APPENDIX B

ANALYSIS USING ANSYS SOFTWARE

(AXIALLY LOADED COLUMNS)

B.1 Introduction

Theoretical study with a finite element package ANSYS is made to ascertain the buckling behaviour of axially loaded thin-walled RC columns. To account for the material nonlinearity an iterative procedure is formulated to perform the analysis by superposing the nonlinear static analysis over the eigen value buckling analysis of the package for axially loaded columns. Of the many finite element analysis packages available, ANSYS 4.4A version developed by Swanson Analysis Systems, USA is used because of its simplicity and versatility. Eigen value buckling analysis available in the package is limited to linear buckling problems. The investigation aims at extending the analysis to inelastic buckling by including the material nonlinearity. The theoretical results are compared with the experimental findings.

B.2 Analysis

B.2.1 Eigen value buckling

Geometric nonlinear problems are accommodated in Finite Element Method by the introduction of geometric
stiffness. At the bifurcation of equilibrium, the effective stiffness of structure vanishes. This is formulated as an eigen value problem. The basic equation is written as [3]

\[
[K] \{U\} + [K_g] \{U\} = \{R\} \quad \ldots \ldots (B.1)
\]

where

\[
[K] = \text{flexural stiffness matrix} \\
\{U\} = \text{displacement vector} \\
[K_g] = \text{geometric stiffness matrix} \\
\{R\} = \text{load vector}
\]

By assuming that throughout the loading range the stress state is written in terms of some function whose intensity is governed by a single parameter, the initial stress state is written as:

\[
[K_g] = \lambda_1 [S] \quad \ldots \ldots (B.2)
\]

where \( \lambda_1 \) = load factor

\[ [S] = \text{stress stiffness matrix} \]

The stability problem is formulated by imposing a variation of displacement at a constant load as

\[
[[K] + \lambda_1 [S]] \{dU\} = \{0\} \quad \ldots \ldots (B.3)
\]

where \( \{dU\} = \text{incremental displacement vector} \)
For nontrivial solution

\[ || [K] + \lambda_1[S] || = 0 \] .... (B.4)

The equation is solved as a standard eigen value problem, where eigen value gives the load factor which is the ratio of buckling load to preload, and eigen vectors give the critical mode of failure.

B.2.2 Nonlinear analysis

As the behaviour of reinforced concrete is nonlinear, the problem cannot be effectively tackled by conventional linear analysis. The physical nonlinearity affects only the conventional stiffness. So the basic equation is written as

\[ [K] \{dU_n\} = \{Fa\} - \{F_{n nr}\} \] .... (B.5)

where

\[ \{dU_n\} = \text{incremental displacement vector for nth iteration.} \]

\[ \{Fa\} = \text{total applied load vector} \]

\[ \{F_{n nr}\} = \text{restoring force vector} \]

and

\[ \{U_{n+1}\} = \{U_n\} + \{dU_n\} \] .... (B.6)

where

\[ \{U_{n+1}\} = \text{new displacement vector} \]

\[ \{U_n\} = \text{previous displacement vector} \]
Restoring force vector [2]

\[ \{F_{n}^{nr}\} = \sum_{m=1}^{N} \{F_{e,m}^{nr}\} \quad \text{.....}(B.7) \]

where

\[ \{F_{e,m}^{nr}\} = \text{element restoring force vector} \]

\[ \int_{V} \{B\}^{T} \{D\} \{\varepsilon_{n}^{el}\} dv \quad \text{.....}(B.8) \]

where

\[ \{B\} = \text{strain-displacement matrix} \]
\[ \{D\} = \text{constitutive matrix} \]
\[ \{\varepsilon_{n}^{el}\} = \text{elastic strain after n iterations} \]

The basic equation is solved by incremental Newton-Raphson method. The total load is applied in several steps. The equations which govern the iterations are

\[ [K]_{m,n} \{dU\}_{n} = \{f^{a}\}_{m} - \{F^{nr}\}_{m,n} \quad \text{.....}(B.9) \]

where

\[ [K]_{m,n} = \text{stiffness matrix for the m\textsuperscript{th} load step and n\textsuperscript{th} iteration.} \]
\[ \{U\}_{n+1} = \{U\}_{n} + \{dU\}_{n} \quad \text{.....}(B.10) \]

Equilibrium is checked during each iteration. When \{dU\} becomes very small, convergence is reached within the load step and the load is stopped. Converged solution at the
final load step gives corresponding displacements and stress distribution.

B.2.3 Nonlinear behaviour of concrete

Nonlinear characteristics of concrete is accounted in the analysis using the stress-strain equation [27]

\[ f_C = (0.85 f_{cc}/e_o^2) (2e_o e - e^2) \] ........(B.11)

where

- \( f_{cc} \) = 28th day cylinder compressive strength
- \( e_o \) = ultimate compressive strain
- \( f_C \) = stress in concrete at strain \( e \)

The curve represented by Eq. (B.11) is idealised into five linear segments for the purpose of analysis.

B.2.4 Element

Eight noded isoparametric shell element STIF 93 is selected for the analysis from the available 99 elements in the package. The element is well suited for curved boundaries. Three translational and three rotational degrees of freedom are presented at all the eight nodes. The following features justify the selection of element [3].

1. Rotational degrees of freedom are available
2. It can take care of material nonlinearity (plasticity).
3. Stress stiffening capacity, which is essential for buckling analysis is present.
4. Large deflections can be accounted. Nodal loads and pressures can be applied.
5. It is an isoparametric element.

The other available three dimensional shell elements are plastic quadratic shell STIF 43 and elastic quadratic shell STIF63. The advantage of STIF 93 over these elements are isoparametry and plasticity.

B.2.5 Analysis of columns

The buckling analysis described is suitable for linear material problems. To perform the analysis a constant constitutive matrix is required based on which load factors are calculated. For concrete having nonlinear material characteristics, the critical stress-strain parameters, strain and tangent modulus at the time of buckling, should be the inputs to derive accurate results from the buckling analysis. A nominal stress distribution according to stress-strain law is also required.

This is achieved by performing a nonlinear analysis where when loads are applied the deflections and stress distribution according to material properties defined, are calculated. Thus the stress distribution at the buckling
load itself becomes input to the nonlinear static analysis, the stress state corresponding to critical strain is obtained. The output from this analysis is the input for the buckling analysis. Hence the load factor (eigen value) which is the ratio of buckling load to preload, obtained from the buckling analysis should be unity. Based on this principle an iterative procedure is formulated to predict the buckling loads.

Inputting buckling load itself to the nonlinear analysis is achieved by iterations. When a higher load is applied, stress will be more and the corresponding tangent modulus will be less and a lesser buckling load is obtained and vice-versa. This property is made use for convergence of the iterative procedure. The steps involved are as follows.

1. A trial load is applied, a nonlinear static analysis is done.
2. The results from the static analysis are submitted to buckling analysis and the load factor is obtained.

The process is continued till load factor becomes unity. For the next trial, the average of previous trial load and buckling load obtained is applied to accelerate the convergence.
B.3 Analysis of Specimens

Columns under investigation are analysed. Analysis is restricted to half the height of the specimen taking advantage of the symmetry of the specimen. A specimen is discretised into elements as shown in Fig. B.1. The columns possess hinged end conditions. A uniformly distributed pressure loading is applied which is converted to consistent loading during execution of the analysis. Master degrees of freedom are selected to reduce computer time and to avoid tension degrees of freedom during buckling analysis. A deformed pattern of a specimen is shown in Fig. B.2.

B.4 Concluding Remarks

Results obtained from the analysis show good agreement with experimental observations with respect to buckling load and buckling mode, validating the idealization and assumptions made in the theoretical investigations.
FIGURE B.1 TYPICAL DISCRETISATION OF COLUMN

FIGURE B.2 TYPICAL DEFORMED PATTERN OF COLUMN