CHAPTER VII

MASS TRANSFER EFFECTS ON TRANSIENT FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE WITH PERIODIC HEAT FLUX

1. Introduction

Because of the importance of free convection flows in industrial applications, this study has received the attention of many research workers during few decades. Among the topics of free convection, transient free convection flows play an important role in industry and hence Illingworth (1950) was the first to study unsteady laminar free convection flow in the vicinity of an infinite vertical plate for fluid of Prandtl number unity. The plate and the fluid were assumed to be initially at the same temperature. Later on, Siegel (1958) solved the problem of unsteady free convection flow near a semi-infinite vertical plate under the condition of step-change in wall temperature or surface heat flux and the non-linear coupled equation's solutions were presented by employing momentum integral method. He pointed out that the initial behaviour of the temperature and velocity fields for the semi-infinite vertical flat plate is the same as for the doubly infinite vertical flat plate and the temperature field is given by the solution of an unsteady one-dimensional heat conduction problem. He also pointed out that the transition from conduction to convection begins only when some effect from the leading edge has propagated up the plate to a particular point depending on the physical situation. These findings were confirmed experimentally by Goldstein and Eckert (1960). Later on, many papers were published on this topic and these are referred in the paper presented by Godstein and Briggs (1964). In most of these papers, the plate temperature was either assumed to be uniform or the plate is supplied heat at constant rate. These physical situations are rather restricted in nature. It is possible that in many industrial applications, the plate temperature need not be receiving heat at constant rate but in a periodic manner and this is represented mathematically by superposing the periodic past on the heat supplied at constant rate. So this study was recently presented by Soundalgekar et al. (to be published) in case of pure fluids. However, in nature, the availability of pure fluids is rather very difficult. The common fluids like air or water are many times contaminated with
foreign gases or soluble substances like salt etc. and in case of flow of such fluids, the theoretical analysis is carried out by considering the presence of a foreign mass like CO₂, O₂, H₂ etc. in air. The governing equations in such cases are derived by Gebhart (1971) by assuming the level of foreign mass to be rather low which enables us to neglect the well-known Soret and Dufour effects. In section 2, the mathematical analysis is presented and the solutions are derived by the Laplace-transform technique and in section 3, the conclusions are set out.

2. Mathematical Analysis

We assume the unsteady free convective flow of an incompressible viscous fluid past an infinite vertical plate. Initially, the fluid and the plate are assumed to be at the same temperature \( T_\infty \) and the concentration level everywhere is also assumed to be \( C_\infty \), which is very low and hence the Soret-Dufour effect can be neglected. We take the \( x' \)-axis along the vertical plate in the upward direction and the \( y' \)-axis is taken normal to the plate. At time \( t' > 0 \), the plate temperature is raised to \( T'_\infty \) and the concentration level near the plate is also assumed to be raised to \( C'_w \) such that \( T'_w - T'_\infty \) and \( C'_w - C'_\infty \) are both greater than zero. Then under usual Bousinesq's approximation, the fully-developed unsteady flow can be shown to be governed by the following system of equations:

\[
\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T'_\infty) + g \beta * (C' - C'_\infty) \quad \text{..(1)}
\]

\[
\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} \quad \text{..(2)}
\]

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad \text{..(3)}
\]

with following initial and boundary conditions:

\[
\begin{align*}
  u' &= 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y' \leq 0 \quad t' \leq 0 \\
  u' &= 0, \quad \frac{\partial T'}{\partial y'} = -\frac{q}{K} \cos \omega t' \cos \omega y', \quad C' = C'_\infty \quad \text{at} \quad y' = 0 \quad t' > 0 \\
  u' &= 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as} \quad y' \rightarrow \infty
\end{align*}
\]

\text{..(4)}

All the physical variables are defined in the Nomenclature.

We now introduce the following non-dimensional qualities:
\[ y' = y' / L_R, \quad u = u' / u_R, \quad t = t' / t_R, \quad \theta = (T' - T_{\infty}') / \Delta T_R \]  \quad \text{(5)}
\[ C = (C - C_{\infty}) / (C_w - C_{\infty}), \quad N = \beta / \Delta C / \beta / \Delta T', \quad Sc = \nu / D, \quad Pr' = \mu / c_p / K \]

in equations (1) - (4) and these are seen to reduce to following system of equations:

\[ \frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} \]  \quad \text{(6)}
\[ \text{Pr} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \]  \quad \text{(7)}
\[ \text{Sc} \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \]  \quad \text{(8)}

with following initial and boundary conditions:

\[ u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad y, t \leq 0 \]
\[ u = 0, \quad \frac{d \theta}{dy} = -1 - \epsilon \cos \omega t \quad \text{at} \quad y = 0 \quad \text{at} \quad t > 0 \]  \quad \text{(9)}
\[ u = 0, \quad \theta = 0, \quad C = 0 \quad \text{as} \quad y \to \infty \]

Here

\[ L_R = \left( \nu^2 k / g \beta q \right)^{1/4}, \quad u_R = \left( \nu^2 g \beta q' / k \right)^{1/4} \]  \quad \text{(10)}
\[ t_R = \left( k / g \beta q \right)^{1/2}, \quad \Delta T_R = \left( \nu^2 q^3 / g \beta k \right)^{1/4} \]

are the reference length, velocity, time and temperature respectively.

These coupled linear partial differential equations are solved by the usual Laplace transform technique and the solutions are as follows:

\[ C = \text{erf} \left( \eta \sqrt{\text{Sc}} \right) \]  \quad \text{(11)}

where \( \eta = y / 2 \sqrt{t} \)

\[ \theta = \frac{1}{\sqrt{\text{Pr}}} \left( 4t \right)^{1/2} \text{erf} \left( \eta \sqrt{\text{Pr}} \right) + \epsilon \frac{e^{i\omega t}}{4\sqrt{\text{Pr}} \sqrt{i\omega}} \left[ \exp \left( -2\eta \sqrt{i\omega t \text{Pr}} \right) \exp \left( 2\eta \sqrt{i\omega t \text{Pr}} \right) \right] \]
\[ \text{erfc} \left( \eta \sqrt{\text{Pr}} - \sqrt{i\omega t} \right) - \exp \left( 2\eta \sqrt{i\omega t \text{Pr}} \right) \text{erfc} \left( \eta \sqrt{\text{Pr}} + \sqrt{i\omega t} \right) \]
\[ + \epsilon \frac{e^{-i\omega t}}{4\sqrt{\text{Pr}} \sqrt{-i\omega}} \left[ \exp \left( -2\eta \sqrt{-i\omega t \text{Pr}} \right) \exp \left( 2\eta \sqrt{-i\omega t \text{Pr}} \right) \right] \]
\[ \text{erfc} \left( \eta \sqrt{\text{Pr}} + \sqrt{-i\omega t} \right) - \exp \left( 2\eta \sqrt{-i\omega t \text{Pr}} \right) \text{erfc} \left( \eta \sqrt{\text{Pr}} - \sqrt{-i\omega t} \right) \]  \quad \text{(12)}
\[ u = \frac{-(4t)^{3/2}}{\sqrt{\text{Pr}(1 - \text{Pr})}} i^3 \text{erfc}(\eta) + \frac{\epsilon}{4\omega^2 \sqrt{\text{Pr}(1 - \text{Pr})}} \left\{ \exp \left( -2\eta \sqrt{i\omega t} \right) + \exp \left( -2\eta \sqrt{-i\omega t} \right) \right\} \]
\[
\begin{align*}
\text{erfc}(\eta - \sqrt{i\omega t}) &= \exp(2\eta + \sqrt{i\omega t}) \text{erfc}(\eta + \sqrt{i\omega t}) \sqrt{i\omega} \\
\exp(i\omega t) + \sqrt{-i\omega} \exp(-i\omega t) &\left[ \exp(-2\eta \sqrt{-i\omega t}) \text{erfc}(\eta - \sqrt{-i\omega t}) \right] \\
&- \exp(2\eta \sqrt{-i\omega t}) \text{erfc}(\eta + \sqrt{-i\omega t}) \right] - \frac{N(4t)}{1 - Sc} \right] t^2 \text{erfc}(\eta) \\
+ \frac{(4t)^{3/2}}{\sqrt{Pr}(1 - \Pr)} &\left[ \frac{\exp(i\omega t) - \exp(-i\omega t)}{4\omega^2 (1 - \Pr) \sqrt{Pr}} \right] \text{erfc}(\eta \sqrt{Pr - \sqrt{i\omega t}} - \exp(2\eta \sqrt{i\omega t} \Pr) \text{erfc}(\eta \sqrt{Pr + \sqrt{i\omega t}}) \\
&+ \sqrt{-i\omega} \exp(-i\omega t) \left[ \exp(-2\eta \sqrt{-i\omega t}) \Pr \text{erfc}(\eta \sqrt{Pr - \sqrt{-i\omega t}}) \right] \\
&- \exp(2\eta \sqrt{-i\omega t} \Pr) \text{erfc}(\eta \sqrt{Pr + \sqrt{-i\omega t}}) \right] + \frac{N(4t)}{1 - Sc} \right] t^2 \text{erfc}(\eta \sqrt{Sc}) .. (13)
\end{align*}
\]

Here \text{erfc} is the complementary error function.

To gain into the understanding of the flow pattern, we have carried out numerical computation for the velocity, temperature and concentration. Since the argument of the error function is complex, we use the following well-known formula (Abramowitz and Stegun [14]) to separate it into real and imaginary parts:

\[
erf(a + ib) = erf(a) + \frac{\exp(-a^2)}{2\pi a} \left[ 1 - \cos(2ab) + \sin(2ab) \right] \\
+ \frac{2\exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-n^2/4)}{(n^2 + 4a^2)} \left\{ f_n(a, b) + ig_n(a, b) \right\} + \epsilon (a, b) .. (14)
\]

where

\[
f_n(a, b) = 2a - 2a \cosh(nb) + n \sinh(nb) \sin(2ab) \\
g_n(a, b) = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab)
\]

\[
|\epsilon (a, b)| \approx 10^{-16} |\text{erf}(a + ib)|
\]

During computation, the numerical values of the Schmidt number are assumed such that they represent a reality. These are given by Gebhart and Pera (1971). Also the Prandtl number \text{Pr} is assumed for air and the values of Sc are assumed as follows:

\[
\begin{tabular}{|l|l|l|}
\hline
\text{Pr} & \text{Species} & \text{Sc} \\
\hline
0.71 & He & 0.30 \\
\hline
\end{tabular}
\]
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<tr>
<td>CO₂</td>
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Again the values of N, the ratio of buoyancy forces, are important. Here N measures the relative importance of these diffusivities which cause the density difference thereby causing the flow. In the absence of species diffusion, N = 0. N is infinite when thermal diffusion is absent, N > 0 corresponds to the flow due to combined effects of these diffusivities i.e. aiding flow and then N < 0 corresponds to these diffusivities opposing each other or opposing flows.

The velocity profiles are plotted on Figs. 1-5. In Fig.1, the effects of the buoyancy ratio parameter N are shown. We observe that for N > 0, and increase in N leads to an increase in the velocity, which is true from the physical point of view. Because, for N > 0, the buoyancy force aid each other which causes a rise in the velocity. But when N < 0, an increase in {-N} leads to a fall in the velocity where the other parametric values of ωt, ω, t Pr or Sc are held constant. On Fig. 2, the variation of velocity with time is shown and we observe from this figure that for N > 0, an increase in time t leads to a rise in the velocity. The effect of the frequency ω on the velocity of air can be seen from Fig. 3 when water vapour is present. We observe from this figure that for both N > 0, an increase in the frequency ω leads to a decrease in the velocity. On Fig. 4, the velocity profiles are shown for different values of the Schmidt number. The effect of Schmidt number depends upon N. When N > 0, i.e. in the presence of aiding type of flow, an increase in the Schmidt number Sc leads to a fall in the velocity whereas when N < 0, i.e. the presence of opposing type of flow, an increase in the Schmidt number leads to a rise in the velocity. On Fig. 5, we have plotted velocity profiles for different values of ωt and we observe from this figure that for both N > 0, an increase in ωt leads to a fall in the velocity of air when water vapour is present.
We now study the effect of such a periodic heat flux on the distance from the leading edge where transition from conduction to convection takes place. The penetration distance from the leading edge is given by

\[ X_p = \int_0^t u(y,t) dt \]  

..(15)

and in terms of the Laplace-transform and inverse-transform with respect to the variable \( t \), it can be expressed as

\[ X_p = L^{-1} \left\{ \frac{1}{S} \bar{u}(S,Y) \right\} \]  

..(16)

Substituting for \( u \) from (13) and carrying out the inverse, we have

\[
X_p = -\frac{1}{\sqrt{Pr(1-Pr)}} (4t)^{5/2} \frac{\varepsilon s^5}{i} \left[ \text{erfc}(\eta) - \text{erfc}(\eta \sqrt{Pr}) \right]
\]

\[
-\frac{\varepsilon}{\sqrt{Pr(1-Pr)} \omega^2} (4t)^{1/2} \left[ \text{erfc}(\eta) - \text{erfc}(\eta \sqrt{Pr}) \right]
\]

\[
+ \frac{\varepsilon}{2 \omega^3 (1-Pr) Pr} \left[ \frac{i \omega e^{i \omega t}}{2i} \left( e^{-2\eta \sqrt{i \omega t}} \text{erfc}(\eta - \sqrt{i \omega t}) \right)
\]

\[
- e^{-2\eta \sqrt{i \omega t Pr}} \text{erfc}(n \sqrt{Pr} - \sqrt{i \omega t}) \left( e^{2\eta \sqrt{i \omega t}} \text{erfc}(\eta + \sqrt{i \omega t}) +
\]

\[
e^{2\eta \sqrt{i \omega t Pr}} \text{erfc}(\eta \sqrt{Pr} + \sqrt{i \omega t}) \left( e^{-2\eta \sqrt{i \omega t}} \text{erfc}(\eta - \sqrt{i \omega t}) + e^{2\eta \sqrt{i \omega t Pr}} \text{erfc}(\eta \sqrt{Pr} - \sqrt{i \omega t}) \right) \right]
\]

\[
- \frac{i \omega e^{i \omega t}}{2i} \left[ e^{-2\eta \sqrt{-i \omega t}} \left( \text{erfc}(\eta - \sqrt{-i \omega t}) + \text{erfc}(\eta \sqrt{Pr} - \sqrt{-i \omega t}) \right) \right]
\]

\[
- e^{2\eta \sqrt{-i \omega t Pr}} \left( \text{erfc}(\eta + \sqrt{-i \omega t}) + \text{erfc}(\eta \sqrt{Pr} + \sqrt{-i \omega t}) \right) \right] \]  

..(17)

Numerical values of \( X_p \) are computed from equation (17) and these are plotted on Figs. 6-10. On Fig. 6 the effects of the buoyancy ratio parameter \( N \) on the distance from the leading edge of the point of transition from conduction to convection viz. \( X_p \), are shown. It is seen from this figure that for both \( N > 0 \), the distance of the point of transition increases with increasing the values of \( \{ \pm N \} \). Hence we conclude that in the presence of aiding or opposing buoyancy forces, the transition from conduction to convection is delayed as the value of \( \{ \pm N \} \) increases.
On Fig. 7, $X_p$ is plotted for different values of the Schmidt number $Sc$ for both $N \geq 0$. We observe from this figure that an increase in the Schmidt number $Sc$ leads to an increase in the value of $X_p$ when $N < 0$ near the plate and opposite is the case for $N > 0$. However, due to the presence of a foreign mass, the transition from conduction to convection takes place at a point, which is nearer to the leading edge as compared to non-existence of a foreign mass.

The effect of time on the distance $X_p$ can be seen in Fig. 8. It is observed that as time increases, the distance $X_p$ of the point of transition also increases, but the increase is more when $N < 0$ i.e. in the presence of opposing buoyancy forces.

The effect of frequency $\omega$ on the point of transition from conduction to convection is shown on Fig. 9 and we observe that the distance of the point of transition from the leading edge decreases as the frequency $\omega$ increases and hence we can conclude that at high values of $\omega$, the transition from conduction to convection may take place at or quite near the leading edge. Hence conduction regime can be maintained near the leading edge if low frequency periodic heat flux is applied at the plate and a convection-regime can be maintained near the leading edge if a low - high - frequency periodic heat flux is applied at the plate.

The influence of periodic heat flux on the transition from conduction to convection in the presence of aiding or opposing flow can be seen from Fig. 10. For both $N \geq 0$, in the absence of periodic heat flux, $\omega t = 0$, the penetration distance is high as compared to that in the presence of $\omega t \neq 0$. Hence imposition of periodic heat flux causes a reduction in the distance of the point of transition from conduction to convection i.e. penetration distance is affected considerably and is reduced due to imposition of periodic heat flux. Also, an increase in $\omega t$ leads to a fall in the distance of $X_p$ from the leading edge for both $N \geq 0$. Hence conduction regime changes to convection regime, nearer and nearer to the leading edge as $\omega t$ increases.

Temperature profiles computed from eqn.(12) are shown on Figs.11-13 and we observe that there is a fall in temperature with an increase in the frequency $\omega$, but temperature is found to rise with increasing $\omega t$ or time $t$. 

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We now study the skin-friction. It is given by

\[ \tau' = \mu \left. \frac{du}{dy} \right|_{y=0} \] ..(18)

Then in view of eqns. (5) and (10), this reduces to

\[ \tau = \tau' / \rho \ u_R^2 = \text{Re}^{-1}_R \left. \frac{du}{dy} \right|_{y=0} \] ..(19)

where \( \text{Re}^{-1}_R = \mu / \rho \ u_R L_R \).

Substituting equation (13) in eqn. (19) and simplifying, we get

\[ \text{Re}^*_R \tau = \frac{(4t)^{3/2}}{4 \sqrt{\text{Pr} (1 + \sqrt{\text{Pr}})}} + \frac{4tN}{\sqrt{\pi} (1 + \sqrt{\text{Sc}})} + \frac{2t}{\omega \sqrt{\text{Pr} (1 + \sqrt{\text{Pr}})}} \] ..(20)

Numerical values of \( \text{Re}^*_R \tau \) are computed from eqn. (20) and these are listed in Table II.

**Table II**

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<th>Pr</th>
<th>T</th>
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We observe from this table that in the presence of aiding buoyancy force (N > 0), an increase in the frequency \( \omega \) leads to a decrease in the skin-friction but an increase in \( \omega t \) leads to an increase in the skin-friction. An increase in the buoyancy ratio parameter N also leads to an increase in the skin-friction. Skin-friction is also found decrease when the Schmidt number is increasing but it increases as time increases.

However, when the buoyancy force parameter N (< 0) is opposing type, the skin-friction is found to be very small as compared to that in the presence of aiding type of buoyancy force parameter. An increase in \( \omega \) or \( \omega t \) or \{-N\} leads to a decrease in the skin-friction. The skin-friction is also found to increase as the Schmidt number Sc or time t increases.

The rate of heat transfer, expressed in terms of the Nusselt number, is given by

\[
Nu = - \frac{1}{\theta (0)} \left. \frac{d\theta}{d\mu} \right|_{\eta = 0} = + \frac{1}{\theta (0)} (1 + \varepsilon \cos \omega t)
\]

(21)

Table III

Values of Nu

We observe from this table that in the presence of aiding buoyancy force (N > 0), an increase in the frequency \( \omega \) leads to a decrease in the skin-friction but an increase in \( \omega t \) leads to an increase in the skin-friction. An increase in the buoyancy ratio parameter N also leads to an increase in the skin-friction. Skin-friction is also found decrease when the Schmidt number is increasing but it increases as time increases.

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\]

(21)
We observe from this table that an increase in $\omega$ or $t$ leads to an increase in the Nusselt number whereas $Nu$ decreases with increasing $\omega t$.

3. **Conclusions:**

1. In the presence of aiding type force $N > 0$, velocity increases and opposite in the case for $N < 0$.

2. For both $N < 0$, velocity increases with increasing time.

3. Velocity decreases with increasing the frequency $\omega$.

4. For $N > 0$, an increase in the Schmidt number leads to a fall in the velocity whereas opposite is the case for $N < 0$.

5. For both $N < 0$, the distance of transition point from conduction to convection increases with an increase in $\{\pm N\}$.

6. An increase in the Schmidt number Sc leads to a delay in the transition for $N < 0$ and opposite is the case for $N > 0$. But due to the presence of a foreign mass, the distance of the point of transition from the leading edge decreases as compared to that in the absence of the foreign mass.

7. Due to an increase in the frequency $\omega$, transition takes place nearer to the leading edge.

8. For $\omega t = 0$, the penetration distance is more as compared to that when $\omega t \neq 0$ for both $N > 0$.

9. For $N > 0$, increase in the frequency $\omega$ or Sc leads to a decrease in the skin-friction but the skin-friction increases with increasing $\omega t$ or $N$. But for $N < 0$, the skin-friction is found to be less as compared to that where $N > 0$.

10. Skin-friction decreases with increasing $\omega$ or $\omega t$ or $\{N\}$ when $N < 0$. Also, for $N < 0$, skin friction increases with increasing Sc or $t$.  

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Nomenclature

$C$ species concentration in boundary layer.

$C_o$ species concentration away from plate.

$C_p$ specific heat of the fluid at constant pressure.

$D$ chemical molecular diffusivity.

$g$ gravitation acceleration.

$K$ thermal conductivity of the fluid.

$L_R$ reference length

$N$ Buoyancy ratio parameter

$Pr$ Prandtl number

$q$ rate of heat transfer

$Sc$ Schmidt number

$t_R$ reference time

$T'$ temperature of the fluid near the plate

$T_{\infty}'$ temperature of the fluid near the fluid

$t'$ time away from the plate

$\Delta t_R$ reference time

$u'$ velocity in the $x'$-direction

$u_R$ reference velocity

$x'$ coordinate axis along the plate

$y'$ coordinate axis normal to the plate

$\nu$ Kinematics viscosity of the fluid

$\beta$ volumetric coefficient of thermal expansion

$\beta^*$ volumetric coefficient of expansion with concentration

$\rho$ density of the fluid

$\varepsilon$ a constant

$\omega'$ frequency

$\eta$ non-dimensional coordinate normal to the plate.
References


Fig. 1 Velocity Profiles $\varepsilon = 1.0$
0.4 = t
0.2 =

wt = 77/33
w = 5
Pr = 0.71
Sc = 0.6

N = 0.4
N = -0.4

Fig. 2 Velocity Profiles $\varepsilon = 1.0$
Fig. 4 Velocity Profiles, \( \varepsilon = 1.0 \)
Fig. 3 Velocity Profiles $\epsilon = 1.0$

- $0.4 = N$
- $5.0 = \omega$
- $10.0 = \omega$
- $15.0 = \omega$
- $N = -0.4$

- $\omega t = \pi/3$
- $Pr = 0.71$
- $t = 0.2$
- $Sc = 0.6$
Fig. 5 Velocity Profiles $\epsilon = 1.0$

- $N = 0.4$
- $N = -0.4$
- $\pi/6$
- $\pi/4$
- $\pi/3$
- $\pi/2 = \omega t$

$\omega = 5$
$Pr = 0.71$
$t = 0.2$
$Sc = 0.6$
\( \omega t = \pi / 3 \)
\( \omega = 5.0 \)
\( t = 0.2 \)
\( Pr = 0.71 \)
\( Sc = 0.6 \)

**Fig. 6 Penetration Distance** \( \epsilon = 1.0 \)
Fig. 7 Penetration Distance, ε = 1.0
Fig. 8 Penetration Distance $\epsilon = 1.0$

- $\omega t = \pi / 3$
- $\omega = 5.0$
- $Pr = 0.71$
- $Sc = 0.61$

$N = 0.4$ $N = -0.4$
Fig. 9 Penetration Distance, $\varepsilon = 1.0$

- $N = -0.4$
- $N = 0.4$

$\omega$: 5.0, 10.0, 15.0

- $t = 0.2$
- $Sc = 0.6$
- $Pr = 0.71$
- $\omega t = \pi/3$
Fig. 10 Penetration Distance $\epsilon = 1.0$

- $N = -0.4$
- $N = 0.4$

- $0.0 = \omega t$
- $\pi/6 = \omega t$
- $\pi/4 = \omega t$
- $\pi/3 = \omega t$
- $\pi/2 = \omega t$

- $Pr = 0.71$
- $\omega = 5.0$
- $t = 0.2$
- $Sc = 0.6$
Fig. 11 Temperature Profiles $\varepsilon = 1.0$

$\omega t = \pi/3$
$Pr = 0.71$
$t = 0.2$
\begin{align*}
\Pi/2 &= \omega t \\
\Pi/3 &= \\
\Pi/4 &= \\
\Pi/6 &= \\
\omega &= 5.0 \\
Pr &= 0.71 \\
t &= 0.2
\end{align*}

\text{Fig. 12 Temperature Profiles } \varepsilon = 1.0^{102}
Fig. 13 Temperature Profiles $\epsilon = 1.0$

$\omega t = \pi / 3$

$Pr = 0.71$

$\omega = 5.0$