6.1.1. Introduction

In recent years the problem of velocity fields in the outer layers of stars has become important because of the stellar winds which cause mass loss, which in turn affects the process of stellar evolution. The ultra-violet absorption in strong lines indicates the magnitude of the terminal velocities of the flows. However, the problem is the interaction of the velocity fields in the deeper layers where the stellar photospheres merge with the outer layers and the stellar winds. The flow gets accelerated through the sonic point and becomes supersonic flow.

The flow becoming supersonic depends on the complex interaction of dynamical processes of the matter and radia-
tion. Therefore one requires the knowledge of the velocity profiles to study the internal dynamics of the matter in the outer layers and in deeper layers.

The line profiles which we observe in early type stars and supergiants are the result of large scale expansion of the matter in the outer layers, rotation of the atmospheres and perhaps the turbulent velocity fields which are responsible for the non-thermal line broadening. The line broadening may be due to microturbulence or macroturbulence or the velocity gradient effects in an expanding atmospheres (Sletteback 1956). There is possibility of non-radial pulsational modes (Stothers and Chin 1977; Vemury and Stothers 1977). This physical effect produces asymmetry and broadening while stellar rotation produces symmetric broadening of the lines.

By means of very sensitive analytical techniques or by constructing physically meaningful models one might be able to unravel the different effects described above. Recently Fourier analysis has been employed to find out the detailed velocity structure in the outer layers. In the case of static atmospheres this technique yields fairly reliable estimates of both stellar radial velocity and turbulent velocity (Gray 1975; 1976, 1978; Smith and Gray 1976). These techniques have been employed by Ebbets (1978) and Duval and Karp (1978) for the expanding atmospheres and however, the accuracy of the results remain open to question.

The calculations of line profiles in expanding atmospheres is more complicated than for the static atmospheres (see Pecker and Thomas (1961) for general discussion.)
Underhill (1947) calculated the profiles of the lines in an atmosphere with plane parallel stratification with a depth-independent expanding velocity. She showed that with such a velocity field the equivalent width of a line does not change although the shape of the flux profiles becomes asymmetric by the variation of the projected line of sight velocity over the stellar disc. Van Hoof and Deurnick (1952) calculated line profiles with a constant expansion velocities by convolving an intrinsic line profile with a broadening function which accounts for the variation of line of sight velocity over a limb darkening disc. They found that weak lines are more affected than strong lines in producing the asymmetry of the lines. None of the above methods have taken velocity gradients correctly into account.

Kubikowski and Ciurla (1965) calculated the equivalent widths in an atmosphere moving with velocity gradients; they showed that the velocity gradients increase the equivalent widths and they also show that velocity gradients tend to desaturate strong lines and prolong the linear part of the curve of growth. They also show that the flat part of the curve of growth is raised and the difference in the raise is attributed to the turbulence, which is actually due to the velocity gradients. Ciurla (1966) has given a more detailed discussion on these things. He showed that in an atmosphere expanding outward with increasing velocity, lines tend to be skewed towards the shorter wavelengths. This result is exactly opposite to the geometrical distortion effect produced by model with constant depth independent expanding velocity. In this case it is assumed that the velocity was increasing linearly with geometrical height and lines
were calculated in LTE which is in pure absorption. This approximation is good only for very thin layers. If one desires to employ thick layers and scattering is to be taken into account (non-LTE effects) one has to study a more complicated transfer problem. Karp (1973) calculated the line profiles assuming LTE and again using hydrodynamic model atmospheres of Cepheids he found that the effects in strong lines are much smaller (1975). More recently Karp (1978) calculated the observed flux from a stellar atmosphere with velocity gradients by using an analytical solution. He used a slowly varying source function which he assumes that it represents scattering process. Worrall (1969) and Canfield (1970) followed the method. These assumptions are not correct conceptually. All the above calculations are done assuming LTE. So one must consider the correct physics in such media. The scattering in the line is important mechanism in formation of the lines. Rybicki (1970), Grant and Peraiah (1972), Simonneau (1973), Noerdlinger and Rybicki (1974), Mihalas, Kunasz, Hummer (1976) have treated the line calculations in non-LTE.

When the velocities are considered in the expanding medium it is important to treat high velocities and solve velocity gradients. If we treat the equation of transfer in the observer's frame (Kunasz and Hummer (1974), Karkofen (1970), Peraiah and Wehrse (1978), Wehrse and Peraiah (1978) it becomes difficult to treat high velocities. Therefore it is necessary to treat such high velocities. In the omov-
ing frame of the gas. Such calculations were first started by Chandrasekhar (1945), Abhyankar (1964a,b and 1965). However, they have treated only coherent scattering by the atom and two stream approximation. The latter assumption is not adequate and gives unphysical results (Noerdlinger and Rybicki 1974). These latter authors have developed a general technique for plane parallel geometry with arbitrary velocity gradients by using a Feautrier type elimination scheme. Based upon Rybicki type elimination scheme, Mihalas and Kunasz (1975) developed the solution of transfer equation in a comoving frame. These methods require large scale computational facilities. Peraiah (1980) developed a solution of the transfer problem in the lines in the comoving frame of the fluid, based on the Discrete space theory and this requires minimum computer time. This method has been employed in calculating the lines in a comoving frame in the moving atmospheres which contains dust (Peraiah, Varghese and M.S.Rao 1987). In the following chapter we shall be using the above technique in estimating the equivalent widths of lines formed in expanding atmospheres with dust.

6.1.2. Hydrogen Lyman Alpha Line

Curve of growth yield important and reasonably good information regarding the chemical abundances in the stellar atmospheres. Generally curves of growth are made use of stationary stellar atmospheres. However observations have established the existence of in many types of stars. The matter in the
stars is in continuous radial motion. Absorption and scattering of radiation by the moving medium creates complications which cannot be revealed through the ordinary curve of growth. Therefore one must study the problem by constructing theoretical models of curves of growth.

The effective optical depth is limited by the velocity gradient in a moving medium. For an electron scattering atmosphere the optical depth is defined as

\[ \tau = \sigma v_{th} \left( \frac{dv}{dr} \right)^{-1} \]  

(6.1.1)

where \( \sigma \) is the electron scattering coefficient, \( v_{th} \) the thermal velocity of the medium in motion and \( \left( \frac{dv}{dr} \right) \) is the velocity gradient. As the radiation force on the lines becomes negligibly small the above relation is used to a fairly good approximation though it is invalid for stellar photospheres. In such situations it is difficult to obtain the information about the number density that is influencing the line formation. We have made investigations based on the effect of expansion velocities of the atmospheres and the presence of dust on the equivalent widths of resonance lines. For this purpose we have chosen Hydrogen Lyman Alpha line whose parameters are well known. We have assumed a spherically symmetric atmosphere containing varying amounts of dust.

6.1.3. Calculations

We have made calculations for a comoving frame and the equation of transfer is written as Peraiah et al.
(1987) and aimed at the investigation of the effects of dust and radial expansion of the outer layers on the equivalent widths. These equivalent widths in turn indicate the total amount of absorption or emission.

Temperature distribution at different radial points of the atmosphere is calculated by assuming that

$$B_\nu(T_r) = \left(\frac{r_0}{r}\right)^2 B_\nu(T_{r_0}) \quad (6-1.2)$$

where $B_\nu$ is the Planck function at frequency $\nu$ and $T_r$ is the temperature at radial point $r$ in the spherical shell. $T_{r_0}$ indicates the temperature at the inner radius of the shell. From the above equation

\[
\left(\frac{2h\nu^3}{c^2}\right) \left(\frac{1}{\hbar \nu/KT_r}\right) = \left(\frac{r_0}{r}\right)^2 \left(\frac{2h\nu^3}{c^2}\right) \left(\frac{1}{\hbar \nu/KT_{r_0}}\right)
\]

\[
\left(\frac{\hbar \nu / KT_r}{e}\right) = \left(\frac{r}{r_0}\right)^2 \left(\frac{\hbar \nu / KT_{r_0}}{e}\right)
\]

\[
h \nu KT_r \left(\frac{e}{r_0}\right) = \left(\frac{r}{r_0}\right)^2 h \nu KT_{r_0} \left(\frac{e}{r_0}\right)
\]

which simplifies to

$$T(r) = \frac{h \nu}{K} \left(\frac{1}{\ln[1+(\frac{r}{r_0})^2(e^{h \nu KT_{r_0} - 1})]}\right) \quad (6-1.3)$$

A temperature of 15,000°K has been assumed at the inner shell $r = r_0$. For the assumed electron density in section 6.1.2 we have obtained the temperature distribution as shown in Figure 6.1.1.
Stellar atmospheres consist of a mixture of atoms at various stages of ionization and excitation. In normal stellar atmospheres hydrogen is the most abundant constituent. At high temperatures hydrogen is appreciably ionised, it contributes most of the electrons to the gas. Hence for a pure hydrogen gas the electron density is given by (Mihalas 1978)

\[
ne(H) = \phi_H^{-1}(N_H + 1)^{1/2} - 1
\]  

(6-1.4)

Where

\[
\phi_H = \frac{U_1(T)}{U_2(T)} C_1 T^{-3/2} e^{X_1/KT}
\]  

(6-1.5)

and

\[
C_1 = 2.07 \times 10^{-16} \text{ CGS units}
\]  

(6-1.6)

\(U_1\) and \(U_2\) are partition functions.

Total number of particles is given by

\[N = \text{Neutral atoms } (N_o) + \text{Number of ions} + \text{number of electrons.}\]

\[N = N_o + n_p + n_e \quad \text{(for hydrogen } n_p = n_e)\]

Hence

\[N = N_o + 2n_e \]  

(6-1.7)

Substituting this in (6-1.4)
\[ n_e(\text{H}) = \frac{1}{\Phi} \left[ \{(N_0 + 2n_e)\Phi + 1\}^{\frac{1}{2}} - 1 \right] \]  

\[ n_e+1 = [(N_0 + 2n_e) + 1]^{\frac{1}{2}} \]

\[ (n_e + 1)^2 = (N_0 + 2n_e) + 1 \]

or

\[ n_e = N_0 \]

Hence

\[ N_0 = \Phi n_e^2 \]  

(6.1.9)

Distribution of neutral atoms in the atmosphere is plotted in Figure (6.1.3).

For obtaining absorption coefficient we need to know the number of hydrogen atoms in level 1 \((n_1)\) . This is done by using Boltzmann law which gives the fundamental relationship of the fraction of atoms in \(r^{th}\) level of excitation in terms of the total number \(N_1\) in the \(i^{th}\) stage of ionization namely

\[ \frac{N_{1,r}}{N_1} = g_{1,r} \frac{\varepsilon_{1,r}}{U_{1}(T)} e^{-\frac{\varepsilon_{1,r}}{KT}} \]  

(6.1.10)

where \(\varepsilon_{1,r}\) is excitation energy of the level above ground level and \(g_{1,r}\) is the statistical weight of that level, \(T\) \(i\) is the temperature \(U_{1}\) is the partition function. If \(N_1\) and \(N_2\) are the number of hydrogen atoms in level 1 and 2 respectively and \(g_1\), \(g_2\) are the corresponding statistical.
weights comparison of population of 2nd level to that in ground level is given by Boltzmann equation (Aller 1963)

\[
\log \frac{N_2}{N_1} = -\theta \epsilon + \log \frac{g_2}{g_1} \tag{6-1.11}
\]

where \( \theta = \frac{5040}{T} \) and \( \epsilon \) the excitation energy in electron volts.

Total number of neutral atoms

\[ N_o = N_1 + N_2 \]

Let \[ \frac{N_2}{N_1} = a \]

hence \[ N_o = N_1 + aN_1 \]

or \[ N_1 = \frac{N_o}{1+a} \tag{6-1.12} \]

Number of neutral atoms \( N_o \) is obtained from Saha's equation by defining the electron pressure and temperature \( T \) (Aller 1963)

\[
\log \frac{N_1}{N_o} P_e = -\frac{5040}{T} I + 2.5 \log T - 0.48 + \log \frac{1}{u_o(T)} \tag{6-1.13}
\]

where

\( I \) = ionization potential in volts

\( P_e \) = Electron pressure in dyne/cm²

\( N_1 \) = density of ionized atoms

\( N_o \) = density of neutral atoms
\( \Upsilon_{1}(T) \) = Partition function for the neutral atoms

\[ N_1 = n_p + n_e = 2n_e \]  

(6.1.14)

for a given \( P_e \) and \( T \) we can calculate \( \left( \frac{N_I}{N_0} \right) \).

For hydrogen Lymann Alpha Line we can calculate the absorption coefficient

\[ \chi_1(\nu) = \frac{n_1B_{12}h\nu}{4\pi} \phi_{\nu} \left( 1 - \frac{n_2}{n_1} \frac{g_1}{g_2} \right) \]  

(6.1.15)

but \( A_{21} = \frac{2h\nu^3}{c^2} B_{21} \)

and

\[ g_2B_{21} = g_1B_{12} \quad B_{21} = \frac{g_1}{g_2} B_{12} \]

or

\[ A_{21} = \frac{2h\nu^3}{c^2} \frac{g_1}{g_2} B_{12} \]

i.e.

\[ B_{12} = A_{21} \frac{g_2}{g_1} \frac{c^2}{2h\nu^3} \]

hence

\[ \chi_1(\nu) = n_1A_{21} \frac{c^2}{2\nu} \frac{\phi_{\nu}}{4\pi} \left( \frac{g_2}{g_1} - \frac{n_2}{n_1} \right) \]  

(6.1.16)

where \( A_{21} \) is Einstein coefficient for spontaneous emission, \( \nu \) is speed of light and \( \nu \) is the central frequency of Hydrogen Lyman Alpha Line and \( \phi_{\nu} \) is the profile function of the line such that
We have employed Doppler profile. Optical depth in each shell of the medium is plotted in Figure 6.1.4 and the total optical depth up to every shell is plotted in Figure 6.1.5. The medium is assumed to contain dust in addition to gas. The amount of dust and its distribution is represented by the dust optical depth. The spherically symmetric medium is expanding radially outwards. We have studied the expansion with and without velocity gradients. Line Transfer equations (5-2) and (5-3) are solved in a comoving frame as explained in the Chapter 5.

In solving the transfer equations phase function of dust is assumed to be isotropic.

6.1.4. Results

We have adopted a non-LTE line with a two-level atom approximation, set $\epsilon = 0$, hence no internal source is assumed.

Following boundary conditions are imposed:

$\epsilon = 0, \quad \beta = 0 \quad B(T(r), x) = 0$

and

$U^+(\tau = 0, \mu_j, x_1) = 0$
$V_A$ and $V_B$ are the velocities in Doppler units at A and B Figure 1.2.

We have considered two cases viz.
(1) $V_A = V_B$ and
(2) $V_A = 0, V_B > 0$

In the first case we have an expanding spherical shell and in the second case we have expanding shell with velocity gradients. The profiles of Hydrogen Lyman Alpha Line are shown in Figures from 6.1.6 to 6.1.10 for specific dust optical depths $t = 0, 0.1, 0.5, 1, 5$. The figures give the relation between the ratio $\frac{F_x}{F_c}$ and $x$ where

$$F_x = 2\pi \int_I x \mu d\mu$$  (6-1.18)

$$F_c = 2\pi \int_I c \mu d\mu$$  (6-1.19)

$I_x$ and $I_c$ being the intensities in the line and continuum respectively.

Figure 6.1.6 is for $V_A = V_B = 0$ i.e. for a static medium for various dust optical depths. It is observed that as the dust optical depth increases more and more photons are scattered into the line centre indicating diminution of absorption depth.

Figure 6.1.7 is for outer velocity $V_B = 5$ mean thermal units while $V_A = 0$ introducing a velocity gradient. Here we notice P-Cygni type profiles. Absorption is being
shifted towards violet side while emission peak remains at the centre of the line. When dust optical thickness is increased the emission is reduced considerably while more photons are scattered by the dust into the absorption core of the line. It may yield an obvious inference that dust has opposite effects in emission wing as compared to that in the absorption core.

In Figure 6.1.8 the expansion velocity is increased to $V_B = 10$ mtu, same effect is observed as in the previous case.

In Figure 6.1.9 and 6.1.10 we have considered the spherical shell to be expanding with constant velocities $V_A = V_B = 5$ mtu and 10 mtu respectively. Here we observe that the emission wing has become broader and the absorption core has been narrowed. Further the shifts of absorption cores in both the cases from the centre of the line are almost the same as that of the expansion velocities used. Almost symmetrical broadening of the emission wing is also noticed.

In Figures 6.1.11 to 6.1.15 we have shown the relation between neutral atoms and the equivalent width of the line which has been explained in Chapter 1.4 in detail.

In Figure 6.1.11 we have the relation plotted for a static medium. This has a close resemblance to the curve of growth. We have a linearly increasing portion then a flat part and further increasing linearly.

In Figure 6.1.12 we have velocity gradient $V_B = 5$ mtu and $V_A = 0$. 
This shows a slight modification in the flat portion of the curve of growth. It may be noticed here that for same equivalent width with a dusty medium results in a larger number of neutral atoms. This is a very important observation. It hints that caution is to be exercised in deriving the number of neutral atoms producing the line if the presence of dust is noticed through infrared observations or by any other means. Figure 6.1.13 is for increased expansion velocity with dust. It is showing similar effect. In Figures 6.1.14 and 6.1.15 we have considered steadily expanding spherical shell with velocities of expansion 5 m\text{tu} and 10 m\text{tu} respectively without velocity gradients. In these cases though the increasing part of the curve of growth is the same as one expects in static medium, however after equivalent width reaches a maximum with the increasing number of neutral atoms, the width falls and the line becomes more of emission, which means that the emission part of P-Cygni type of profiles is larger than the absorption part. The presence of dust increases the equivalent width.

6.1.5. Conclusion

Calculations have been performed to show the effect of velocity of expansion of the stellar atmospheres and the presence of dust in the atmosphere on the equivalent width of Hydrogen Lyman Alpha line simultaneously. It is found that there is noticeable change in the equivalent widths due to presence of dust in the expanding medium.
Figure 6.1.1. Temperature at various points in the spherical shell in which hydrogen Lyman $\alpha$ line is forming. Here $T(r_0) = 15000$ K. Shell No.1 is at $r = r_{\text{max}}$ and shell No.100 is at $r = r_0$. 
Figure 6.1.2. Electron density ($N_e$) at different shells $N$.

Figure 6.1.3. Number of neutral atoms ($N_0$) at different layers ($N$) of the stellar atmosphere in Hydrogen Lyman $\alpha$ line calculations.
Figure 6.1.4. Optical depth in each shell of the stellar medium.

Figure 6.1.5. Total optical depth at various layers of the medium.
Figure 6.1.6 Hydrogen Lyman α line profiles formed in a static medium with different amounts of dust ($\tau_d$). Distribution of dust is constant throughout the atmosphere.

Figure 6.1.7 Line profiles of hydrogen Lyman α formed in a medium moving with velocity of expansion $v_b = 5$ mtu (with velocity gradient).
Figure 6.1.8. Same as in Figure 6.1.7 but the velocity of expansion is equal to 10 mtu.

Figure 6.1.9. Hydrogen Lyman α line profiles formed in a medium expanding with velocity 5 mtu and without velocity gradients.
Figure 6.1.10. Same as in Figure 6.1.9. but the velocity of expansion is equal to 10 mtu.

Figure 6.1.11. Variation of equivalent widths of the hydrogen Lyman α line with the increasing number of neutral atoms (log No) for a static medium and for dust optical depths (τ_d).
Figure 6.1.12. Same as in Figure 6.1.11 but the atmosphere is expanding with velocity gradient, expansion velocity is equal to 5 mtu.

Figure 6.1.13. Same as in Figure 6.1.12 but velocity of expansion is equal to 10 mtu.
Figure 6.1.14. Same as in Figure 6.1.12 but without velocity gradient i.e. $V_A = V_B = 5$.

Figure 6.1.15. Same as in Figure 6.1.12 but the velocity ex in 10 m/s.
6.2.1 Dust in the Spherical Shells

Presence of dust in the outer layers of stars has been inferred from the infrared observations of these stars. Br_α Alpha line radiation was observed in Becklin-Neugebauer (BN) object in Orion by Grasdalen (1976). These objects show the presence of ionised gas and several of such objects as BN and CRL 490 seen to possess dusty shells around a hard ionised gas core. The majority of Be stars have circumstellar dust to give rise to an infrared excess over and above that of free-free emission. Br_ν, Br_ν Br_ν infrared lines have been observed in the compact molecular clouds by Simon et al (1981, 1985). Persi et al (1983) derived mass loss rates for 15 O-type stars based upon infrared photometric observations from 2.3 μm to 10 μm. Infrared observations of several objects such as gaseous nebulae, active galactic nuclei, T-Tauri stars, atmospheres of cool super giants show the presence of dust in the outer layers of these objects. Allen's study (1973) of several early type emission line stars has revealed that Bremsstrahlung and thermal emission from dust grains are the causes of infrared excess. Giesel (1970), Allen and Pol Swings (1972) have found that almost all stars with prominent infrared continua appear to have circumstellar envelopes. Schwartz et al (1983) studied the far infrared and submillimeter mapping of S140 IRS and have concluded that there is a good coupling between the dust and gas. Huggins et al (1984) derived abundances in the envelopes of IRC+10216 line analysis following the approach of Kwan and Hill (1977).
and Henkel et al (1983). Felli studied the infrared emission from extended stellar envelopes.

Line formation calculations have taken into account of the velocity of expansion, presence of dust, geometrical extension of outer shell and the chemical species present. We have considered non-LTE, two-level atom approximation and the dust is assumed to scatter isotropically. The envelope is divided into 100 shells and the total optical depth is about 300. We have neglected absorption due to dust by setting albedo \( \omega = 1 \). We have tried two types of expanding shell (1) with velocity gradient and (2) without velocity gradients. Recently there have been calculations including dust and velocity of expansion (Peraiah and Wehrse 1978, Wehrse and Peraiah 1979, Hummer and Kunasz 1980, Wehrse and Kalkofen 1985, Peraiah et al 1987). Observer's frame method of solving is disadvantageous if scattering is to be considered because of large number of angles and frequencies that have to be used to obtain accurate results. For such flows, a solution in the comoving frame is better suited.

6.2.2. Calculations

Radiative transfer equation in a comoving frame were first explored by S. Chandrasekhar (1945), Abhyankar (1964, 1965), Mihalas et al (1975) and Mihalas (1978). Here we perform the calculations taking the equation of transfer written as (Peraiah et al 1987)
Similarly for an oppositely directed beam we have

$$\begin{align*}
-\mu \frac{3I(r,\mu,x)}{dr} + \frac{1-\mu^2}{r} \frac{3I(r,\mu,x)}{d\mu} &= K_L(r)[\phi(x)+\beta][S(r,\mu,x)-I(r,\mu,x)] \\
+ \{(1-\mu^2) \frac{V(r)}{r} + \mu^2 \frac{dV(r)}{dr}\} \frac{I(r,\mu,x)}{dx} \\
+ K_{dust}(r)\{S_{dust}(r,\mu,x)-I(r,\mu,x)\} \quad (6\cdot2.2)
\end{align*}$$

where $\mu = \cos \theta \epsilon(0,1)$ and $K_L(r), \beta, \phi(x), S$ etc. have usual meaning as explained in Chapter V.

$\epsilon$ the probability per scatter that a photon is thermalized due to collisional de-excitation is given by

$$\epsilon = \frac{C_{21}}{C_{21}^0 + A_{21}[1-e^{-\frac{-hv \epsilon_0/\nu}{KT_e}}]} \quad (6\cdot2.3)$$

where $C_{21}$ is collisional de-excitation rate and $A_{21}$ is spontaneous emission rate. $I(r, \mu, x)$ is the specific intensity of the ray making an angle $\theta = \cos^{-1} \mu$ with the radius vector $r$ at a standardised frequency $x$ given by.
\[ X = \frac{\nu - \nu^o}{\Delta} \]  

(6-2.4)

\( \Delta \) being mean thermal Doppler width. \( \nu(r) \) is the velocity of expansion of gases in Doppler units. Normalised line profile (Doppler) \( \phi(x) \) is employed

\[ \int_{-\infty}^{+\infty} \phi(x) dx = 1 \]  

(6-2.5)

For a two-level atom approximation the statistical equilibrium equation is given by

\[ N_1 \{ B_{12} \int_{-\infty}^{+\infty} \phi(x) J(x) dx + C_{12} \} = N_2 \{ B_{21} \int_{-\infty}^{+\infty} \phi(x) J(x) dx + C_{12} + A_{24} \} \]  

(6-2.6)

where \( B_{12} \) and \( B_{21} \) are Einstein coefficients and \( N_1, N_2 \) are the population densities in levels 1 and 2 respectively.

The quantity \( S_{\text{dust}}(r, \mu, x) \) is the source function due to dust and is given by

\[ S_{\text{dust}}(r, \mu, x) = (1 - \omega) B_{\text{dust}} + \frac{\omega}{2} \int_{-\infty}^{+\infty} P(\mu, \mu', r) I(r, \mu', x) d\mu' \]  

(6-2.7)

Where \( B_{\text{dust}} \) is the Planck function for the emission, \( \omega \) is the albedo for single scattering and \( P(\mu, \mu', r) \) is the phase function. Emission from dust is not having much influence on the profile shape hence is neglected. Equivalent width is calculated by using the formula

\[ W = \int_{-\infty}^{+\infty} \left( 1 - \frac{\nu - \nu^o}{\Delta} \right) dx \]
Where $a$ is half band width of the line i.e. $a = \max |x|$, $F_c$ is the flux in the immediate neighbourhood continuum of the line and $F_x$ is the flux at point $x$.

Solution to equations (6-2.1 and 6-2.2) is obtained on the lines explained in detail in Chapter 5.

6.2.3 Results and Discussions

The equation of transfer is solved as described earlier. We have chosen trapezoidal points for frequency integration and Gauss-Legendre points for angle integration. We have employed nine frequency points with one point always at the line centre and two angles: i.e. $I = 9$ and $m = 2$ in each half space. This brings the working matrix of calculations (18x18). This is accurate enough to give precision permitted by the computer (Mighty Frame II located at the Indian Institute of Astrophysics, Bangalore). We have several physical situations to study, hence the number of parameters will be quite large and therefore we have to restrict to physically meaningful parameters. We have set the dust density constant throughout the medium.

Boundary Conditions: In the case of a purely scattering medium it is assumed that there is no radiation entering the shell from outside where the radius $r = R$ and the optical depth at this point is 0 ($\tau = 0$) while unit intensity is incident at $r = A$ and the optical depth
here is maximum i.e. \( \tau_{\text{max}} = T \). Further when there is emission from within the medium radiation is assumed to be incident neither at A nor at B. Boundary conditions imposed here are

\[
U^{-}(\tau = T, \mu) = 1 \\
(\epsilon = 0, \beta = 0) \quad (6-2.9)
\]

\[
U^{+}(\tau = 0, \mu) = 0
\]

and for internal emission

\[
U^{-}(\tau = T, \mu) = 0 \\
(\epsilon > 0, \beta > 0) \quad (6-2.10)
\]

\[
U^{+}(\tau = 0, \mu) = 0
\]

In the case of the boundary condition of the frequency derivative it is taken

\[
\frac{\partial U^{+}}{\partial x} (\text{at } x = |x_{\text{max}}|) = 0 \quad (6-2.11)
\]

and the velocities at A and B are set as \( V_A \) and \( V_B \) respectively. For the case of shell which is expanding with constant velocity we have

\[
V_A = V_B \\
(6-2.12)
\]

for a static medium

\[
V_A = V_B = 0 \\
(6-2.13)
\]
while for the expanding medium with velocity gradients we have taken
\( V_A = 0 \) and \( V_B > 0 \) (5, 10, 20, 30, 40 and 50 mtu)

Profiles are described in terms of the integrated fluxes over the whole disc versus the normalised frequency. We have varied the parameters in the following way and the corresponding results have been presented graphically.

\( \frac{B}{A} \) = Ratio of the outer to inner radii of the spherical shell.

\( \varepsilon \) is the probability per scattering that a photon is thermalised by collisional de-excitation.

\( \beta \) is the ratio of the absorption coefficient per unit frequency interval in the continuum to that at the line centre.

\( V_A \) = Velocity in Doppler units at the point \( r = A \).

\( V_B \) = Velocity of the expanding spherical medium at a point with radius \( r = B \).

\( V \) = Velocity of expansion of the shell.

\( \tau_d \) = Total dust optical depth.

\( T \) = Total gas optical depth at the line centre.

The spherical shell has been divided into 100 subshells. Optical depth of each shell and the total optical depth upto any internal point as well as the emergent and the internal radiation fields are calculated by using the algorithm suggested by Peraiah (1984). We have chosen the ratio \( B/A \) to be 20 and is kept constant for all cases.
Red. Total gas optical depth at the line centre has worked out to be 300 m all the calculations.

Figures 6.2.1 to 6.2.4 show the line profiles plotted with respect to the normalised frequency $x$ in a static medium ($V_A = V_B = 0$). Dust optical depths 0, 0.5, 2, and 5 are considered. It is observed that the emission in the wings falls rapidly for increased dust optical depth. For dust optical depth $\tau_d = 5$, there is hardly any emission and the absorption core is reduced to almost half of that when there was no dust, and the absorption core is very wide extending up to 3 Doppler units on either side of the line center. Substantial emission is observed in the wings when there is no dust. This shows that dust scatters photons mostly into the cores of the absorption lines and removes them from the wings.

In Figures 6.2.5 to 6.2.8, expansion velocity $5 m tu$ is given with velocity gradients. Lines have become asymmetric. Here also we observe the wing emission eroding rapidly with the presence of dust in the atmospheres. For dust optical depth 5 the emission in the wings is almost nil and the absorption core becomes too narrow indicating that the photons have been removed due to scattering by dust and they have been added to the cores. These lines are for a medium in which the radiation is scattered by dust and gas without any emission.

For Figures 6.2.9 to 6.2.12, thermal emission is included by setting $\epsilon = 10^{-4}$ in the static medium. These curves show us how the dust scatters radiation when we
have thermalisation of photons. Substantial amount of emission is seen on either side of the absorption core at the centre. Though the presence of dust reduces the emission in the wings gradually and vanishes completely for $\tau_d = 5$, the absorption core width is observed to be unaffected. In Figures 6.2.13 to 6.2.16, we have considered expansion velocity with velocity gradient. This brings in a sudden asymmetry in the lines. In case of dust free medium we see two peaks of unequal heights, the larger among them is observed on the lower frequency side of the centre. P-Cygni type profiles develop when $\tau_d = 0.5$ with substantial reduction in emission. Absorption core developed is observed to be shifted towards violet side while emission peak remains on the red side of the line centre. On increasing the dust further ($\tau_d = 2$) emission is found to reduce further while absorption core persists. For dust optical depth 5 the absorption core deepens to such an extent that it becomes almost dark with hardly any emission in the wings.

In Figures 6.2.17 to 6.2.20 the velocity of expansion has been increased to 10 m\(\text{tu}\). We observe the same effect as observed for lower velocity of expansion 5 m\(\text{tu}\) with a difference of lowering of absorption depth comparatively.

In figures 6.2.21 to 6.2.24 we have plotted the profiles for an expanding spherical shell with constant velocity without velocity gradients i.e. $V_A = V_B = 5$ m\(\text{tu}\).
As observed in the previous case we see two peaks of emission. The peak on the red side is larger in comparison with that on violet side but it is flat-topped. When dust is introduced \( \tau_d = 0.05 \) emission reduced to 1/3 the previous retaining its flat-top. Further increase in dust causes disappearance of emission peak and deepening of absorption core.

Increasing the velocity of expansion to 10 m.s\(^{-1}\) is showing almost the same behaviour which has been shown in Figure 6.2.25 to 6.2.28.

Figures 6.2.29 to 6.2.44 are drawn for the profiles formed in a medium which has continuum emission together with the line emission. These have similar features as seen in the earlier figures 6.2.9 to 6.2.28.

In Figures 6.2.45 and 6.2.46 we have plotted the equivalent widths of different lines against different dust optical depths. While Figure 6.2.45 shows the equivalent widths corresponding to the lines formed in a medium without velocity gradients those in Figures 6.2.46 are for a medium expanding with velocity gradients. In both these cases we have considered scattering of radiation by both dust and gas. We have observed quite a noticeable difference in the equivalent widths of the lines formed in a medium without velocity gradients as compared to those formed in a medium with velocity gradients. In 6.2.45 we see that the equivalent width increases with the increase in dust optical depth while in Figure 6.2.46
it is the reverse i.e. the equivalent width is reduced with increasing optical depth when velocity gradients are considered.

Figure 6.2.47 is to show how the ratio of the heights of two emission peaks varies with expansion velocities. We have studied this behaviour for different dust optical depths. In Figures 6.2.48 and 6.2.49 we have shown how this behaviour changes with the constant velocity of expansion of the spherical shell \((V_A = V_B)\). We observe that this ratio gets reduced as the dust optical depth is increased.

In Figure 6.2.50 we have shown the relation between the velocity of expansion and equivalent widths of lines for expanding atmospheres with velocity gradients. We have compared dust optical depth \(\tau_d = 2\) with that of dust free atmosphere. It may be noticed that the equivalent width increases steadily with the velocity of expansion.

In Figure 6.2.51 we have considered the same relation for a medium moving without velocity gradients \((V_A = V_B)\). In this case we see that the equivalent width is decreasing with velocity of expansion up to 15 m/s and thereafter it increases linearly with velocity.

Figure 6.2.52 is similar to those of 6.2.50 with a difference that there exists a thermal emission in the line \((\epsilon = 10^{-4})\). Here we notice that all the lines are in emission and the emission reduces as the amount of
dust is increased. Figure 6.2.53 shows the relation between equivalent width and expansion velocities of the expanding medium without velocity gradients. We observe similar behaviour of the plots.

Figures 6.2.54 and 6.2.55 are to show the relation between velocities of expansion and equivalent widths of the lines formed in medium expanding with and without velocity gradients respectively. In this case there is emission both in the line and also in the continuum. There is similarity in behaviour between this and for the medium in which emission was considered only in the line and not in the continuum. We observe that the widths of the lines are different in the two cases.
Figure 6.2.1  Line profiles formed in a static medium without dust ($\tau_d=0$).

Figure 6.2.2  Same as in Figure 6.2.1 for $\tau_d=0.5$ dust being distributed uniformly.
Figure 6.2.3  Same as in Figure 6.2.2 with dust optical depth, $\tau_d=2$.

Figure 6.2.4  Same as in Figure 6.2.2 with dust optical depth, $\tau_d=5$. 
Figure 6.2.5 Line profiles formed in an expanding medium with velocity gradients and velocity of expansion \( V_x = 5 \) m/s and with no dust.

Figure 6.2.6 Same as in Figure 6.2.5 with dust distributed uniformly having \( r_d = 0.5 \).
Figure 6.2.7 Same as in Figure 6.2.5 with dust distributed uniformly having $\tau_d = 2$.

Figure 6.2.8 Same as in Figure 6.2.5 with dust distributed uniformly having $\tau_d = 5$. 
Figure 6.2.9  Line profiles formed in a static medium in which there is line emission and no dust included.

Figure 6.2.10  Same as in Figure 6.2.9 with uniform dust distribution giving a total $\tau_d = 0.05$. 
Figure 6.2.11 Same as in Figure 6.2.9 with $\tau_d = 0.5$.

Figure 6.2.12 Same as in Figure 6.2.9 with $\tau_d = 5.0$. 
Figure 6.2.13 Line profiles formed in an expanding medium with velocity gradients and velocity of expansion $V_b = 5$, $\tau_d = 0$.

Figure 6.2.14 Line profiles formed in an expanding medium with velocity gradients and velocity of expansion $V_b = 5$, $\tau_d = 0.5$. 

\[ \epsilon = 1 \times 10^{-4}, \beta = 0 \]

\[ V_a = 0, V_b = 5 \]

\[ \tau_d = 0.5 \]
Figure 6.2.15 Line profiles formed in an expanding medium with velocity of expansion $V_b = 5 \text{ m} \text{tu}$, $\tau_d = 2.0$.

Figure 6.2.16 Line profiles formed in an expanding medium with velocity of expansion $V_b = 5 \text{ m} \text{tu}$, $\tau_d = 5.0$. 
Figure 6.2.17  Same as in Figure 6.2.13 but velocity of expansion is equal to 10 mtu.

Figure 6.2.18  Same as in Figure 6.2.14 with velocity of expansion $v_b = 10$ mtu.
Figure 6.2.19  Same as in Figure 6.2.15 with velocity of expansion $V_b = 10$ mtu.

Figure 6.2.20  Same as in Figure 6.2.16 with velocity of expansion $V_b = 10$ mtu.
Figure 6.2.21 Line profiles formed in an expanding medium with out velocity gradients and velocity of expansion $v_b = 5$ m/s, medium containing no dust ($\tau_d = 0$).

**Figure 6.2.22** Same as in Figure 6.2.21 but dust content is given as optical depth $\tau_d = 0.05$. 

\[ \frac{F_x}{F_C} \]

\[ \epsilon = 1 \times 10^{-4}, \beta = 0 \\
V_a = 5, V_b = 5 \\
\tau_d = 0 \]
Figure 6.2.23 Same as in Figure 6.2.21 but $\tau_d = 0.5$.

Figure 6.2.24 Same as in Figure 6.2.21 but $\tau_d = 0.5$. 

\[ F_x/F_C \]

$\epsilon = 10^{-4}, \beta = 0$

$V_a = 5, V_b = 5$

$\tau_d = 0.5$
Figure 6.2.25  Same as those in Figure 6.2.21 with velocity of expansion $V_b = 10$ m/s without dust contents.

Figure 6.2.26  Same as in Figure 6.2.25 but uniform dust distributed offering an optical depth $\tau_d = 0.05$. 
Figure 6.2.27 Same as in Figure 6.2.25 but dust is more \( r_d = 0.5 \).

Figure 6.2.28 Same as in Figure 6.2.25 with \( \tau_d = 5.0 \).
Figure 6.2.29 Line profiles in a static medium without dust.

\[ F_X / F_C \]

\[ \epsilon = 1 \cdot 10^{-4}, B = 1 \cdot 10^{-5} \]
\[ V_a = 0.0, V_b = 0.0 \]
\[ \tau_d = 0.0 \]

Figure 6.2.30 Line profiles formed in a static medium with dust, \( \tau_d = 0.02 \).
Figure 6.2.31  Line profiles formed in a static medium with dust, \( \tau_d = 0.5 \).

Figure 6.2.32  Line profiles formed in a static medium with dust, \( \tau_d = 5.0 \).
Figure 6.2.33 Line profiles formed in an expanding medium with velocity gradients and velocity of expansion $V_b = 5 \text{ mtu.}$

Figure 6.2.34 Same as in Figure 6.2.33, with dust contained in the medium $\tau_d = 0.02.$
Figure 6.2.35 Same as in Figure 6.2.33, with dust contained in the medium $\tau_d = 0.05$.

Figure 6.2.36 Same as in Figure 6.2.33, with dust contained in the medium $\tau_d = 0.5$. 
**Figure 6.2.37** Line profiles formed in an expanding medium without velocity gradients, velocity of expansion is equal to 5 mtu.

**Figure 6.2.38** Same as those of Figure 6.2.37 but with dust.
Figure 6.2.39  Same as those of Figure 6.2.37 but with dust $\tau_d = 0.5$.

$\epsilon = 1 \cdot 10^{-4}, \beta = 1 \cdot 10^{-5}$
$\nu_a = 5.0, \nu_b = 5.0$
$\tau_d = 0.5$

Figure 6.2.40  Same as those of Figure 6.2.37 but with dust $\tau_d = 5.0$.

$\epsilon = 1 \cdot 10^{-4}, \beta = 1 \cdot 10^{-5}$
$\nu_a = 5.0, \nu_b = 5.0$
$\tau_d = 5.0$
Figure 6.2.41 Line profiles formed in an expanding medium without velocity gradients, velocity of expansion of the medium $v_b = 10$ mtu.

Figure 6.2.42 Same as in Figure 6.2.41 but with dust $\tau_d = 0.05$. 
Figure 6.2.43  Same as in Figure 6.2.41 but with dust $\tau_d = 0.5$.

Figure 6.2.44  Same as in Figure 6.2.41 but with dust $\tau_d = 5.0$. 
Variation of equivalent width with dust optical depth \((\tau_d)\) for various velocities of expansion (without velocity gradients).

Figure 6.2.45

Same as in Figure 6.2.45 but with velocity gradients.

Figure 6.2.46
Figure 6.2.47 Variation of the ratio of two emission peak heights ($\frac{h_1}{h_2}$) in an expanding medium with velocity ($v_b$) (with velocity gradients).

Figure 6.2.48 Same as those of Figure 6.2.47 but without velocity gradients.
Figure 6.2.49  Same as in Figure 6.2.48 for $\beta = 10^{-5}$

Figure 6.2.50  Variation of equivalent width of lines with velocity of expansion (with velocity gradients).
Figure 6.2.51  Same as those in Figure 6.2.50 (without velocity gradients).

Figure 6.2.52  Same as those of Figure 6.2.50 but for $c = 10^{-4}$. 
Figure 6.2.53  Same as those of Figure 6.2.51 but for $c = 10^{-4}$. 

$V_a = V_b$
$\epsilon = 10^{-4}$
$\beta = 0$

$\tau_d = 0$
Figure 6.2.54  Same as those in Figure 6.2.50 but with $c = 10^{-4}$ and $\beta = 10^{-5}$. 
Figure 6.2.55 Same as those in Figure 6.2.51 but with $\epsilon = 10^{-4}$ and $\beta = 10^{-5}$. 