EXTENSION OF SOLUTIONS TO SOME NONLINEAR DIFFERENTIAL EQUATIONS

ABSTRACT

1. INTRODUCTION :

Differential equations are necessary in scientific modeling of physical problems, which find relevance in almost every sphere of human endeavor from Agricultural Sciences, Engineering, Medical Sciences, Physical Sciences to Social Sciences.

The subject ‘Differential Equations’ is a well established part of mathematics and its systematic development goes back to the early days of the development of Calculus.

Many recent advances in mathematics, paralleled by a renewed and flourishing interaction between mathematics, the sciences, and engineering, have again shown that many phenomena in the applied sciences, modeled by differential equations will yield some mathematical explanation of these phenomena.

For finding exact solutions of nonlinear Initial Value Problems (IVPs) is a goal for mathematicians, engineers, and scientists, and it plays an important role in real world applications. In recent years, first and second order nonlinear IVPs were considered by many authors. For instance, Adomian Decomposition Method (ADM) is used to solve nonlinear differential equations such as Dung-Vanderpole equation, solved nonlinear IVPs by the Laplace Adomian decomposition method (LADM), obtained approximate solutions by the method of differential transforms (DTM), and the variation iteration methods (VIM) were used by many authors.

2. OBJECTIVES :

The main objectives are carried out as follows :

1. To study the existence and uniqueness of solution of nonlinear differential equations and its extension of solutions.
2. To discuss nonlinear ordinary differential equations for their different behavior of the solutions.

3. To analyze some applications of nonlinear ordinary differential equations studied in the present work to some concrete problem of the other areas of mathematics.

4. To develop the method for existence and uniqueness of solutions by using fixed points theorem.

5. To study the existence and uniqueness of solutions of nonlinear ordinary differential equations in various aspects.

3. BRIEF DESCRIPTION OF THE THESIS:

The thesis entitled “EXTENSION OF SOLUTIONS TO SOME NONLINEAR DIFFERENTIAL EQUATIONS” has five chapters and each chapter is divided into sections and subsections. The chapter wise scheme of the thesis is as follows.

· First Chapter is introductory in nature and deals with preliminaries and some basic terminologies of the probabilistic nonlinear analysis and the different orders of nonlinear ordinary differential equations with initial conditions.

· Second chapter explains an extension of solutions of examples of first order nonlinear ordinary differential equations. The solutions are obtained by using explicit and implicit formulas for the solution of various ordinary differential equations and by using basic existence and uniqueness theorem.

· Third Chapter focuses on the initial value problem of second order nonlinear ordinary differential equation. The existence theorem is proved by using Lipchitz condition and Weierstrass M-test. By using uniqueness theorem the solution of the given problems is extended.

· Fourth Chapter discusses the initial value problem of third order nonlinear ordinary differential equation. The Picard’s existence and uniqueness theorem is proved by using the Banach Fixed Point Theorem.
The initial value problem of fourth order nonlinear ordinary differential equation is discussed in Fifth Chapter. The Picard’s existence and uniqueness theorem is proved by using the Banach Fixed Point Theorem.

Chapter –II: It is studied that the first order nonlinear ordinary differential equations with initial conditions. It discusses an extension of solutions of examples of nonlinear ordinary differential equation. The solutions are obtained by using explicit and implicit formulas for the solution of various ordinary differential equations and by using basic existence and uniqueness theorem.

Let $x_0, y_0$ be real number in the domain of $f(x, y)$. Suppose that there exist positive numbers $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that $f(x, y)$ and $f_y(x, y)$ are both continuous on the rectangle $\{(x, y) : |x - x_0| < \varepsilon_1 \text{ and } |y - y_0| < \varepsilon_2 \}$.

Consider the first order nonlinear ordinary differential equation:

\[
\begin{align*}
y' &= f(x, y) \\
y(x_0) &= y_0
\end{align*}
\]

(1)

on the interval $|x - x_0| < \delta$, where $\delta > 0$.

And some examples are discussed.

Chapter - III: It discusses the initial value problem of second order nonlinear ordinary differential equation.

The existence theorem is proved by using Lipchitz condition and Weierstrass M-test.

Weierstrass M-test:

Let $\{f_n\}$ be a sequence of functions defined on a set $E$. Suppose that for all $n \in \mathbb{N}$, there exists $M_n \in \mathbb{R}$ such that

$|f_n(x)| \leq M_n$ for all $x \in E$

Then if $\Sigma M_n$ converges, $\Sigma f_n$ must converge uniformly on $E$.

By using uniqueness theorem, the solutions of the given problems are extended.
Chapter – IV : This Chapter discusses the initial value problem of third order nonlinear ordinary differential equation.

In Ordinary Differential Equations, Picard’s existence and uniqueness theorem is one of the most important theorems. Because it can be generalized to established existence and uniqueness results for higher order ordinary differential equations.

The Picard’s existence and uniqueness theorem is proved by using the Banach Fixed Point Theorem for operators.

**Banach Fixed Point Theorem for operators :**

Let S denotes the set of continuous functions on \([a,b]\) that lie within a fixed distance \(\alpha > 0\) of a given function \(y'(x) \in C[a,b]\),

i.e. \(S = \{y \in C[a,b] : ||y - y'|| \leq \alpha \}\).

Let \(G\) be an operator mapping \(S\) into \(S\) and suppose that \(G\) is a contraction on \(S\),

i.e. there exists \(k \in \mathbb{R}, 0 \leq k < 1\) s.t. \(||G[\omega] - G[z]|| \leq k||\omega - z||\) for all \(\omega, z \in S\).

Then the operator \(G\) has a unique fixed point solution in \(S\).

Finally some examples are discussed.

Chapter – V : In this chapter the initial value problem of fourth order nonlinear ordinary differential equation are discussed.

The Picard’s existence and uniqueness theorem is proved by using the Banach Fixed Point Theorem.

The solution of the fourth order IVP with initial conditions Picard’s iteration for IVP method is used.

Finally some examples are discussed.

**Conclusion :**

In this work, an efficient simple method is presented for solving different orders of nonlinear ordinary differential equations. Some examples are given to show the simplicity and easiest way for computation.

By using different conditions, formulas, and theorems, the computations associated with the examples are performed. The solutions of nonlinear ordinary differential equations are extended by using existence and uniqueness.