots, z_{N-1}, 0) \neq 0, \left\{ \bigcap_{i=1}^{N-2} |z_i| = 1 \right\} \cap \left\{ |z_{N-1}| \leq 1 \right\} \)

(ii) \( B(z_1, z_2, z_3, \ldots, z_{N-2}, 0, 0) \neq 0, \left\{ \bigcap_{i=1}^{N-3} |z_i| = 1 \right\} \cap \left\{ |z_{N-2}| \leq 1 \right\} \)

\[ \vdots \]

\[ \vdots \]

(N-2) \( B(z_1, z_2, 0, 0, \ldots, 0) \neq 0, \left\{ |z_1| = 1 \right\} \cap \left\{ |z_2| \leq 1 \right\} \)

(N-1) \( B(z_1, 0, 0, \ldots, 0) \neq 0, |z_1| \leq 1 \)

**Sufficiency Condition :**

\( B(z_1, z_2, z_3, \ldots, z_N) \neq 0, \left\{ \bigcap_{i=1}^{N-1} |z_i| = 1 \right\} \cap \left\{ |z_N| \leq 1 \right\} \)

All the above necessary and sufficient conditions given by the respective equations can be translated in such a way that the roots of the reciprocal polynomials are constrained to be within the polydisk for stability.
For the characteristic polynomial given by the equation the corresponding reciprocal polynomial is written as

$$B(z, z_2, z_3, ..., z_N) \neq 0, \left\{ \bigcap_{i=1}^{N-1} |z_i| = 1 \right\} \cap \{z_N \geq 1\}$$

In a similar way the remaining polynomials defined for necessary conditions can be translated to respective reciprocal polynomials.

Equivalently, all the above conditions can also be written using reciprocal variables viz.,

$$1/z_1, 1/z_2, ..., 1/z_{N-1}$$

1.4 STABILITY THEOREMS AND STABILITY TESTING

1.4.1 Stability of system [6]

A system is stable if its output is well bounded for all bounded input. Any transient behaviour due to abrupt changes in the input should be localized and the steady state behaviour of the systems should be practicable. To make this concept more normal, researchers have defined different types of stability; but so far as linear time invariant systems are concerned, all these essentially bound the impulse response in some fashion. The most extensively studied stability criterion is the BIBO criterion.

A Linear time invariant discrete system will be stable if and only if its impulse response is absolutely summable. To check this for stability test, an infinite sum must be evaluated, and the impulse response will also be available. Moreover, system design algorithms provide either the coefficients of a difference equation or a transfer function. So, it would be able to determine the stability of a system directly from transfer function. Numerous methods are available for testing the stability of 1-D digital systems without resorting to the exact computation of actual roots [23-34].
1.4.2 Stability Theorems for 2-D systems and N-D systems:

If a 2-D sequence is summable, its z-transform will be analytic on the unit surface $|z_1| = |z_2| = 1$. In other words, if the z-transform of a sequence is analytic on the unit surface, then the sequence is absolutely summable, and the system is stable. For the practical case of rational transfer function

$$H(z_1,z_2) = \frac{A(z_1,z_2)}{B(z_1,z_2)}$$

guaranteeing the analyticity of $H$ for $|z_1| = |z_2| = 1$ is equivalent to ensuring $B(z_1,z_2) \neq 0$ for $|z_1| = |z_2| = 1$.

The stability of one-dimensional system is related to the location of its poles. In the multi-dimensional system, the stability is likewise related to the zero set of the denominator polynomial $B(z_1,z_2)$.

The earliest stability theorems, which is due to Shanks [21], extend the idea of examining pole locations to the 2-D systems. Shanks theorem states that the system represented by its transfer function

$$H(z_1,z_2) = 1 / B(z_1,z_2)$$

will be stable if and only if $B(z_1,z_2) \neq 0$ for every point $(z_1,z_2)$ such that $|z_1| \geq 1$ and $|z_2| \geq 1$. It requires that the whole exterior of the unit bicircle be searched for points of singularity. An equivalent result was implied by Shanks, stating that the system will be stable if and only if the following conditions are true:

(i) $B(z_1,z_2) \neq 0, \quad |z_1| \geq 1, \quad |z_2| = 1$

and

(ii) $B(z_1,z_2) \neq 0, \quad |z_1| = 1, \quad |z_2| \geq 1$
A similar theorem was stated by Huang [35] also. Huang's theorem states that if and only if $B(z_1, z_2)$ satisfies the following two conditions for stability

(i) $B(z_1, z_2) \neq 0, \quad |z_1| \geq 1, \quad |z_2| = 1$

and

(ii) $B(a, z_2) \neq 0, \quad |z_2| \geq 1$ for any $a$ such that $|a| \geq 1$

The second condition of Huang's theorem is a 1-D stability condition; the first condition is 2-D, but is confined to its unit circle. The role of $z_1, z_2$ can be interchanged. DeCarlo et. al. [36] and Strintzis [37] independently showed that Huang's test could be simplified as written below:

(i) $B(z_1, z_2) \neq 0, \quad |z_1| \geq 1, \quad |z_2| = 1$

(ii) $B(a, z_2) \neq 0, \quad |z_2| \geq 1$ for any $a$ such that $|a| \geq 1$

(iii) $B(z_1, b) \neq 0, \quad |z_1| \geq 1$ for any $b$ such that $|b| \geq 1$

Condition (ii) and (iii) are corresponding to 1-D stability condition and (i) is a 2-D condition.

Anderson and Jury [22] generalised the Huang's stability theorem to the multi dimensional systems and gave $N-1$ necessary conditions and one sufficiency condition as follows:

**Necessary conditions (N-1):**

(i) $B(z_1, z_2, z_3, ..., z_{N-1}, 0) \neq 0, \quad \{ \bigcap_{i=1}^{N-2} |z_i| = 1 \} \cap \{ |z_{N-1}| \leq 1 \}$

(ii) $B(z_1, z_2, z_3, ..., z_{N-1}, 0, 0) \neq 0, \quad \{ \bigcap_{i=1}^{N-3} |z_i| = 1 \} \cap \{ |z_{N-1}| \leq 1 \}$

... 

(N-2) $B(z_1, z_2, 0, 0, ..., 0) \neq 0, \quad \{ |z_1| = 1 \} \cap \{ |z_2| \leq 1 \}$

(N-1) $B(z_1, 0, 0, ..., 0) \neq 0, \quad |z_1| \leq 1$
1.4.3 STABILITY TESTING

Several Contributions [35-61] have been made on stability tests for 2-D and N-D system based on the classical tests proposed by Huang for the transfer function

\[ H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} \]

devoid of non-essential singularities of the second kind. To test a 2-D polynomial

\[ B(z_1, z_2) \neq 0, |z_1| \leq 1 \text{ and } |z_2| \leq 1, \]

the necessary and sufficient conditions for stability as proposed by Huang are

(i) \( B(z_1, 0) \neq 0, |z_1| \leq 1 \) and

(ii) \( B(z_1, z_2) \neq 0, |z_1| = 1 \text{ and } |z_2| \leq 1 \)

Siljak [15] tested the first condition using Marden's table. For testing the second condition, he applied Marden's test along with Schur-Cohn matrix. These results are simpler than Anderson and Jury, and Maria and Fahmy[38]. In this method, only determinant of the Schur-Cohn matrix needs to be tested for positivity on unit circle, instead of all the leading principal minors of the Schur-Cohn matrix, as proposed by Maria and Fahmy.

Methods described by E.I. Jury, have been suggested by N.K. Bose to test the first condition and Peter Steffen[28] described a new algorithm for testing the stability of discrete systems using methods exclusively in z-Domain. Moreover Bistritz presented a stability test for linear discrete
systems in the table form [31] for testing 1-D condition.

M.N.S. Swamy et al. [41-43] modified Anderson and Jury's test and suggested an alternative stability test based on transformation variable for testing the condition (iii).

Karan and Srivatsava [44] presented a method for testing the conditions (i) and (ii) of Huang. The test for the first condition follows the method given by Bistritz. To test the second condition they presented another method simpler to the method proposed by Siljak and N.K.Bose.

P.Karivaratha Rajan and H.C. Reddy [12] described a test for a 2-D polynomial to be a Discrete Scattering Hurwitz polynomial employing Schur-Cohn matrix associated with the necessary and sufficient condition proposed by Basu and Fettweis.

X.Hu and T.S. Ng [45,46] presented a method to test the 2-D characteristic polynomial. In this method real polynomial has to be formed first, then appropriate polynomial array has to be constructed subsequently. They also developed and extended array techniques for polynomials with literal coefficients removing the conjugate polynomial.

M.Wang et al [47] proposed a numerical algorithm for testing the stability of the two-dimensional digital systems and estimating their marginal stability.

P.Agathoklis et al [48] presented algebraic necessary and sufficient conditions for the stability analysis of 2-D systems based on the frequency dependent formulation of the Lyapunov equation using Kronecker products.

Anderson and Jury proposed a generalised stability test to the N-D digital system. DeCarlo et al.[31] and Strintzis [32] proposed a similar test to analyse the stability of N-D digital systems.

In all the above methods, the characteristic polynomial is taken for
analysis of a 2-D and N-D digital systems to perform the test. Only few algebraic stability tests are available.

Reviewing the above methods, the main objective of the thesis work is to formulate certain new generalised algebraic stability criteria, by suitably interpreting the graphical criterion due to Mikhailov, De Carlo et al and Pontryagin.

1.5 Scope of the thesis work

In chapter 2, the well known Routh Stability Test is extended suitably for 2-D digital system. Further using the theorems given by Karivaratha Rajan & Reddy [14] the singularity situations are also analysed employing the proposed scheme. Also, the stability analysis for 2-D digital system is carried out by suitably adopting Hurwitz Determinants. The schemes are applied to certain illustrations.

The Mikhailov's graphical stability criterion for 1-D system is suitably interpreted and deduced two new algebraic criteria for 1-D digital system and extended to 2-D digital systems in chapter 3. These two newly deduced algebraic schemes are also extended to N-D in chapter 6. These criteria are applied to typical examples.

In chapter 4, the 2-D graphical stability criterion proposed by Decarlo et al is suitably interpreted and two new algebraic stability criteria are formulated and termed as RPSC and IPSC. The two schemes are extended to N-D in Chapter 7. For Illustrations, examples are provided.

The graphical stability criterion developed for delay differential system by Pontryagin is suitably extended for 2-D digital system. Further, this new algebraic test is also extended to N-D digital systems in chapter 8.

Further, a new scheme is suggested for stabilisation of certain class of 2-D digital system with the help of Pontryagin's inequality results along with a magnitude error criterion in chapter 9. Summary of the thesis work and further suggestions are given in chapter 10.