CHAPTER XV

FULLY DEVELOPED MIXED CONVECTION FLOW OF WATER AT 4°C BETWEEN TWO LONG VERTICAL PARALLEL PLATES

1. Introduction

Fully developed mixed convection flow between two long vertical parallel plates is studied because it has good practical applications in the industry. It was observed experimentally by Sparrow et al. (1984) that flow reversal takes place near the wall where temperature is maintained at a lower level as compared to the temperature of the other wall.

In all these studies, the fluid considered was air or water at normal temperature 20°C and in this case, it was observed that the variation of density is a linear function of temperature, viz. \( \nabla \rho = \rho \beta (T - T_\infty) \) where \( T \) is the temperature, \( \rho \) the density and \( \beta \) is the coefficient of volume expansion. However, when the temperature of water falls to 4°C, it was observed that the density of water is maximum and at this temperature, the density does not vary linearly. The variation of density \( \rho \) is adequately represented by \( \nabla \rho = \rho \gamma (T - T_\infty)^2 \) where \( \gamma \) is the thermal expansion coefficient and is given by \( \gamma = 0.8 \times 10^{-5} \text{ (°C)}^2 \). This is accurate when \( T \approx 4°C \). So Goren (1966) studied the free convection flow of water at 4°C past a semi-infinite vertical plate. Soundalgekar (1973) established the existence of similarity solution of free convection flow of water at 4°C in the presence of a variable wall temperature.

The first theoretical study of fully-developed mixed convection flow was published by Ostrach (1954). Later on, Lietzke (1954), Cebeć et al. (1982) studied different types of mixed convection flow between two long vertical parallel plates. However, Aung and Worku (1986) presented the systematic study of such a flow wherein they showed the conditions under which flow reversal near the cooled plate occurs. As viscous dissipative heat was neglected by Aung and Worku (1986), in their study, the temperature viz., Prandtl number, did not appear explicitly. So it is

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very difficult to predict the flow reversal in terms of Prandtl number. Aung and
Worku's case is application for fluids at normal room temperature of 20°C. If the
temperature of room falls to 4°C, how the flow of water at 4°C through two vertical
long parallel plates is modified? This is an important topic which is not studied in the
literature. Hence we now devote this paper to study this aspect. As the governing
equations are modified for flow of water at 4°C, without bringing the Prandtl number
in the picture, we can study the behaviour of mixed convection flow of water at 4°C.
In Section 2, the mathematical analysis is presented and in Section 3, the conclusions
are stated.

2. Mathematical Analysis

Consider the flow of water at 4°C between two long vertical parallel plates. The x-axis is taken along the left-side vertical plate in the upward direction and the y-
axis is taken normal to the plate. The other plate is held at y = b. Let \( T_1 \) and \( T_2 \) be
the temperatures of left and right plate respectively and \( T_0 \) is the temperature of the
fluid at the entrance of the channel which is 4°C. Then the fully developed flow of
water at 4°C can be shown to be governed by

\[
O = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \rho g \gamma (T - T_0)^2 
\]  

..(1)

\[
O = \frac{d^2T}{dy^2} 
\]  

..(2)

The boundary conditions are

at \( y = 0 \), \( u = 0 \), \( T = T_1 \) \[ \{ \]

at \( y \to b \), \( u = 0 \), \( T = T_2 \) \[ \}

..(3)

All the physical variables are defined in the Nomenclature.

If \( u_0 \) is the velocity of water at the entrance, we now define the following non-
dimensional quantities:
\[ P = \frac{p}{\rho_0 u_0^2}, \quad U = \frac{u}{u_0}, \quad Y = \frac{y}{b}, \quad X = \frac{x}{b Re}, \]
\[ \text{Re} = \frac{u_0 b}{v}, \quad \theta = \frac{T - T_0}{T_2 - T_0}, \quad r_T = \frac{T_1 - T_0}{T_2 - T_0}. \]
\[ Gr = \frac{g r_T (T_2 - T_0)^2 b}{v^2} \]

Substituting eqns. (4) into eqns. (1) - (3), we have
\[ -\frac{dP}{dX} + \frac{d^2 U}{dY^2} + \frac{Gr}{Re} \theta^2 = 0 \]
\[ \frac{d^2 \theta}{dY^2} = 0 \]

with the following boundary conditions:
\[ Y = 0, \quad U = 0, \quad \theta = r_T \]
\[ Y = 1, \quad U = 0, \quad \theta = 1 \]

The solution of eqn. (6) satisfying conditions (7) is
\[ \theta = (1 - r_T)Y + r_T \]

The solution of eqn. (5) on taking into account (7) and (8) is now given by
\[ U = -\frac{Gr}{Re} \left[ \frac{1}{12} y^4 + \frac{1}{3} r_T (1 - r_T) y^3 + \frac{1}{2} r_T^2 y^2 \right] \]
\[ + \frac{Gr}{Re} \left[ \frac{1}{12} (1 - r_T)^2 + \frac{1}{3} r_T (1 - r_T) + \frac{1}{2} r_T^2 - \frac{\alpha}{2} \right] Y + \frac{\alpha}{2} Y^2 \]

where \[ \alpha = -\frac{dP}{dX}. \]

To determine \( \alpha \), we assume the conservation of mass at any cross-section in the channel, which is given by
\[ \int_0^1 UdY = 1 \]

Substituting (9) into eqn. (10) and simplifying, we have
\[ \alpha = \frac{Gr}{Re} \left[ \frac{12}{40} (1 - r_T)^2 + r_T (1 - r_T) + r_T^2 - 12 \right] \]

We have computed numerical values of \( \alpha \) for different values of Gr/Re and \( r_T \) and these are shown in Fig. 1. We observe from this figure that the pressure-gradient decreases with increasing Gr/Re or \( r_T \).
We have now computed numerical values of $U$ on taking into account the values of $\alpha$ computed from eqns. (11) and these are plotted in Figs. 2 and 3. We observe from Fig. 2 that for $r_T = 0.2$, the flow reversal will take place in the left-half of the channel when $Gr/Re$ increases beyond 100, but in the right half space, the velocity increases with increasing $Gr/Re$, with maximum velocity not exceeding 2 at $Gr/Re = 100.00$. In Fig. 3, the variation of velocity for $Gr/Re = 100.00$ is shown for different values of $r_T$. It is seen from this figure that for $Gr/Re = 100.00$, the velocity increases in the left-half of the channel when $r_T$ increases from 0.2 onward. The reverse-type of flow may occur at $Gr/Re = 100.00$ when $r_T$ decreases below 0.2. However, in the right-half space, the velocity increases with increasing $r_T$.

It is interesting to study the effects of these parameters on the bulk temperature $\theta_b$, which is given by

$$\theta_b = \frac{1}{U\theta} \int_0^1 U\theta \, dY$$

Substituting for $U$ and $\theta$, we have carried out the integration and it is given by

$$\theta_b = \frac{(1-r_T) \left[ \frac{1}{72 \, Re} \frac{Gr}{(1-r_T)^2} + \frac{2}{45 \, Re} \frac{Gr}{r_T(1-r_T)} + \frac{1}{24 \, Re} \frac{Gr}{(r_T)^2 - \frac{\alpha}{24}} \right] + r_T \left[ \frac{1}{40 \, Re} \frac{Gr}{(1-r_T)^2} + \frac{1}{40 \, Re} \frac{Gr}{12 \, Re} \frac{r_T}{r_T(1-r_T)} + \frac{1}{12 \, Re} \frac{Gr}{(r_T)^2 - \frac{\alpha}{12}} \right]}{10 \left[ \frac{1}{40 \, Re} \frac{Gr}{(1-r_T)^2} + \frac{1}{40 \, Re} \frac{Gr}{12 \, Re} \frac{r_T}{r_T(1-r_T)} + \frac{1}{12 \, Re} \frac{Gr}{(r_T)^2 - \frac{\alpha}{12}} \right]}$$

$(13)$

The numerical values of $\theta_b$ are evaluated and these are plotted on Fig. 4. We observe from this figure that the bulk-temperature increases with increasing $r_T$ but it is not significantly affected by $Gr/Re$. 

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3. Conclusions

1. The pressure gradient decreases with increasing Gr/Re or \( r_T \).
2. For \( r_T = 0.2 \) the flow reversal will take place in the left half of the channel when Gr/Re increases beyond 100, but in the right-half space, the velocity increases with increasing Gr/Re.
3. For Gr/Re = 100.00 the velocity increases in the left half of the channel when \( r_T \) increases from 0.2 onward. The reverse type of flow may occur at Gr/Re = 100.00 when \( r_T \) decreases below 0.2.
4. The bulk-temperature increases with increasing \( r_T \) but it is not significantly affected by Gr/Re.

Nomenclature

- \( b \) - spacing between plates
- \( \beta \) - coefficient of volume expansion
- \( \rho \) - density
- \( \gamma \) - thermal expansion coefficient
- \( T_1, T_2 \) - temperature of left and right plate respectively
- \( T_0 \) - temperature of fluid at the entrance
- \( D_e \) - hydraulic diameter
- \( g \) - acceleration due to gravity
- \( \text{Gr} \) - Grafoil number
- \( h \) - heat transfer coefficient
- \( K \) - thermal conductivity
- \( \text{Nu} \) - Nusselt number
- \( p \) - pressure difference
- \( P \) - dimensionless pressure difference
- \( r_T \) - wall temperature difference ratio
- \( \text{Re} \) - Reynolds number
- \( T \) - temperature
- \( u_0 \) - velocity of water at the entrance
- \( u \) - axial velocity
- \( U \) - dimensionless axial velocity
- \( x, y \) - axial and transverse coordinate, respectively
$X$ - dimensionless axial coordinate
$Y$ - dimensionless transverse coordinate
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u$ - Kinematic viscosity
$\theta$ - dimensionless temperature
$\theta_b$ - bulk temperature

References


Fig. 1 Pressure Gradient vs $\text{Gr/Re}$

$R_T = 0.2$
$= 0.5$
Fig. 2 Velocity Profiles
Fig. 3: Velocity Profiles

$R_T = 0.2$
$= 0.5$
$= 0.7$

$Gr/Re = 100$
Fig. 4 Bulk Temperature

\( \Theta_b \)

Gr/Re

\( R_T = 0.5 \)
\( R_T = 0.2 \)