5.1 INTRODUCTION

Experiments are the most reliable means of investigations. But due to the high cost and long durations associated with them, the range of design variables over which experiments can be conducted becomes limited. In this context, numerical methods are a very powerful tool using which, virtual experiments can be conducted covering a wider range of design and operating variables. In this module of the work, a mathematical model is developed to investigate the transient thermal behaviour of worm gearboxes from the basic principles of heat transfer, which can be used for investigations over a larger range of variables.

The heat transfer mechanism occurring in a gearbox is highly complex and is influenced by a large number of design and operating variables. These variables have an unpredictable influence on the temperature rise in a gearbox and hence, it is not an easy task to accurately model the gearbox for its thermal behaviour. Hence, a large margin of safety is usually incorporated in the design and most of the worm gear units are run at less than the allowable temperature rise levels.

A simple and quick method of predicting the operating temperatures in gearboxes will aid designers to have an approximate first estimate of the thermal characteristics of the units without conducting laborious experiments or computer simulations using expensive software. As a step towards this objective, a lumped-mass mathematical model is developed from the basic principles of heat transfer, using which, the temperature rise of
the major elements of the gear units under different operating conditions can be predicted. The model is developed for single reduction worm gear boxes. The results can be obtained very quickly and the method does not require the use of any commercial software.

5.2 HEAT TRANSFER IN GEARBOXES

In order to develop a thermal model of any system, the major sources of heat generation and the modes of heat dissipation in the system should be understood clearly. The various sources of heat generation and the method by which the heat is dissipated in a gearbox are discussed below.

5.2.1 Heat Generation in Gearboxes

There are four principal sources of heat generation in a gearbox. These are gear tooth friction, bearing resistance, oil resistance and seal resistance. The power transmission efficiency is a measure of the above resistances. In a worm gear box, a large proportion of the total losses is due to the tooth friction at the meshing zone. This is because of the high degree of sliding occurring between the worm and the worm wheel.

5.2.2 Heat Dissipation in Gearboxes

Heat transfer between the components of a structure can occur by conduction, convection and radiation. In a gearbox, the heat generated is first transferred to the lubricant from the meshing zone by the worm, which is partially immersed in oil. The lubricant carries this heat to the inner surfaces of the casing by conduction and convection, from where, it is conducted through the walls of the casing to the outer surface of the casing. Dissipation of heat from the outer casing to the surrounding air is effected by convection and radiation. A portion of the heat generated in the bearings and
seals and the gear teeth escapes to the atmosphere by conduction along the shafts and through the casing and then convection. The heat transfer processes taking place in gearboxes is illustrated in Figure 5.1.

![Figure 5.1 Heat transfer processes in a gearbox](image)

5.3 CONCEPTS OF EQUILIBRIUM

A typical thermal history from start up to the shut down of any system consists of two modes of thermal equilibrium, viz, the steady state and transient modes.

At steady state, the temperatures at different parts of the system are steady and do not vary with time. The steady state energy balance is given simply by

\[
\text{Rate of heat generation} = \text{Rate of heat dissipation}
\]

\[
Q_{\text{gen}} = Q_{\text{cond}} + Q_{\text{conv}} + Q_{\text{rad}}
\]  

...(5.1)

This means that the rate of generation of heat and the rate of dissipation are the same and that, there is no change of temperature of the different parts of the system.
Chapter 5

Lumped Mass Thermal Model of Gearbox

An analysis of a system based on the steady state energy balance aims at predicting the steady state temperatures of the various parts of the system.

In the more general case during warming up or cooling down, the temperatures of the different parts of the system change with time. Assuming that these temperatures are uniform over an infinitesimal time interval $dt$, an energy balance for the time interval $dt$ is given by

$$(\text{Rate of heat generation} - \text{Rate of heat dissipation}) \times dt = \text{Rate of absorption of heat by different components of the system}$$

5.4 METHODOLOGY ADOPTED IN THIS WORK

In this work, a simple mathematical model is developed based on the basic equations of heat transfer to determine the temperatures of the lubricating oil and the major components of the gear units. The model adopts a lumped mass energy balance approach. The energy balance is applied over an infinitesimal time step $dt$ iteratively until steady state is reached. Starting from the initial condition, the temperatures are estimated at each time step and the results obtained in a time step form the input for the next time step. The properties of the oil such as the kinematic viscosity, thermal conductivity and the specific heat capacity are modeled as temperature dependent. Hence, these properties are recalculated for each time step based on the temperature of oil during the start of the time step, and the relevant convective heat transfer coefficients estimated.
5.5 MATHEMATICAL MODEL OF HEAT TRANSFER IN GEARBOX

The transient energy balance of the gearbox unit in a small time interval $dt$ is given by

$$(\text{Rate of heat generation in gearbox} - \text{Rate of heat dissipation due to convection and radiation}) \times dt = \text{Rate of absorption of heat by the different components of the gearbox}$$

The total frictional heat generated in a gearbox is given by

$$Q_{gen} = H_{tooth} + H_{brg} + H_{churn} \quad \ldots (5.2)$$

neglecting the losses in seals.

The amount of heat that goes to raise the temperature of the different elements of the gearbox can be modeled as

$$Q_{abs} = \sum mC_p dT \quad \ldots (5.3)$$

where, the summation includes the different components of the gearbox.

The heat dissipation process in the gearbox starts with the removal of heat from the worm surface by the oil. This can be modeled as forced convection, since, the worm, which is partly immersed in the oil, is in dynamic motion and churns the oil. The heat is then transferred to the inner casing, from where, it is conducted through the walls of the casing, to the outside surface of the casing. The heat from the outer casing is dissipated to the atmosphere by convection and radiation.

The mathematical model of the above heat transfer process is developed as follows, by assuming a two-stage process. First, the heat generated at the meshing zone is
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assumed to be partly absorbed in the gear pair and partly to be convected into the lubricant oil. This is modeled for an infinitesimal time interval $dt$ as

$$Q_{gen} dt = Q_{abs} + Q_{conv} dt$$  \hspace{1cm} \ldots (5.4)$$

where, $Q_{abs}$ is the heat absorbed by the gear pair during the time interval $dt$.

Expanding,

$$Q_{gen} dt = m_g C_{pg} (T_{fg} - T_{ig}) + h_{wo} A_w (T_{fg} - T_{io}) dt$$  \hspace{1cm} \ldots (5.5)$$

Rearranging, the temperature of the gear pair at the end of the time-step $dt$ may be determined as

$$T_{fg} = \frac{Q_{gen} dt + m_g C_{pg} T_{ig} + h_{wo} A_w T_{io} dt}{m_g C_{pg} + h_{wo} A_w dt}$$  \hspace{1cm} \ldots (5.6)$$

Thus, knowing the temperature of the gear pair and that of the oil at the beginning of the time-step, the temperature of the gear pair at the end of the time-step can be determined.

Now, the second stage of the heat dissipation process is modeled as follows. The heat transferred to the oil from the worm surface by convection, i.e., $Q_{conv}$, is assumed to be partly absorbed by the lubricant oil and the rest of it is assumed to be transferred to the outside air through the oil and the wall of the casing. The corresponding energy balance equation is then,

$$Q_{conv} = Q_{abs} + Q_{conv}$$  \hspace{1cm} \ldots (5.7)$$

Thus, the second stage of heat balance can be modeled as

$$Q_{conv} dt = m_o C_{po} (T_{fo} - T_{io}) + U_{ovl} A_{ovl} (T_{fo} - T_o) dt$$  \hspace{1cm} \ldots (5.8)$$
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The transfer of heat from the oil to the outside atmospheric air is modeled using an overall heat transfer coefficient which consists of convection from the oil to the inside casing, conduction through the casing wall to the outside of the casing and convection from the outside casing to the atmosphere. Thus, the overall heat transfer coefficient $U$ is defined as

$$U_{ovl} = \frac{1}{h_{oic}} + \frac{x_{wall}}{K_{wall}} + \frac{1}{h_{ext}}$$

...(5.9)

Thus, during every time step $dt$, the bulk temperature of the gear pair is obtained in the first stage, followed by the oil and the casing temperatures in the second stage. The steps involved in the modeling of the heat transfer and the solution methodology are shown in Figure 5.2.
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Fig. 5.2 Steps involved in heat transfer analysis of gearbox
5.6 ESTIMATION OF INPUTS TO THE THERMAL MODEL

5.6.1 Frictional Heat Input

The sources of internal heat generation are the power losses in the gearbox due to friction between the components in relative motion. As mentioned supra, the major sources of frictional heat generation in a gearbox are gear tooth friction, bearing resistance, oil resistance and seal resistance. The friction due to seal resistance is found to be very small and hence neglected. The total power loss and hence the heat generated in the gearbox is given by equation (5.2) as

\[ Q_{gen} = H_{tooth} + H_{brg} + H_{chum} \]  \hspace{1cm} (5.2)

5.6.1.1 Tooth losses

The power loss due to friction between the meshing gear teeth under loaded condition is computed as

\[ H_{tooth} = V_s W_f \]  \hspace{1cm} (5.10)

with the sliding velocity \( V_s \) given by,

\[ V_s = \frac{V_w \cos \lambda}{\cos \phi \sin \lambda \mu \cos \lambda} \]  \hspace{1cm} (5.11)

The frictional force \( W_f \) is given by

\[ W_f = \mu W \]  \hspace{1cm} (5.12)

where,

\[ W = \frac{W_x}{(\cos \phi \sin \lambda \mu \cos \lambda)} \]  \hspace{1cm} (5.13)

The coefficient of friction at the mesh is dependent on the sliding velocity and the viscosity of the lubricant. It is estimated as [68]

\[ \mu = \frac{1.6}{\left(700U_{e}^{0.15} F_{r}^{0.15} F_{s}^{0.35} R_{r}^{0.5}\right)} \]  \hspace{1cm} (5.14)
with the entraining velocity $V_e$ and the rolling radius $R_r$ determined as

$$V_e = 0.524 \times d \times n \times \sin(\lambda) \sin(\phi) \quad \ldots \ (5.15)$$

$$R_r = 0.5 \times D \times \left(\frac{1}{\cos \lambda}\right)^2 \times \sin(\phi) \quad \ldots \ (5.16)$$

The frictional heat generated at the gear tooth-meshing zone under no load condition is estimated as [69]

$$H_{tooth} = (10^4)^{2.5} \times \left(\frac{(\nu \times 10^6 + 90) \times \left(\frac{n}{1000}\right)^{1.33}}{1800}\right) \quad \ldots \ (5.17)$$

5.6.1.2 Bearing losses

The bearing losses are estimated as [68]

$$H_{brg} = \frac{T_f \times n}{9542.895} \quad \ldots (5.18)$$

where, the frictional torque $T_f$ is

$$T_f = \mu \times D_b \times W_b \quad \ldots (5.19)$$

and,

$$W_b = \sqrt{w_{r1}^2 + w_{r2}^2 + w_{r3}^2} \quad \ldots (5.20)$$

$$w_{r1} = w_x \times \frac{a}{a+b} \quad \ldots (5.21)$$

$$w_{r2} = w_y \times \frac{a}{a+b} \quad \ldots (5.22)$$

$$w_{r3} = w_z \times \frac{r_1}{a+b} \quad \ldots (5.23)$$

$$w_x = 999.66 \times \frac{P}{V_w} \quad \ldots (5.24)$$
\[ w_y = w \cdot \sin \phi \quad \ldots (5.25) \]
\[ w_z = w (\cos \phi \cdot \cos \lambda - \mu \sin \lambda) \quad \ldots (5.26) \]

The radial and axial loads on the bearings, which support the worm and the worm wheel, are determined as per Table 5.1

**Table 5.1 Bearing load calculations:**

<table>
<thead>
<tr>
<th>Forces</th>
<th>Worm Bearing Loads</th>
<th>Worm-gear Bearing Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bearing A</td>
<td>Bearing B</td>
</tr>
<tr>
<td>Tangential Force, ( W_x )</td>
<td>( w_{r1} = \frac{a_y}{a_y + b_y} )</td>
<td>( w_{r4} = \frac{b_y}{a_y + b_y} )</td>
</tr>
<tr>
<td>Separating Force, ( W_y )</td>
<td>( w_{r2} = \frac{a_y}{a_y + b_y} )</td>
<td>( w_{r5} = \frac{b_y}{a_y + b_y} )</td>
</tr>
<tr>
<td>Thrust Force, ( W_z )</td>
<td>( w_{r3} = \frac{r_1}{a_y + b_y} )</td>
<td>( w_{r6} = \frac{r_1}{a_y + b_y} )</td>
</tr>
<tr>
<td>Total Radial Load</td>
<td>( W_{r1} = \sqrt{\left(W_{r1}\right)^2 + \left(W_{r2} - W_{r3}\right)^2} )</td>
<td>( W_{r4} = \sqrt{\left(W_{r4}\right)^2 + \left(W_{r5} - W_{r6}\right)^2} )</td>
</tr>
<tr>
<td>Total Thrust Load</td>
<td>( W_z ) (may be applied to either Bearing A or B)</td>
<td>( W_z ) (may be applied to either Bearing C or D)</td>
</tr>
</tbody>
</table>

Equivalent Load (\( W_b \)) for each of the bearing is found out using the above load calculations. Bearings A and B are on worm shaft. Bearings C and D are on worm wheel shaft.
5.6.1.3 Churning Losses in Oil

Since the worm is partly immersed in oil in the splash type of lubrication, losses occur in the oil due to churning. The power loss due to churning of the lubricating oil is estimated as [70]

\[ H_{chum} = 0.075 \sqrt{\nu b(Ed)^{0.5}} \quad \text{(5.27)} \]

Since the viscosity of lubricating oils is generally known in centistokes, a conversion chart has been used to obtain the equivalent viscosity in Engler's units, as equation (5.27) requires the oil viscosity in Engler's degrees.

The frictional losses due the three sources mentioned above, namely, the tooth losses, bearing losses and the churning losses, are estimated for various operating conditions on single reduction worm gearboxes of different sizes. As a sample set of values, the tooth losses, bearing losses and the churning losses in an 8" gearbox for different conditions of load, input speed and oil viscosity are shown in Figure. 5.3 to Figure. 5.5. The figures show that around 75%-85% of the total losses are due to tooth friction while the remaining losses are due to bearing friction and churning of oil. The proportions of these losses change with the power being transmitted by the gearbox.

![Figure 5.3 Variation of frictional losses with applied load](image-url)
5.6.2 Convective Heat Transfer Coefficients

The enclosed type of gears are usually lubricated by splash system of lubrication, in which case, the oil level in the gear box is maintained so that the teeth of the bottom wheel just dips into the oil. The arrangement is as shown in Figure.5.6. Alternatively a pressure circulating system may be used, in which, oil is sprayed on the teeth close to the point of engagement and is re-circulated either directly from the bottom of the gearbox or through an oil tank. But, due to the simplicity of the arrangement, splash lubrication is the most commonly employed method in gearboxes.
Splash lubrication is suitable where pitch line speeds are low, up to 5 m/sec for spur, helical and bevel gears and up to 7 m/sec for worm gears. With splash lubricated gears, it is most important that the oil level is not too high; otherwise excessive churning of oil will occur with consequent rise in oil temperature and power loss. The depth, to which the bottom wheel should dip into the oil, when stationary, is generally between 20 mm to 40 mm depending upon the size of the gear. Usually twice the tooth depth is sufficient for splash lubrication to minimize excessive churning. Where high-powered gear sets running at high speeds are used, pressure-circulating systems with oil coolers are preferred to reduce churning [71].

For the splash system of lubrication, the transfer of heat from the teeth-meshing zone to the atmospheric air occurs in three different stages. The first is the heat transfer from the worm surface to the oil in the gearbox, the second is from the oil to the inside casing, and the third is from the inside casing to the outer surfaces of the casing by conduction through the wall of the casing, from where, it is dissipated to the atmosphere.

The heat transfer from the surface of the worm to the oil contained in the gearbox is modeled using the principle of rotating disks as [46]

\[ h_{wo} = NuK \left( \frac{\omega}{\nu} \right)^{0.5} \quad \text{...(5.28)} \]

\[ Nu = 0.335(Re)^{0.5} \quad \text{...(5.29)} \]

Since the worm is partly immersed in oil, an average coefficient, arrived at based on the properties of air and those of the oil is used.
The heat transfer from the oil to the inner wall of the casing by convection is assumed to be effected by the splash of the oil and hence the convective coefficient used for the heat transfer between the worm and the oil is used here also.

On the outer surface of the casing, the heat transfer coefficient corresponds to that of natural convection. Thus, it is modeled as

$$h_{ext} = \frac{Nu K}{Z}, \quad \text{...(5.30)}$$

where, the Nusselt number $Nu$ is given by [74]

$$Nu = \left[0.825 + \frac{0.38 \chi Gr Pr^{0.167}}{[1 + (\frac{0.492}{Pr})^{0.602}]^{0.296}}\right]^2 \quad \text{...(5.31)}$$
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The convective heat transfer coefficients as explained above are estimated for different operating conditions in single reduction gearboxes of different sizes. Since the properties of the lubricating oil such as density, viscosity, thermal conductivity and specific heat capacity are temperature dependent, the variations of these parameters with temperature are accounted while estimating the convective coefficients. Figure.5.7 shows the variation of the convective heat transfer coefficients with oil temperature. The variations of the coefficients with viscosity of oil and the input speed are shown in Figure.5.8 and Figure.5.9.

Figure.5.7 Variation of convective coefficients with oil temperature

Figure.5.8. Variation of convective coefficients with oil viscosity
5.7 RESULTS AND DISCUSSIONS

The thermal model developed for investigating the thermal behaviour of worm gearboxes was tested on gearboxes of different sizes under different operating conditions to predict the bulk temperature of the gears, the bulk temperature of the oil and the casing temperature. The results of the analysis are presented here.

5.7.1 Comparison with Experimental Data

In order to validate the results obtained by the lumped mass approach, the methodology is first applied to the gearboxes tested in the experimental setup. The temperature data are obtained from the lumped mass approach by setting the time interval $dt$ to 600 s. These results are compared with the respective experimental results.

Figure 5.10 to Figure 5.12 show the transient temperature variations obtained using the lumped mass method in comparison with those obtained from experiments. The figures show the variation of the oil temperature with time in three different experiments of the L27 array against those predicted by the lumped mass approach. It may be observed that the time taken to attain steady state as predicted by the lumped mass method is slightly
longer than the actual time observed in the experiments. But the general trend is found to be similar in the two cases.

Figure 5.10 Comparison of transient oil temperatures in 3" gearbox

Figure 5.11 Comparison of transient oil temperatures in 4" gearbox

Figure 5.12 Comparison of transient oil temperatures in 5" gearbox
A comparison of the steady state oil temperatures obtained by the model developed with the corresponding experimental results is given in Table 5.2. It is found that the steady state temperatures predicted by the lumped mass approach are lesser than the corresponding experimental values in all the cases. Within the experimental range, the there is a maximum deviation of 7.6°C, leading to the percentage error ranging from 9 to 14. The lumping of masses, and the assumption that the entire quantity of oil contributes to the transfer of heat from inside the gearbox to the casing by convection might have contributed to the lower values of predicted temperatures. In the actual case, the oil, very near to the rotating worm only transfers heat to the casing by convection.

Since the results obtained by the lumped mass model developed are in reasonably good agreement with the experimental values, the model developed is further used for investigations on the effect of various parameters on the temperature rise of the different elements of the gearbox and the model simulation results are presented in the forthcoming sections.
Table 5.2 Comparison with experimental results

<table>
<thead>
<tr>
<th>Expt. No</th>
<th>Steady state oil temperature °C</th>
<th>Deviation °C</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expt</td>
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</tr>
<tr>
<td>1</td>
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</tr>
<tr>
<td>27</td>
<td>50</td>
<td>43.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>
5.7.2 Temperature Variations in Different Elements

The increase in the temperatures of the gear tooth, the oil in the gearbox and the gearbox casing with time in gearboxes of different sizes is shown in Figure 5.13 to Figure 5.16.

Figure 5.13 Temperature variations in 3-inch gearbox

Figure 5.14 Temperature variations in 4-inch gearbox

Figure 5.15 Temperature variations in 6-inch gearbox
From the above figures, it is found that the gear tooth bulk temperature is higher than the oil temperature, in all the gearboxes for the same applied load conditions. The oil temperature and the casing temperature are found to be almost the same in all the cases. The difference between the gear tooth bulk temperature and the oil temperature is very high in smaller gearboxes and the difference reduces as the gearbox size increases. It may also be observed that it takes a longer time to reach steady state in gearboxes of larger sizes for the same conditions of output torque, input speed and grade of the lubricating oil. The time taken for attaining steady state in different gearboxes under the same conditions of output torque, input speed and lubricating oil is shown in Figure.5.17.

The variation of the bulk temperature of the gear tooth and the temperatures of the lubricating oil and the gearbox casing for different applied load conditions are shown in Figure.5.18 and Figure.5.19. It is observed that with increase in the output torque of the gearbox, the temperatures increase proportionally and the time taken for attained the steady state also increases.

\[ \text{Temperature, } ^\circ \text{C} \]
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Figure 5.17 Time to reach steady state in different gearboxes

Figure 5.18 Temperature variations in gear tooth with output torque

Figure 5.19 Temperature variations in oil with output torque
5.7.3 Effects of Various Parameters

The effects of various parameters on the steady state temperatures prevailing in the different zones of the gearboxes are presented here.

5.7.3.1 Effect of Applied load

The effect of the load applied on the steady state tooth bulk temperature and the oil and casing temperatures of the gear box are shown in Figure.5.20 to Figure.5.23. It is observed that the steady state temperatures increase linearly with an increase in the torque on the output shaft. This is in agreement with the results presented for spur gear by Townsend and Akin [47]. But the rate of increase of the gear tooth temperature with load is higher compared to the rate of increase of oil and casing temperatures. But the rate with which the gear tooth bulk temperature increases with increase in load decreases in gearboxes of larger sizes. Table 5.3 shows the increase in steady state temperature of the bulk of the gear tooth and the oil/casing in the different gearboxes for an increase of 1 kW of power, when the input speed is 1440 rpm and the lubricating oil used has a viscosity of 150 cSt. Thus, every kW of input power causes the steady state gear tooth temperature to increase by 46° C in a 3-inch gearbox, 24° C in a 4-inch gearbox, 8.5° C in a 6-inch gearbox and 3.3° C in an 8-inch gearbox. It also results in the increase of the oil and the casing temperatures by 11° C in a 3-inch gearbox, 8.5° C in a 4-inch gearbox, 3.5° C in a 6-inch gearbox and 1.2° C in an 8-inch gearbox.
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Figure 5.20 Variation of steady state temperature with output torque in 3-inch gearbox

Figure 5.21 Variation of steady state temperature with output torque in 4-inch gearbox

Figure 5.22 Variation of steady state temperature with output torque in 6-inch gearbox
Figure 5.23 Variation of steady state temperature with output torque in 8-inch gearbox

Table 5.3 Increase in steady state temperature with increase in input power

<table>
<thead>
<tr>
<th>Gearbox size</th>
<th>Increase in steady state temperature, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gear tooth</td>
</tr>
<tr>
<td>3-inch</td>
<td>46</td>
</tr>
<tr>
<td>4-inch</td>
<td>24</td>
</tr>
<tr>
<td>5-inch</td>
<td>8.5</td>
</tr>
<tr>
<td>6-inch</td>
<td>3.3</td>
</tr>
</tbody>
</table>

5.7.3.2 Effect of input speed

The variation of the steady state gear tooth bulk temperature and the oil and casing temperatures with variations in the input speed of the gearboxes is shown in Figure 5.24. From the figure, it is found that the steady state temperatures of the different elements of the gearbox increase almost linearly with increase in the input speed of the gearbox. This is similar to the trends shown by Townsend and Akin [47] for spur gears. But the effect of the input speed on the temperatures is found to be very small compared to the
effect of load applied. This could be due to the reason that even though the heat generation rates increase with increase in speed under the same output torque conditions, the heat dissipation rates also increase due to the increased convective heat transfer coefficients at higher speeds of the rotating elements. Hence, the increase in temperatures is very small with increase in the input speed.

![Figure.5.24 Effect of input speed on steady state temperatures](image)

5.7.3.3 Effect of viscosity of lubricating oil

The effect of the viscosity of the lubricating oil used, on the steady state temperatures is shown in Figure.5.25. It is seen that the gear tooth bulk temperature remains unaffected with increase in the viscosity grade of the lubricating oil, whereas, there is a slight reduction in the oil and the casing temperatures. In the figure shown, in a 4-inch gearbox operating at an input speed of 710 rpm transmitting a power of 1.5 kW, the reduction in the oil temperature achieved for an increase of 100 cSt in the kinematic viscosity of oil is 1°C. Thus, very small reductions in oil temperatures are observed for the all gearboxes analysed under different operating conditions.
5.7.3.4 Effect of external forced convection by fan cooling

One of the effective ways of heat dissipation from the gearbox to the external air is the application of forced convection on the external surface of the casing by means of an external fan. In this part of the investigations, the effectiveness of using a fan to externally cool the gearbox is studied. The convective heat transfer coefficient for the forced convection by fan is estimated using the relationship [72]

\[
h_f = f(0.919 + 0.00007(T_m - T_a))^{0.15} \tag{5.32}
\]

The effect of fan cooling on the steady state temperature of the gear tooth and that of the oil for various gearboxes are shown in Figure.5.26 to Figure.5.29. In all the cases, it is quite clear that forced convection by fan cooling is effective in reducing the steady state temperatures prevailing in the gearbox. The reduction in the steady state temperatures is significant up to a fan velocity of about 20 m/s, beyond which, the effectiveness of the cooling decreases.
Figure 5.26 Effect of fan cooling in 3-inch gearbox

Figure 5.27 Effect of fan cooling in 4-inch gearbox

Figure 5.28 Effect of fan cooling in 6-inch gearbox
5.7.3.5 Effect of level of oil

In order to predict the effect of the level of oil in the gearbox on the steady state thermal behaviour of the elements of the gearbox, the level of the oil in the gearbox is varied and the thermal analysis carried out. The results obtained from this analysis are shown in Figure 5.30. It is found that the height of the lubricating oil from the bottom of the casing, and hence, the quantity of oil contained in the gearbox has no effect on the steady state temperatures prevailing in the gearbox. This can be attributed to the reason that even though the increased quantity of oil helps in reducing the temperature rise, it also results in an increased churning loss and hence offsets the advantage obtained by the larger quantity of oil absorbing the heat. Hence, it is desirable to have the least possible quantity of oil in the gearbox so as to minimise the quantity of oil required and the weight of the unit, which is an important factor in mobile systems. Therefore, it is sufficient to maintain an oil level such that a part of the worm remains dipped in the oil.
5.7.3.6 Effect of fins

To study the effectiveness of fins in reducing the equilibrium temperature of the gearbox units, the fin area is increased in steps and the thermal analysis carried out to determine the steady state temperatures. The variation in the steady state temperatures of the gear tooth, the oil and the casing with increase in fin area is shown in Figure 5.31. With increase in the area of the fins, these temperatures are found to decrease slightly. The percentage reduction in the tooth temperature and the oil temperature with increase of fin area in a 4-inch gearbox is shown in Figure 5.32. Thus, we see that every 10% increase in the fin area results in a 38% reduction in the oil and the casing temperatures and a 15% reduction in the gear tooth bulk temperature. But at the same time, from Figure 5.33, we find that every 10% increase in fin area leads to a 30% increase in the weight of the gear box. Hence, to achieve a few degrees of reduction in the gear tooth and the oil temperatures, the weight of the gearbox needs to be increased by a significant value. This is particularly of more concern in gearboxes used in mobile systems.
Figure 5.31 Effect of fin area on steady state temperatures

Figure 5.32 Reduction in temperature with increase in fin area

Figure 5.33 Effect of fin area on gearbox weight
5.7.3.7 Effect of casing wall thickness

The effect of varying the thickness of the walls of the gearbox casing on the steady state temperatures of the various components of the gearboxes is shown in Figure 5.34. It is clear that the wall thickness does not have any influence on the gear tooth bulk temperature or the temperatures of the oil or the casing. Therefore, the thickness of the walls of the gearbox casings may be decided based on the mechanical requirements. Hence, the casings may be provided with the minimum thickness required to withstand the mechanical loads imposed on them during the transmission of power.

![Figure 5.34 Effect of wall thickness on steady state temperatures]

5.8 CONCLUDING REMARKS

The methodology adopted for investigations on gearboxes for their thermal characteristics based on a lumping of masses approach has been presented in this chapter. The method adopts a simple heat balance and does not need special skills or sophisticated software. The transient bulk temperatures of the gear pair, the lubricating oil and the casing are predicted by this technique. The temperatures estimated by this approach are compared with those obtained using experiments and the following are the major conclusions arrived at:
• the general trend observed in the transient temperatures matches with that observed in the experiments, but the temperatures predicted by the proposed method are lesser than the corresponding actual temperatures measured in experiments. This can be attributed to the lumping of masses and the assumption that the entire depth of oil participates equally in the transfer of heat from inside the gearbox to the casing by convection.

• the method predicts longer time to attain steady state than that was practically observed.

• the method results in very closely the same temperatures for the bulk of the oil and the casing, whereas, there is quite some difference in reality.

• the investigations of the effect of operating parameters show that the steady state temperatures of the bulk of the tooth, the oil and the casing increase linearly with increase in the applied load and increase slightly with increase in the input speed; these trends are in general agreement with those published in literature for spur gears; these temperatures decrease slightly with increase in the viscosity of the lubricating oil, and again decrease slightly with increase in fin area; the level of oil in the gearbox and the thickness of the wall of the casing do not affect the steady state temperatures significantly; cooler operation can be achieved by force cooling with the help of external fan.

• the difference between the gear tooth bulk temperature and oil temperature is higher in smaller gearboxes and reduces as the gearbox size increases. In larger gearboxes, for the same load, the temperature rise of the gear pair will be less. Moreover, larger hub surface is available for heat dissipation by convection. The above two facts contribute to the smaller difference between the gear tooth temperature and the oil temperature in larger gearboxes.
the lumped mass approach assumes that the heat generated at the meshing zone is immediately absorbed into the bulk of the gear, thus resulting in smaller temperature rises than the actual case. As time progresses, in the actual case, the rate at which heat is absorbed by the gears will slow down resulting in slower rate of rise in temperature with time. This results in larger difference between the results predicted by the lumped approach and the experiments in the beginning and the difference between the two sets of results reduce with increase in time.