CHAPTER 6

VARIATION OF BREATHING RATE UNDER EXERCISE

6.1 Introduction

Determination of the respiratory characteristics under physical exercise has been the subject of research through the years. Both the ventilatory and cardiovascular systems are stressed during physical exercise and the ability to support adequately the increased metabolic rate during exercise reflects their efficiency. For this reason, tests designed to assess ventilation, gas exchange, and cardiovascular function during exercise can provide information not obtainable with static tests of cardiopulmonary function. Variation of breathing rate under exercise is an important parameter for describing the performance of respiratory system.

This chapter looks at the variation of breathing rate under exercise from the point of view of optimal control of minimum work rate criterion.

6.2 Respiratory System under Exercise

Unlike the study of pathology and disease, the study of exercise physiology leads to a wonderful understanding of the way the body is supposed to work while performing at its healthy best. Cognizance of acute and chronic responses to exercise gives an understanding of the physiological stresses to which the body is subjected.
6.2.1 Relevance and Scope of the Study

Many measurements made at rest inadequately reflect the reserve capacity of physiological systems that contribute to exercise tolerance. Exercise testing uses the increased metabolic demands of exercise to stress the systems and so establish their capacity to respond. The reserve capacity that is normally available in the circulatory and respiratory systems is so large that the demands of daily living do not become compromised until function has been greatly impaired; thus exercise testing may reveal abnormality at an earlier stage of cardiopulmonary disease. For the medical practitioner, exercise study helps in the objective assessment of patient's symptoms, assessment of impaired function, and diagnosis of certain diseases such as myocardial ischemia, peripheral vascular disease, dyspnea, exercise induced asthma, vasoregulatory asthenia, and muscle phosphorylase deficiency [77].

Exercise testing may help evaluate fitness for certain types of work or sports. It will also evaluate the effects of modifications in therapeutic and rehabilitation programs. Knowledge of exercise physiology imparts the engineer the ability to design devices to be used by humans, or to borrow ideas from human physiology to apply to other situations. There is need for engineers to design the many equipment used by sports and health enthusiasts, to modify prostheses or devices for the handicapped, to alleviate physiological stresses caused by personal protective equipment, and to design human powered machines that are compatible with the capabilities of operators [78].

A great deal of research has been performed investigating the nature of human ventilatory responses to exercise [78–82]. But still difficulty exists in understanding the complex nature of respiratory control and the multitude of possible inputs and outputs. Because of this, respiratory responses are difficult to reproduce, and there appear to be significant influences of the degree of sophistication of the subjects, prior work history,
age of the subjects, and individual variation. Due to these complexities, considerably less amount of work has been done to explore the changes of respiratory system under exercise.

When a person is exercising, the increased physical activity raises the energy demand of the human body, thus increasing consumption of oxygen and nutrients. As a result, the breathing rate of the person increases. In this work, study of variation in breathing rate under exercise is focussed. The variation in the energy expended for breathing with breathing rate, is also analysed. The responses described in this work are based on experiments conducted with normal, healthy, young, adult males aged between 20 and 25. Application of these ideas to young females can probably be made without much reservations, and application to aged people must take into account changes of mechanical properties and responsiveness that occur with age.

Many investigators have assumed the concept of optimal control of work rate in establishing a theory for respiratory system under exercise [79,82]. In a meritable work in this area [82], optimum values of breathing rate were determined when the subject is under exercise assuming the pattern of the inspiratory flow to be rectangular wave. This problem has been approached in a different perspective in this thesis.

6.2.2 Variation of Airflow Shape during Exercise

The pattern of airflows under different levels of exercise were closely studied. Airflow was observed as the subject performs exercise on a magnetic resistance bicycle ergometer [83]. The bicycle ergometer allows the workload to be varied by adjustment of the resistance to pedalling. The resistance is provided with the help of magnetic action. As the resistance to flywheel is already calibrated, amount of work performed can be accurately calculated.
It was observed that the inspiratory air flow pattern is almost sinusoidal in shape during normal breathing. As the subject continues exercise, the pattern was observed to be changing to the trapezoidal shape, and finally it was approximately rectangular wave. Based on these changes in airflow shape, equations for breathing rate have been derived under each condition, viz. when airflow is sinusoidal, trapezoidal, and rectangular wave. Fig. 6.1 shows breathing airflow waveshapes at increasing levels of exercise for the subject S.R., measured using a pneumotachograph.

The dependence of respiratory resistance and compliance on tidal volume is considered in our study. Values of breathing rate obtained using the numerical formulae are compared with the experimentally obtained values and they show good agreement in majority of the cases. Apart from this, changes in elastic and non-elastic components of the energy spent for breathing with variation in breathing rate were also studied. It can be seen that the work rate due to elastic and non-elastic elements of the respiratory system exhibits opposite trends, and the breathing occurs at the rate corresponding to the total minimum work rate.
6.3 Mathematical Model

Determination of respiratory characteristics has been the object of much study through the years. The concept that respiration is controlled by an optimisation process has been assumed by most of the researchers; only the parameter being optimised has been under investigation. Most of the optimisation models for the breathing rate of the respiratory system are based on either minimum work rate criterion or the minimum average respiratory muscle force criterion [84]. Later studies have substantiated the theory of minimum work criterion. This study is based on the minimum work rate criterion.

Researches were also focussed on mechanisms connected with increased ventilation demand during exercise [85-87]. During exercise, a manifold increase in oxygen (O$_2$) utilization and carbon dioxide (CO$_2$) production occur. It has been reported that ventilation follows CO$_2$ delivery to the lungs. That is, the cardiac output (the rate of blood flow to the lungs) times blood concentration of CO$_2$ must be matched by an appropriate alveolar ventilation rate, otherwise a change in blood pCO$_2$ (partial pressure of CO$_2$) would result. This change would then act as a powerful stimulant to the respiratory controller to correct the pCO$_2$ error. Therefore,

$$CO_2 \text{ delivered to the lungs} = CO_2 \text{ removed by the blood} + CO_2 \text{ removed by air.}$$

$$Q_{pulm}C_{CO_2} = Q_{pulm}C_{aCO_2} + V_A F_{CO_2alv} \quad (6.1)$$

where

$Q_{pulm}$ = cardiac output

$C_{CO_2}$ = carbon dioxide concentration

$V_A$ = alveolar ventilation rate

$F_{CO_2alv}$ = fraction of CO$_2$ in alveolar air

$a,v$ = subscripts denoting arterial and venous values.
From Eq. 6.1,

\[ V_A = \frac{\dot{Q}_{pulm}(C_{\text{ICO}_2} - C_{a\text{CO}_2})}{F_{CO_2,alv}} \]  

(6.2)

It is evident from Eq. 6.2 that oxygen demand sets the alveolar ventilation in order to control \( C_{a\text{CO}_2} \). However, for the given value of \( \frac{V_A}{V_D} \) (\( V_D \) is dead space volume) the minimum work rate criterion enables calculation of optimum breathing rate. For a given level of exercise, \( V_A \) can be regarded as a constant, however it changes with load.

Of the total pressure generated by the respiratory muscles, part is required to overcome the elastic properties of the respiratory system in order to change lung volume and part is dissipated to overcome the resistive characteristics of the respiratory system in order to generate flow. The sequence of events translating muscle activation into ventilation, and the factors involved in each step is shown in Fig. 6.2.
The total pressure generated for inspiration can be expressed by the equation

\[ p(t) = R \dot{v}(t) + \frac{v(t)}{C} \]  (6.3)

where \( R \) is the respiratory resistance, \( C \) is the respiratory compliance, and \( v(t) \) is the lung volume. It may be noted that an additional pressure component is required to accelerate the gas, and its magnitude depends on the inertia of the gas and respiratory structures. Because this component is usually a small portion of \( p(t) \), it is often neglected.

If \( V_T \) is the tidal volume and \( T \) is the respiratory period, the work done in respiratory action,

\[ W = \int_0^{V_T} p(t) dv = \int_0^{V_T} \frac{v}{C} dv + \int_0^{T/2} R \left[ \frac{dv}{dt} \right]^2 dt. \]  (6.4)

The tidal volume is given by

\[ V_T = \frac{V_A}{f} + V_D \]  (6.5)

where \( V_D \) is the dead space volume and \( f \) is the breathing rate.

An empirical formula was obtained for the value of \( V_D \) as a function of \( V_A \) based on experimental values as well as physiological conditions as [88]

\[ V_D = 0.1698V_A + 0.1587 \]  (6.6)

The study described in this chapter is based on the following assumptions:

1. Breathing rate is adjusted in such a way as to minimise the inspiratory work rate
2. Alveolar ventilation rate does not vary with respiratory period.
3. Inertia of tissues and air is neglected.
4. Expiration does not enter into the determination of respiratory period.
6.3.1 Sinusoidal Airflow

Inspiratory airflow rate $v(t)$ is assumed to be sinusoidal,

$$v(t) = V \sin \frac{2\pi t}{T}$$

(6.7)

where $V$ is peak inspiratory flow rate.

As the accumulated inspiratory airflow is equal to the tidal volume during inspiration,

$$V_T = \int_0^{T/2} V \sin \frac{2\pi t}{T} dt$$

(6.8)

Hence,

$$V = \frac{\pi V_T}{T}$$

(6.9)

Substituting expression for sinusoidal flow in Eq. 6.4 and using Eq. 6.9, we get

$$W = \frac{V_T^2}{2C} + \frac{\pi^2 RV_T^2 f^2}{4T}$$

(6.10)

Average inspiratory work rate is

$$W = \frac{W}{f} = \frac{V_T^2 f}{2C} + \frac{\pi^2 RV_T^2 f^2}{4}$$

(6.11)

When tidal volume is substituted by its equivalent alveolar ventilation rate and dead space in Eq. 6.11

$$W = \frac{f}{2C} \left( \frac{VA}{f} + V_D \right)^2 + \frac{\pi^2 R}{4} \left( V_A + V_D f \right)^2$$

(6.12)

Eq. 6.12 is differentiated to find the breathing rate that minimises the average work rate. An equation involving variable $f$ is obtained.

$$R \pi^2 V_D C f^2 + V_D f - V_A = 0$$

(6.13)

Solution of Eq. 6.13 results in the expression for optimum breathing rate, $f_{opt}$

$$f_{opt} = -1 + \frac{\sqrt{1 + 4\pi^2 RC(V_A/V_D)}}{2R\pi^2 C}$$

(6.14)
6.3.2 Trapezoidal Airflow

Considering airflow to be of trapezoidal shape with parallel sides of lengths $\frac{T}{8}$ and $\frac{T}{4}$,

$$V_T = \int_{\frac{T}{8}}^{\frac{3T}{8}} \frac{8VT}{T} dt + \int_{\frac{T}{8}}^{\frac{3T}{8}} \frac{VT}{T} dt + \int_{\frac{3T}{8}}^{\frac{T}{2}} (4V - \frac{8VT}{T}) dt$$

Therefore,

$$V_T = \frac{3VT}{8} \quad (6.15)$$

Substituting the expression for trapezoidal flow in Eq. 6.4 and using Eq. 6.15, inspiratory work can be shown to be,

$$W = 2.37RV_T^2f + \frac{V_T^2}{2C}$$

Average inspiratory work rate is,

$$W = 2.37RV_T^2f^2 + \frac{V_T^2f}{2C} = 2.37R (V_A + V_Df)^2 + \frac{f}{2C} (\frac{V_A}{f} + V_D)^2 \quad (6.16)$$

Differentiating Eq. 6.16 and equating to zero, a quadratic equation involving variable $f$ is obtained.

$$9.48V_DRCf^2 + V_Df - V_A = 0 \quad (6.17)$$

Solution of Eq. 6.17 yields the expression for optimum breathing rate.

$$f_{opt} = \frac{-1 + \sqrt{1 + 37.92RC(V_A/V_D)}}{18.96RC} \quad (6.18)$$

6.3.3 Rectangular Airflow

As the degree of exercise increases, airflow pattern changes to rectangular wave. When airflow is rectangular, tidal volume is,

$$V_T = \frac{\dot{V}T}{2} \quad (6.19)$$
Assuming a rectangular airflow in Eq. 6.4 yields inspiratory work to be

\[ W = \frac{V_f^2}{2C} + \frac{V^2RT}{2} \]  \hspace{1cm} (6.20)

Substituting Eq. 6.19 in Eq. 6.20,

\[ W = 2RV_f^2f + \frac{V_f^2}{2C} \]

Average inspiratory work rate is,

\[ W = 2RVT^2 + \frac{fV_f^2}{2C} \]  \hspace{1cm} (6.21)

Optimum breathing rate for minimum work rate can be obtained as,

\[ f_{opt} = \frac{-1 + \sqrt{1 + 32RC(V_A/V_D)}}{16RC} \]  \hspace{1cm} (6.22)

The respiratory resistance, \( R \) is the sum of chest wall resistance \( R_{cw} \) and airways resistance \( R_{aw} \). \( R_{cw} \) is calculated using the relation given by Eq. 6.23 and \( R_{aw} \) is taken as 2 cm H\(_2\)O sec/litre [89].

\[ R_{cw} = 2.9 - 0.75 \log(V_T) \]  \hspace{1cm} (6.23)

The respiratory compliance, \( C \) is the series combination of chest wall compliance \( C_{cw} \) and lung tissue compliance, \( C_{lt} \). \( C_{cw} \) is given by the relation given by Eq. 6.24 and \( C_{lt} \) is taken as 0.24 litre/(cm H\(_2\)O) [89].

\[ C_{cw} = \frac{1}{22 - 5.3 \log(V_T)} \]  \hspace{1cm} (6.24)

The changes in resistance and compliance with breathing rate are not considered in this work as the changes are not very considerable.
6.4 Experimental Validation

Theoretical values of the breathing rate are compared with the experimental data collected from volunteer subjects to assess the validity of the control procedure described in Section 6.3.

Theoretical values of breathing rate at different levels of exercise were evaluated applying Eq. 6.14, 6.18, and 6.22. A computer program written using the technical computing software MATLAB was used for simulation of the equations at different values of $\nu_p$. The subjects have undergone exercise on a magnetic resistance bicycle ergometer. The laboratory set-up for exercise study is shown in Fig. 6.3.

All subjects were aged between 20 and 25 and were in good health condition. Underweight subjects, overweight subjects or subjects with height values above or
Fig. 6.4. Comparison of theoretical breathing rates with experimental values for subject S.R.

below the 90% confidence limits of those predicted for the South Indians were not considered for this study. Subjects who had any medical history of cardiac, respiratory or neuromuscular diseases were excluded. No subject had a present or past history of smoking or significant occupational exposure to ambient hazards.

The subjects were submitted to the exercise test considerable time after any heavy meal or exertion. They were instructed to ride the ergometer at 50 rpm with the riding resistance fixed at a particular level, to ensure that exercise was done in a steady fashion, not hastily. Spirometry tests were conducted at frequent intervals to determine the breathing rate as well as the tidal volume. This exercise testing was performed under the supervision of the research consultant Dr. Rm. Pl. Ramanathan.
Experimental readings were taken for ten different subjects. Out of the ten subjects, seven showed breathing rate changes as per the proposed theory. Two best results are presented here. Variation of breathing rate with $\frac{V_A}{V_D}$ for the subjects S.R. and V.K. are shown in Fig. 6.4 and Fig. 6.5 respectively. It can be observed that at low values of $\frac{V_A}{V_D}$, breathing rate roughly follows the curve corresponding to sinusoidal and then that of trapezoidal flow shape. At higher levels of exercise, breathing rate follows the rectangular wave curve. Hence this study substantiates the minimum work criterion of breathing rate.

The total work rate consists of elastic and non-elastic parts. Fig. 6.6 gives the variation of elastic and non-elastic work rates with breathing rate. It can be observed
that the elastic work rate, represented by the amount of work rate needed to stretch the lungs and chest wall, increases as depth of breathing increases. In order to minimise the non-elastic work rate and still maintain the required minute volume, long, slow breath rates with low flow rates are implied. To minimise elastic work rate, rapid, shallow breaths with relatively high flow rates are required. Since these requirements oppose each other, a minimum is obtained. The optimum control of human respiration adjusts in such a way that the breathing takes place at a rate where the total work rate is minimum. It can be seen that in the case shown in Fig. 6.6, minimum work rate occurs at around 45/minute. It can also be observed that the minimum point is not well defined, hence the breathing may occur at a rate within the minimum region without severe penalty of increase in work rate. The work done during each inspiratory cycle is given in Fig. 6.7. It is observed that the work done varies slowly at low values of $\frac{V_A}{V_D}$, but at higher exercise the variation is very steep.
6.5 Summary

An optimal control system, based on the minimum work rate criterion to control the breathing rate and inspiratory flow shape under exercise, is presented. Equations for breathing rate when the inspiratory air flow shape is sinusoidal, trapezoidal, and rectangular waves were derived. The validity of these equations were verified experimentally and the results are encouraging. Variation of elastic and non-elastic parts of breathing energy were studied and it was found that at a particular breathing rate the sum of these become minimum. Human body performs breathing at this rate.

There could be criticism that why inspiratory work rate is considered in this study instead of the total work rate. The elastic work stored during inspiration is completely used during expiration. If energy spent for expiration is also considered, the expiratory elastic work cancels inspiratory elastic work with a net result of no elastic work term appearing in total cycle work expression. Without elastic work, there can
be no optimal respiratory period. However, due to the activation of expiratory muscles during exercise some additional elastic and non-elastic work is done during expiration also. This work has not been considered in this study. It may also be noted that at elevated breathing rates, tissue and gas inertia may also become significant.

In this work, a preliminary study of variation in breathing rate under exercise, based on the change in inspiratory flow shape, has been proposed. More exhaustive investigations may be required to ascertain the results more conclusively. However, this work could be considered as a major step in the study of respiratory system under exercise.