CHAPTER II

TRANSIENT FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE WITH OSCILLATING HEAT FLUX

1. Introduction

Transient free convection flows past an infinite vertical plate under different physical conditions were studied in 60's by many researchers and all the papers are referred in Goldstein and Briggs (1). The case of constant heat flux at the plate was also one such studied in the past. However, the condition of constant heat flux is rather a very restricted boundary condition. Many times the heat flux at the plate is of oscillatory type and this affects free convection flow. It is therefore the object of this paper to study the effects of oscillatory heat flux on the transient free convection flow past an infinite vertical plate.

2. Mathematical Analysis

Consider an infinite vertical plate surrounded by an infinite mass of fluid initially maintained at the temperature $T_\infty$. At time $t' > 0$, the plate is supplied heat which oscillates in time with frequency $\omega$. The $x'$-axis is taken along the vertical plate in the upward direction and the $y'$-axis is taken normal to the plate. Then under usual Boussinesq's approximation, the unsteady free convection flow can be shown to be governed by the following equations in non-dimensional form:

\[ \frac{\partial u}{\partial t} = \theta + \frac{\partial^2 u}{\partial y^2} \]  \hspace{1cm} (1)

\[ Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \]  \hspace{1cm} (2)

With following initial and boundary conditions:

\[ u = 0, \quad \theta = 0 \text{ for all } y, t \leq 0 \]

\[ u = 0, \frac{\partial \theta}{\partial y} = -\cos \omega t \text{ at } y = 0 \quad \text{at } t > 0 \]  \hspace{1cm} (3)

\[ u = 0, \quad \theta = 0, \quad \text{as } y \to \infty \]

The non-dimensional quantities are defined as follows:
\[ \Delta T = \left( \frac{\nu^2 q^3}{g \beta K^2} \right)^{1/4}, \quad U_R = \left( \frac{\nu^2 g \beta q}{K} \right)^{1/4} \] 
\[ T_R = \left( \frac{K}{g \beta q} \right)^{1/2}, \quad L = \left( \frac{\nu^2 K}{g \beta q} \right)^{1/4} \] 
\[ u = u'/U_R, \quad y = y'/L, \quad t = t'/T_R, \quad \theta = (T' - T^\infty)/\Delta T \]
\[ \text{Pr} = \mu c_p / K \]

All the quantities have their usual meaning.

The solutions of equations (1) and (2), satisfying the conditions (3) are derived by the usual Laplace-transform technique and these are as follows:

\[ \theta = \frac{1}{4 \sqrt{\text{Pr}}} \frac{e^{i\omega t}}{\sqrt{i \omega}} \left[ e^{-2\eta \sqrt{i \omega \text{Pr}}} \text{erfc} \left( \eta \sqrt{\text{Pr}} - \sqrt{i \omega t} \right) - e^{2\eta \sqrt{i \omega \text{Pr}}} \text{erfc} \left( \eta \sqrt{\text{Pr}} + \sqrt{i \omega t} \right) \right] \]  
\[ u = \frac{1}{2 \sqrt{\text{Pr}(1 - \text{Pr})}} \left[ \sqrt{i \omega} e^{i\omega t} \right] \left[ e^{-2\eta \sqrt{i \omega t}} \text{erfc} \left( \eta - \sqrt{i \omega t} \right) - e^{2\eta \sqrt{i \omega t}} \text{erfc} \left( \eta + \sqrt{i \omega t} \right) \right] \]

Where \( \eta = y/2 \sqrt{t} \)

\( \ldots \ldots \)  

We have computed numerically the values of \( u \) and \( \theta \) by using the well-known formulae for separation of real and imaginary parts in \( \text{erfc} \) function and these are shown for air in Figs. 1 and 2. We observe from these expressions that both temperature and velocity field are affected by the frequency of the oscillating plate temperature. We observe from these two figures that the transient temperature and
velocity decrease with increasing the frequency \( \omega \) but increase with increasing \( \omega t \) values.

We have as usual computed the skin-friction and the Nusselt number and these are listed in Table 1.

<table>
<thead>
<tr>
<th>( \omega t/\omega )</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/2 )</td>
<td>0.1152</td>
<td>0.0576</td>
<td>0.0384</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>0.0997</td>
<td>0.0498</td>
<td>0.0332</td>
</tr>
<tr>
<td>( \pi/4 )</td>
<td>0.0814</td>
<td>0.0407</td>
<td>0.0271</td>
</tr>
<tr>
<td>( \pi/6 )</td>
<td>0.0576</td>
<td>0.0288</td>
<td>0.0192</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nusselt Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/2 )</td>
</tr>
<tr>
<td>( \pi/3 )</td>
</tr>
<tr>
<td>( \pi/4 )</td>
</tr>
<tr>
<td>( \pi/6 )</td>
</tr>
</tbody>
</table>

3. Conclusions

We conclude from this Table that the skin-friction decreases and the Nusselt number increases with increasing the frequency \( \omega \), but owing to an increase in \( \omega t \), the skin-friction decreases and the Nusselt number increases.

References

Fig. 1 VELOCITY PROFILES, Pr = 0.71
Fig. 2 TEMPERATURE PROFILES, Pr = 0.71