2. SURVEY OF LITERATURE

2.1. GENERAL

Shell structures have come to wide use in the twentieth century. Although the Pantheon of Rome, built in the year A.D. 1 and the Mosque of Santa Sophia in Istanbul, built in the year A.D. 538 feature shell roofs spanning large distances, they are relatively thick in cross section (16). It was not until the 1920's that thin shell roof emerged as a practical means for spanning large distances.

Analytical tools for thin shell structures were first developed much earlier, over a century ago. G. Lame and E. Clapeyron established the fundamental theory for shell membrane action in 1828 (11). Aron considered bending behaviour in 1874. The problem was more accurately analysed in the late nineteenth century by G.B. Airy and A.E.H. Love, who developed both bending and membrane theory for the analysis of shells (17). Subsequent theoretical efforts have been directed towards improvements of Love's formulation and the solution associated with differential equations. Such solutions were initially hard-won. Analytical formulations have followed rapidly in the interim period and papers dealing with shell analysis number in thousands.

On the experimental side, the first attempt to design a shell roof proper seems to have been due to Dischinger in 1923 (17). The analysis apparently failed because of difficulties in the mathematical computations involved. This was followed by a
series of experimental shells, on all of which extensive loading tests were carried out. More recently, the works of Felix Candella have attracted considerable attention (17).

Available analytical solutions to thin shell structural problems are nevertheless limited in scope and in general do not apply to arbitrary shapes, load conditions, irregular stiffening and support conditions, cutouts and many other aspects of practical design. The finite element method has consequently come to the fore as an approach to thin shell structural analysis because of its facility to deal with these complications. Like shell structures themselves, the finite element method has come to fruition in the twentieth century.

2.2. THE EVOLUTION OF FINITE ELEMENT THIN SHELL ANALYSIS

The finite element method in thin shell structural analysis began with applications to aerospace vehicles (1). The plate elements comprise a 'faceted' representation of the aircraft shell surface. The formative years of the finite element method (1950-1960) were occupied in a large measure, with the continual refinement of such idealizations.

The early 1960's saw the shift from aircraft to space vehicles in the aerospace industry and with it a change to highly curved unstiffened thin shell structures. R.H. Gallagher (16) undertook the development of the simplest form of shell element, a circular cylindrical segment in 1966. Displacement fields which had previously been used for flat plate elements were adopted. Subsequent contributors to the literature demonstrated
that such an approach is inadequate for realisation of an acceptable level of accuracy in solution.

By 1969 a larger number of curved thin shell finite elements had appeared in the literature. These encompass a large proportion of the ideas which still hold promise for acceptable formulations.

Bogner, Fox and Schmidt (13) described a cylindrical shell element, which used interpolation functions defined in shell co-ordinates. Nodal values involved the three displacements and derivative of these displacements with respect to local coordinates resulting in an element with twelve degrees of freedom for each of the four nodes. Utku (13) had represented a shallow curved triangular shell element with three displacements and two rotations at each node. During the 1970's there has been considerable activity in the design of nuclear reactor structures. These are generally solid or thick-walled and the use of three dimensional (3-D) elements for the purpose of F.E.M. has been widespread (18).

2.3. DEGENERATED SHELL ELEMENTS (14)

In using 3-D solid isoparametric elements to model thick shells, the dimension of the element in the thickness direction will be smaller than the other two dimensions. We may have to use a larger number of element layers in the thickness direction to capture adequately the strain and stress variation in this direction. Though the idea looks simple there are two main difficulties in such an analysis as listed below:
i) Since the thickness of the element is small as compared to the other two dimensions, the stiffness coefficients for displacements in this direction may become numerically very large. This introduces numerical difficulties in the solution of the global stiffness equation.

ii) The total number of degrees of freedom required for an adequate analysis may become too large, thus making the analysis too expensive.

However ways have been devised to overcome the above difficulties. The contributions of Ahmed et.al (14) have introduced certain modifications in the 3-D analysis, so as to make the resulting specialised theory useful in analysing a wide range of shell structures. In order to overcome the first difficulty, the strain energy corresponding to stresses perpendicular to the middle surface is neglected. The second difficulty is minimised by introducing the assumption that straight lines normal to the middle surface before deformation remain straight after deformation. This assumption is valid even for thick shells. Since only straightness and not normality to the middle surface is required, shear deformation is permitted which is important in thick shell situations. In using this element for thin shell situations, reduced integration of shear energy term is necessary.
2.4. EFFECTS OF EDGE BEAMS ON CYLINDRICAL SHELLS (19,17)

Edge beams are a necessity for "long" barrel shells. Acting as supports to the shell in the longitudinal direction, they stiffen the structure throughout its length. Being monolithically connected to the shell they allow transverse moments to develop at the junction and thereby help relieve the shell of flexure to a great extent. By deepening the cross-section, they help distribute the longitudinal compressive forces over a wider area and also accommodate the longitudinal steel which is required to resist the longitudinal tension.

The stresses in the shell, especially the flexural perturbations originating from the longitudinal edges as well as the forces and moments in the edge beam itself, are controlled by the structural properties of the edge beam, namely, the extensional, flexural and torsional rigidities which are functions of edge beam dimensions.

The introduction of edge beams normally reduce the value of \( M_\theta \) and \( N_x \) at the crown. It also reduces the bending moment \( M_x \), which, however, is not critical in shells without edge beams.

The longitudinal force \( N_x \) in the body of the shell is mostly compressive and the edge beam will have to be deep enough to keep this stress within limits.

In the case of rectangular edge beams of the downstand type, depth is known to be a far more significant dimension than the width of the edge beam (19). A width of two to three times the thickness of the shell would usually suffice (8).