CHAPTER 2

THERMAL ANALYSIS

2.1 INTRODUCTION

Whenever an electrical or electronic circuit is designed, it creates a thermal system as well. In most of the cases, as long as the power output is small, thermal system can be neglected. But when the power handling capabilities of the devices are increased with high packaging densities, thermal system plays a major role. Reliability of a system then depends totally upon the heat dissipation techniques employed. Thermal techniques and design for thermal testability are ideas now accepted by the electrical testing community as part of the overall testing concept. The exact knowledge about the heat sinking properties of the semiconductor device mounts is very useful from the point of determining the limits of static or dynamic power load and developing good mount designs. Measuring junction to ambient or junction to case static thermal resistance is a conventional method but gives only a single value rating without more information about the heat flow details.

2.2 PRINCIPLE OF FINE STRUCTURE OF HEAT FLOW PATH

In 1988, Vladimir Szekely and Tran Van Bien [15] have developed a method to identify the thermal environment of a semiconductor device chip. The identification algorithm operates on the thermal transient response of the device recorded during one short pulse measurement. A deconvolution operation performed in the logarithmic time domain gives the “time-constant spectrum” of the chip case ambient thermal structure. A further transformation leads to the “structure-function” i.e. the cross-sectional area of the heat conducting material vs thermal resistance (related to the heat source). The structure function has good and quantitatively evolvable correspondence to the physical chip environment and heat conducting structure. Separating the different regions of the heat flow path (corresponding to the chip, bond, header, case) as well as the detection of eventual heat – transport irregularities (mounting errors) is possible.

The measured function a (t) is the step function response of the thermal one-port represented by the device and its mount. The device is excited by a step of the dissipating
power and its temperature rise function is measured. One of the simplest $a(t)$ response function is that of the single time constant system. It has mathematical form of $(1-e^{-t/\tau})$.

The response of more complex thermal structures can be considered as the sum of many such individual exponential terms with different $\tau$ (time constants) and different magnitudes. Thus it is possible to characterize a thermal system by the (discrete or
continuous) distribution of time constants occurring in its response and by the related magnitudes.

Let’s introduce the logarithmic time variable as;

\[ z = \ln t \]  
\[ (2.1) \]

and the logarithmic time-constant distribution of the response

\[ R(z) = \lim_{\Delta z \to 0} \frac{\text{magnitude related to the time constants between } z \text{ and } z + \Delta z}{\Delta z} \]

Now the a(t) response can be expressed as:

\[ a(t) = \int_{-\infty}^{\infty} R(\zeta)(1 - e^{-t/(1+\zeta)})d\zeta \]  
\[ (2.2) \]

or using the logarithmic time variable as:

\[ a(z) = \int_{-\infty}^{\infty} R(\zeta)(1 - e^{-z/(1+\zeta)})d\zeta \]  
\[ (2.3) \]

This is a convolution-type differential equation for the unknown R(\zeta) function. Differentiating both sides with respect to z we obtain:

\[ \frac{d}{dz}a(z) = R(z) \otimes W(z) \]

where \( W(z) = \exp [z - \exp(z)] \)  
\[ (2.4) \]

and \( \otimes \) is the symbol of the convolution operation. Transforming the response function to the logarithmic time variable, differentiating it and finally deconvolving it by the fixed function equation (2.4) gives the R(z) time-constant spectrum of the investigated thermal one-port. The Foster and the Cauer equivalent are shown in Fig. 2.1 (a) and (b).

2.3 Trait Method

In 1998, Paolo Emilio Bagnoli et al. in their paper [16] carried out careful theoretical analyses of the thermal dynamics of the electronic devices and its package. The device temperature evaluation in time is ruled by an infinite and consequent series of time constant. The knowledge of the first n terms of the time constant spectrum obtained from the temperature transient measurement allows the complete characteristics of a suitable and reliable equivalent thermal circuit structured as a Cauer low-pass network with n cells. The total thermal resistance is therefore evaluated as function of several contributions due to given parts of whole spectrum. This is the one dimensional approach.
TRAIT is expanded as Thermal Resistance Analysis by Induced Transient for thermal resistance and capacitance evolution. Two important features of TRAIT method are spatial resolution and dynamic characterization. These features can be achieved by taking into account the evolution in time of the device internal temperature after a power dissipation step variation. In fact, the transient behavior is generally ruled by a set of theoretically infinite exponential time constants which can be in any way related to the thermal properties of the various physical parts of the system. Therefore, an accurate equivalent thermal circuit composed by low-pass RC cells can be achieved.

2.3.1 Theoretical Background

For a homogeneous and isotropic medium, heat conduction equation can be written as follows:

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{2.5} \]

where \( T \) is the temperature, \( t \) the time, and \( \alpha \) the thermal diffusivity defined as

\[ \alpha = \frac{k}{\rho C_p} \tag{2.6} \]

in which the constant parameters \( k, C_p \) and \( \rho \) are the thermal conductivity, the specific heat at constant pressure, and material density, respectively.

Kadambi and Abuaf [17] analyzed the steady-state and transient conduction of simple homogeneous solids as cylinders and parallelepipeds with convective heat-transfer condition at the bottom surface, adiabatic lateral sides, and with heat source on the top surface. It is particularly meaningful, the case in which the power chip package is modeled by a homogeneous cylinder with base radius \( r_b \), thickness \( d \), heat-transfer coefficient \( h \) of the bottom surface and bearing at the top surface a circular heat source centered at the vertical axis with circle area radius \( r_c < r_b \), and uniform heat flux \( q \). The aforementioned authors, assuming a switch on, of the dissipating thermal power, examined the subsequent transient response of this body and reported the temperature of the heat-source center as a time function. On the basis of these results, for the same system the thermal corresponding to the power switch-off case is obtained as
The expression (2.7) can sometimes evolve toward a single series of exponential functions, and consequently, the system becomes thermally 1-D. This occurs, for instance, when \( r_c \rightarrow r_b \), or the heat-source area is approaching the whole top surface, so
that according to (2.8) all the terms of the series contained in the square brackets of (2.7) become negligible at any time. Another possible case is when the radius of the cylinder is smaller than its thickness. Under such condition, the first term of the \( V_n \) sequence which is greater than \( \lambda_1 \) generally occurs at an index \( n^* \gg 1 \), and, therefore, all the series within the square brackets of (2.7), corresponding to index \( n < n^* \), are practically negligible with respect to the ratio \( r_c / r_b \).

Therefore, in the above cases the expression of the switch-off transient response of the heat source can be expressed as

\[
T(t) = T_0 + q_0 \cdot \sum_{i=1}^{\infty} A_i \cdot \exp \left( \frac{-1}{\tau_i} \right), \quad \text{with} \quad \tau_{i+1} < \tau_i \quad \forall_i \quad (2.9)
\]

Where \( q_0 \) is the total dissipating thermal power, \( \tau_i \) are the time constants related to the Eigen value sequence \( V_n \) of (2.7), and corresponding amplitude factors. From a general point of view, under the assumption of 1-D conduction, an equation similar to (5a) can be applied to the analysis of any spatially limited solid with a heat source localized at a certain abscissa [19], The temperature of any point of this solid is given by

\[
T(x, t) = T_0 + q_0 \cdot \sum_{i=1}^{\infty} A_i(x) \cdot \exp \left( \frac{-t}{\tau_i} \right), \quad \text{with} \quad \tau_{i+1} < \tau_i \quad \forall_i \quad (2.10)
\]

More precisely, the time constants are the natural characteristic parameters which rule the temperature dynamic behavior of whatever point of the system and do not depend on the point, or better on the point abscissa, while on the contrary the amplitude factors \( A_i \) are a function of the point, depending on its position along the system. It must be pointed out that in the case of the 1-D solid, the values of \( \tau_i \) and \( A_i \) can be considered as the lumped "time-constant spectrum" which is composed by an infinite number of lines whose values are determined by the boundary conditions and by the thermal properties of the whole system, being both homogeneous or multilayered. The particular forms [(2.2a) and (2.2b)] of the temperature transient are suitable for applying the TRAIT method for the thermal resistance measurement exposed in the next section.

### 2.3.2 Thermal Transient Approximation

This section deals with the method for obtaining the knowledge of the total thermal impedance and its components from the analysis of the temperature transient response in the 1-D case. Fig. 2.2.a shows a schematic view of a thermally 1-D solid
composed by several layers of the different materials, which has only one heat source located at X=0 and one heat sink. Fig. 2.2.b shows the temperature transient when a constant power is switched off and calculated for many positions along the distance axis. All the curves in the figure have the same analytical expression $A_i$ are all positive only in the case of $x=0$ (i.e., the heat-source position) in which the curve has everywhere positive concavity. In this case, the series of the coefficients $A_i(0)$, which is the whole temperature variation of the transient, converges to a finite value and which can be related to total thermal resistance $R_{th}$ of the system. Therefore, the following two relations hold

$$R_{th} = \frac{T(x=0, t=0) - T_0}{q_0}$$

$$R_{th} = \sum_{i=1}^{\infty} A_i(0)$$

Where $T(0,0)$ is the steady-state temperature of the heat source before the power is switched off and $T_0$ is the heat-sink temperature. From a dynamic point of view it can be observed that as the order $i$ increases and the time constant $\tau_i$ becomes progressively smaller (the exponential function decays progressively faster), the corresponding factor $A_i(0)$ rapidly decreases in amplitude because of the convergence of the series of nonnegative elements. Despite the fact that the function $T(0,t)$ is the sum of an infinite number of exponential functions, the above statements imply that it can be described in an approximated way only by taking into account the first $n$ terms of the series, so that the following relationship stays valid:

$$T(0,t) \approx T_0 + q_0 \sum_{i=1}^{n} A_i(0) \exp \left( \frac{-t}{\tau_i} \right)$$

(2.12)

The error due to the approximation, which is a decreasing function of the parameter, can be described as a function of time as

$$\text{Err}(t) = q_0 \sum_{i=n}^{\infty} A_i(0) \exp \left( \frac{-t}{\tau_i} \right) \leq q_0 \sum_{i=n}^{\infty} A_i(0)$$

(2.13)

2.3.3 Equivalent Thermal Circuit

A solid system can usually be described from the thermal point of view by means of a discrete element electrical circuit, composed by resistances and capacitances, in
which the temperatures and the thermal powers are retained as voltages and currents, respectively. Therefore, the transient behavior of the heat-source of the heat-source temperature $T(0,t)$ expressed by (2.12) can be recognized as the response of the equivalent thermal circuit to an internal power stimulus, and the time constants $\tau_i$ are nothing else than the poles of the circuit. The two alternative equivalent thermal circuits which can represent the above-described system are shown in Fig. 2.3 a, b and c shows the equivalent detail of a top commercial device. In accordance with the labels used by Szekely [20], [21], they can be addressed as the Foster network and the Cauer network [22]. The current generator $T_0$ corresponds to the thermal power in the heat source, and the ideal voltage generator represent the heat sink. The switch operates the step variation and the electrical ground is the environmental temperature.

Both the circuits represent a solid which is thermally one directional: this means that when there is only one heat source and one heat sink, the thermal energy from the heat source flows to the heat sink through a single path both in static and dynamic regimes. As far as the electronic systems are concerned, this property is practically satisfied when the thermal losses toward the environment by radiation or convection through the lateral walls or the top surface of the packaging are negligible in comparison with the 1-D heat flow due to the conduction mechanism.

![Diagram of Foster network](image)

a) Foster network

![Diagram of Cauer network](image)

b) Cauer network
According to the above considerations, if the temperature cooling transient can be approximated by decreasing exponential functions, it implies that the system can be represented by a circuit composed by \( n \) RC cells. In the Foster network, each time constant is simply equal to the product of the resistance and the capacitance of the cell, while in the Cauer, the values of the time constants cannot be directly calculated since they generally depend on all the resistances and the capacitances of the circuit.

It must be pointed out that the circuit form which is suitable for representing the system from the physical point of view is just the Cauer one. The passive elements of the circuit, thermal resistances, and the capacitances have a real physical meaning: each RC cell represents the contribution to the thermal impedance of a given part of the whole solid which can be addressed as “thermal domain”. In particular, the value of each resistance \( R_i \) is generally determined by the material thermal conductivity of the solid part and by its size. While the capacitances \( C_i \) is related to the mass and to the specific heat of the sector.

2.3.4 Circuit Parameter Calculation

The parameters of the Cauer equivalent thermal circuit can be calculated by applying the following procedure. Once the temperature transient of the heat-source cooling has been experimentally recorded, the curve must be numerically analyzed by using a multi exponential fitting in order to calculate the time constants \( \tau_i \) and the corresponding pre exponential coefficients \( A_i(0) \) [23]. The experimental knowledge of the total thermal resistance \( R_{th} \) given by (2.11) and of the parameters \( \tau_i \) and \( A_i(0) \) allows us to build the Laplace transform \( \Delta T_c(s) \) of the cooling transients \( T(0,t) - T_0 \) caused by a falling step of the thermal power.
\[
\Delta T_C(s) = q_0 \sum_{i=1}^{N} \frac{A_i(0) \tau_i}{1 + s \tau_i} \tag{2.14}
\]

The sample Laplace transform \(\Delta T_C(s)\) can also be expressed by the following equation where \(Z(s)\) is the input thermal impedance of the Cauer circuit as seen from the heat source:

\[
\Delta T_C(s) = \frac{q_0}{s} \left[ R_{TH} - Z(s) \right] \tag{2.15}
\]

Therefore, by equating the second members of (2.14) and (2.15), the expression \(Z(s)\) can be uniquely obtained.

\[
Z(s) = R_{TH} - s \sum_{i=1}^{N} \frac{A_i(0) \tau_i}{1 + s \tau_i} \tag{2.16}
\]

The calculation of all the circuit thermal resistance \(R_i\) and capacitances \(C_i\), of the circuit, in their proper order from the heat source toward the heat sink, can be performed starting from the expression of the \(Z(s)\) by applying the Cauer method for the synthesis of passive networks. This technique consists of \(2 \times n\) polynomial divisions between the numerator and denominator alternatively of the admittance \(Y_i(s)\) and the impedance \(Z_i(s)\) in agreement with the following algorithm:

\[
i = 1, \ldots, n
\]

\[
Z_i(s) = z(s)
\]

\[
\frac{1}{Z_i(s)} = sC_i + Y_i(s)
\]

\[
\frac{1}{Y_i(s)} = R_i + Z_{i+1}(s)
\]

\[
Z_{n+1}(s) = 0 \tag{2.17}
\]
Thus, it was shown that the knowledge of the heat source temperature transient, the equivalent thermal circuit of the whole structure can be completely characterized both from the static and dynamic point of view. Furthermore, since all the thermal resistances $R_i$ are known, steady state temperature distribution across the nodes of the thermal circuit can be now calculated for any theoretical power dissipation $q_0$, applying the following relationship, where $T_i$ is the temperature of the $i^{th}$ node of the Cauer circuit.

$$T_i = T_0 + q_0 \sum_{j=1}^{N} R_i$$  \hspace{1cm} (2.18)

All the steps of the TRAIT analysis are schematically summarized in Fig. 2.4.
Fig. 2.5 Method for tracing the thermal domain boundaries using (a) Simulated steady-state temperature spatial distribution and (b) the temperature plot at the nodes of the thermal circuit.

2.3.5. Identification of the Contributions

The first technique, by means of which a physical correspondence can be established, is based on the comparison between a simulation (analytical or by finite elements) of the steady-state temperature field along the system and the results of the trait performed on the same system [24].

In steady-state condition, (2.18) gives the temperature $T_i$ at the isothermal surfaces which are the borders of the physical sectors. In the simple case shown in Fig. 2.2, the surfaces are parallel planes perpendicular to the solid axis because of the uniformity of the thermal flux. If the steady state temperature spatial distribution along the symmetry axis of the system $T(x)$ is also known from simulation and for a given power dissipation $q_0$, the positions $X_i$ of the domain borders on this axis can be calculated using the following implicit equation:

$$T(x_i) = T_i \quad (2.19)$$

In Fig. 2.5, this identification approach is graphically explained by drawing, with the same vertical scale, on the right side the calculated temperatures at the circuit nodes and, on the left one, the simulated temperature distribution along the symmetry axis as a function of the spatial dimension $x$.

An experimental, although qualitative, identification of the contributions can be performed by applying the TRAIT analysis to a standard sample and to samples having only one defect or failure purposely induced in any layer of the system [25], [26]. The
standard sample must be controlled by means of any other techniques (for instance, X-ray inspection) in order to avoid the presence of defects within the structure or the connecting layers. By comparing the results from the failed samples with those from the defect-free ones, we obtain information about the amount and the localization of the defect. By detecting disagreements in thermal resistance values, we can establish which RC cells receive the contribution of the failed layer.

2.4 3-D THERMAL SYSTEMS

In the TRAIT method for the three dimensional analysis, ANSYS (FEM) software is used. The TRAIT method has been able to calculate a completely characterized multiple RC equivalent circuit for the structure and provide a simple and accurate SPICE model for the simulation of the device temperature dynamics under whatever shaped electrical power stimulus, as for instance the short pulsed regime used in power-controlling systems.

In all the above measurements techniques only heat transfer techniques through conduction is considered. In most of the equipment the Power transistor and SCRs are mounted on the external surface of the equipment and is exposed to atmosphere. Heat transfer in such case will not be due to conduction alone. But will also be due to convection and radiation. If these affects are also taken into account, the efficiency of heat sink may be better than predicted by theoretical analysis. If CFD analysis is used, these effects also may be taken into account and the results obtained will closely match with that of the experimental results. Hence the Power electronic components may be loaded in a better way so as to get maximum efficiency and output. CFD analysis gives a total heat transfer concept and hence it is the best method available.

Ramachandran Vijayalakshmi et. al, in their paper [27] described an improved lumped circuit model of power Bipolar junction transistor that can predict the turn-off fall time to a greater accuracy. The model presented is based on the charge dynamics of the device. The device was subjected to zero current switching and zero voltage switching and results are compared with the measured results.

2.5 A REVIEW OF IGBT MODELS

The Insulated Gate Bipolar Transistor (IGBT) is an established replacement for the power bipolar junction transistor (BJT), Darlington transistor, metal oxide semiconductor field effect transistor (MOSFET), and GTO thyristor in medium...
frequency, medium power applications. As IGBT voltage and current ratings increase, its application range is extending to high power applications.

Various circuit simulators including Saber, the simulation program with integrated circuit emphasis (SPICE) family, and some others are commercially available for IGBT modeling. The majority of IGBT models are used in one (or several) of these simulators. Based on the same core program, simulators in the Spice family (PSpice, Spice, HSpice, IG-Spice, Contec-Spice, SmartSpice etc.) have different features for various applications. Saber is a comprehensive multi technology circuit simulator which has strong program capability. Numerical simulators, including the SILVACO package and MEDICI, etc., are also available for IGBT device and circuit simulations. Although giving accurate results, they are not suitable for normal IGBT users because they require device manufacturing parameters and are costly and time consuming. However, these numerical simulation packages are useful for device manufacturers, since they offer a reduced design cost and period.

Since the invention of the IGBT in 1982 [28], more than fifty papers have been published on IGBT models. In this chapter, some selected IGBT models published in the literature are reviewed, analyzed, compared and classified as shown in Table 2.1, into different categories according to mathematical type, objectives, complexity, accuracy and speed. A number ranging from 1 to 5 is assigned to each model to indicate complexity, with increasing complexity from 1 to 5. Short comments describe the modeling method used, model features and achievements. Although the majority of models are developed for device users to simulate device behavior in circuits, some other models (mostly mathematical models) are developed for understanding device operation mechanism and for structure optimization.
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Type</th>
<th>Complexity</th>
<th>Comments</th>
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<tr>
<td>Baliga</td>
<td>1985</td>
<td>Mathematical</td>
<td>4</td>
<td>Pin(BJT)-MOSFET Connection used for static characteristic, tail current analyzed</td>
</tr>
<tr>
<td>Metzner</td>
<td>1993, 1994</td>
<td>Semi-Numerical</td>
<td>4</td>
<td>Wide base region discretized to obtain accurate carrier for different conditions</td>
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<tr>
<td>Kevin, Undel and, Ronge</td>
<td>1993</td>
<td>Semi-Mathematical</td>
<td>2</td>
<td>Not well described, incomplete</td>
</tr>
<tr>
<td>Clemente &amp; Dapkus</td>
<td>1993</td>
<td>Behavioral</td>
<td>1</td>
<td>Curve-fitting method used, only for calculating switching losses in circuit design, no DC and dynamic behavior</td>
</tr>
<tr>
<td>Kim, Cho, Kim, Choi and Han</td>
<td>1993</td>
<td>Semi-Mathematical</td>
<td>2</td>
<td>Simply combines Spice models of BJT and MOSFET, non linear Cgd approximated polynomial</td>
</tr>
<tr>
<td>Li, Lafore, Arnold &amp; Raymond</td>
<td>1993</td>
<td>Mathematical</td>
<td>3</td>
<td>Linear carrier distribution assumed, other aspects similar to Hefner's model</td>
</tr>
<tr>
<td>Hefner</td>
<td>1993</td>
<td>Mathematical</td>
<td>4</td>
<td>His previous model extended to PT-IGBTs</td>
</tr>
<tr>
<td>Kuo, &amp; Chaing</td>
<td>1994</td>
<td>Mathematical</td>
<td>4</td>
<td>For turn-on transient only, incomplete</td>
</tr>
<tr>
<td>Goebel</td>
<td>1994</td>
<td>Semi-Numerical</td>
<td>5</td>
<td>Combines one-dimensional numerical and analytical methods, difficult to implement into a normal circuit simulator</td>
</tr>
<tr>
<td>Kuzmin, Yurkov, &amp; Pomorteseva</td>
<td>1994</td>
<td>Mathematical</td>
<td>4</td>
<td>PT-IGBT analyzed, negative differential resistance model</td>
</tr>
<tr>
<td>Kovac &amp; Kovacova</td>
<td>1994</td>
<td>Behavioral</td>
<td>2</td>
<td>Piece-wise modeling of Cgd</td>
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<tr>
<td>Allard, Morel, Lin, Helali &amp; Chante</td>
<td>1994</td>
<td>Mathematical</td>
<td>4</td>
<td>Systematical modeling using bond graph method, implementation requires complex programming, accurate results given</td>
</tr>
<tr>
<td>Besbes</td>
<td>1995</td>
<td>Mathematical</td>
<td>4</td>
<td>Using bond graph method</td>
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</table>
Although satisfactory IGBT models are available for various circuit simulations levels, comprehensive physical models for device mechanism understanding are not available. So we shall conclude that some problems persist in IGBT modeling.

2.6 CONCLUSION

Based upon the survey mentioned above, the following problems have been addressed in this thesis work.

In all the above models, they are realized and validated in the room temperature i.e., when the devices are not thermally loaded. In our thesis we are evaluating the variation of the junction temperature under various operating conditions with natural convection.

The simulated results are verified experimentally and validated. Hence for various operating conditions simulation can be carried out and the junction temperature can be predicted. Steady state and transient analysis are performed and a more accurate RC equivalent circuit can be realized in the Cauer and Foster forms.

The analysis is performed on individual components like power transistor and IGBT and also on board level with different VLSI components.

When considering computer board level, the effect of the temperature of the individual components and their complex effects on the neighboring components are also analyzed, thus predicting the temperature of the more sensitive and important component.