CHAPTER 4
THE STABILITY THEORY OF MHD SHOCK WAVES

There are six MHD shocks (the fast shock, the slow shock and four intermediate shocks) satisfying the requirements of the conservation law and the second law of thermodynamics. Raghunathan S. and Mabey [1] studied passive shock wave boundary layer control on a wall-mounted model. They evaluated the effect of oriented, normal forward facing and backward facing. Ragonathan [2] studied pressure fluctuation with positive shock position showed an appreciable decrease in drag compared with solid surface model. It is therefore proposed that study of such non-linear problems nil come out in presence of conducting and non-conducting medium where conditions may be adiabatic as well isothermal both. These phenomenon usually occur in stellar atmosphere where turbulence in medium is more often. Many more scientists discover that in stanleous energy release along a line cylindrical shocks or along point, spherical shocks may propagate with increasing strength without limit. The non-dimensional similarly conditions define the pattern of propagation of such motions and hence, it will be a matter of great interest if it is analysed whether the motion of these discontinuities strong or weak really propagate with high energy yields in the problems solar photosphere rocket reentry fission fusion reactions etc.

Carrus et al [3], Sedov [4] and later on Verma and Singh [5] obtained the numerical solutions for self similar adiabatic flows in self-gravitating compressible fluids. But these solutions does not describe the flow behavior in general. Since the phenomena associated with heat transfer in radiating fluid are expressively complex whether the fluid is at rest or in motion, steady or unsteady. Wang [6] considered the piston problem with thermal radiation for one-dimensional unsteady shocks using similarly method of Sedov[4]. Hellewell [7] took a more general case of piston problem with radiation heat transfer general optically and transparent limit. One or several plane monochromatic small disturbance waves are introduced to both sides of discontinuity. The shock wave is stable if the amplitudes of diverging waves (reflection and refraction waves) are uniquely determined by the conditions on discontinuity. Landau and Lifschitz [8] discussed the stability of shock wave and
obtained the stability of MHD shock. One or several plane monochromatic small disturbance waves introduced to both sides of discontinuity are incident upon the discontinuity. The shock wave is said to be stable (or evolutionary) if the amplitudes of the diverging waves (reflection and refraction waves) are uniquely determined by the conditions on discontinuity with the incident wave amplitudes given.

Akhiezer et al. [9] generalized the stability concept of gasdynamic shock for MHD shock. He discussed the interaction of MHD shock with normal small disturbance waves and concluded that only when the fast and slow shocks are stable, the intermediate shocks are unstable. Unsteady hydromagnetic flows are of great interest in practical metallurgical processing where quenching rates exert a significant effect on the final constitution of products. With the combined effect of heat transfer many challenging flow problems have been studied in transient magneto-hydrodynamic convection flows. Singh [10] examined the unsteady hydromagnetic laminar free convection flow past a vertical infinite flat plate whose temperature or heat flux varies as some power of time under a constant horizontal magnetic field, obtaining solutions for several values of Prandtl numbers for stepwise variations either in the plate temperature or in heat flux.

Chandra et al. [11] investigated the influence of viscosity, magnetic field and buoyancy force on the unsteady free magneto-hydrodynamic convection flow generated by the uniformly accelerated motion of a vertical plate subject to constant heat flux. They showed that an increase in velocity results from a larger heat flux which may be regulated via increasing the magnetic field strength and that skin friction at the boundary increases for larger Prandtl and Hartmann numbers, and decreases with increasing Grashof number. Jha [12] analyzed the effects of magnetic field and permeability of the porous medium on unsteady forced and free convection flow past an infinite vertical porous plate in the presence of a temperature-dependent heat source. A study of unsteady laminar hydromagnetic flow and heat transfer in a porous channel with temperature-dependent properties was presented by Umavathi [13]. Chaudhary and Jain [14] studied the magnetohydrodynamic effect on transient convection flow past a vertical surface embedded in a porous medium with oscillating temperature. Kontorovich [15] discussed the interaction of MHD shock with oblique small disturbances, defining the incident and diverging waves according to the direction of group velocity and concluded with a graphical method that the number of
diverging waves is independent of the changes of the incident angle and that can not be applied to the study on the stability theory of MHD shock waves. The investigation of interaction of MHD shock (fast shock) with oblique small disturbance is required to explain the phenomenon. The interaction of gas-dynamic shock wave with oblique small disturbance waves are generalized to the case of MHD shock wave in this chapter. The incident and diverging waves are defined according to the phase velocity as well as group velocity. An analytical method is used for a special kind of MHD shock waves and the interaction of MHD shock with oblique small disturbances is discussed. The stability condition is dependent on the frequency when the disturbances are entropy wave and fast and slow magneto-acoustic waves. As a limiting case when the small disturbances are normal incident (reflection and refraction) the fast and slow shocks are unstable.

4.1 FORMULATION OF SHOCK WAVE
We assumed the shock front on the yz-plane

\[ u = (U,0,0) \]  \hspace{1cm} (4.1)

\[ B = (B_x,0,0) \]  \hspace{1cm} (4.2)

Where \( u \) and \( B \) are velocity and magnetic field in front of and behind the shock lie on the xy-plane.

The plasma is ideal and compressible.
The relations between parameters in front of and behind the shock front are

\[ B_{x1} = B_{x2} = B, \quad \text{(4.3)} \]
\[ \rho_1 U_1 = \rho_2 U_2, \quad \text{(4.4)} \]
\[ \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} + \frac{1}{2} U_1^2 = \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2} + \frac{1}{2} U_2^2, \quad \text{(4.5)} \]

Where \( \gamma \) is the ratio of specific heats. So this kind of MHD shock wave is only a gasdynamic shock wave with a uniform magnetic field \( B \) perpendicular to the shock front.

### 4.2 OBLIQUE SMALL DISTURBANCE WAVES

In case some oblique small-disturbance waves appear in front of and behind the shock wave, the governing equations are

\[ \rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) u_x + \frac{\partial P}{\partial x} = 0, \quad \text{(4.6)} \]
\[ \rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial y} \right) u_y + \frac{\partial P}{\partial y} = -\frac{B}{4\pi} \frac{\partial B_x}{\partial y} + \frac{B}{4\pi} \frac{\partial B_y}{\partial x}, \quad \text{(4.7)} \]
\[ \rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right) u_z = \frac{B}{4\pi} \frac{\partial B_z}{\partial x}, \quad \text{(4.8)} \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \rho + \rho \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) = 0, \quad \text{(4.9)} \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \left( P - c^2 \rho \right) = 0, \quad \text{(4.10)} \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) B_x + B \frac{\partial u_x}{\partial y} = 0, \quad \text{(4.11)} \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) B_y + B \frac{\partial u_y}{\partial x} = 0, \quad \text{(4.12)} \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) B_z - B \frac{\partial u_z}{\partial x} = 0 \quad \text{(4.13)} \]
\[ \frac{\partial B}{\partial x} + \frac{\partial B_y}{\partial y} = 0, \quad (4.14) \]

Where sound speed \( a = \sqrt{\frac{\gamma P}{\rho}} \). To find the solution, we assume that the forms of plane progressive waves all disturbed quantities are proportional to \( \cos(\lambda_x x + \lambda_y y + \omega t) \), where \( \lambda = (\lambda_x, \lambda_y, 0) \) is the wave vector and \( \omega \) is the frequency. Some small disturbances waves are

1) Entropy wave

For Entropy wave the dispersion relation is
\[ \omega + U\lambda_x = 0, \quad (4.15) \]
And the solution is
\[ P = u = B = 0, \quad \rho \neq 0, \quad (4.16) \]

2) Alfven wave

For Alfven wave, the dispersion relation
\[
\left( \omega + U\lambda_x \right)^2 - \frac{B^2}{4\pi\rho} \lambda_x^2 = 0, \quad (4.18)
\]
And the solution is
\[ P = \rho = u_x = u_y = B_x = B_y = 0 \quad (4.19) \]

3) Fast and slow magneto-acoustic waves

For fast and slow magneto-acoustic waves, the dispersion relation is
\[
\left[ \frac{(\omega + U\lambda_x)^2}{\lambda_x^2 + \lambda_y^2} \right] = \frac{1}{2} \left( \frac{B^2}{4\pi\rho} + a^2 \right) \left[ \frac{B^2}{4\pi\rho} + a^2 \right] - \frac{a^2 B^2}{4\pi\rho} \cdot \frac{\lambda_x^2}{\lambda_x^2 + \lambda_y^2}, \quad (4.20)
\]
And the solution is
\[ u_x = -\frac{\lambda_x}{\rho(\omega + U\lambda_x)} P, \quad (4.21) \]
\[ u_y = \frac{\lambda_y}{\varepsilon - \rho(\omega + U\lambda_x)} P, \quad (4.22) \]
Where
\[ \varepsilon = \frac{B^2}{4\pi} \frac{\lambda_x^2 + \lambda_y^2}{\omega + U\lambda_x}, \quad (4.23) \]
$$B_x = -\frac{B\lambda_y}{(\omega + U\lambda_y)}u_y,$$ (4.24)

$$B_y = -\frac{B\lambda_x}{(\omega + U\lambda_x)}u_y,$$ (4.25)

$$\rho = \frac{1}{a^2}P,$$ (4.26)

$$u_x = B_x = 0,$$ (4.27)

Thus in front of and behind the shock front we get seven small disturbance waves. If in static system of coordinate the dispersion relation is known we can get formula of frequency

$$\omega_0 = \omega + \lambda.u$$ (4.28)

Where $\omega, \lambda_y$ are positive and identical in all small disturbance waves.

### 4.3 CONSERVATION LAW ON THE DISTURBED SHOCK WAVE

We assume that the form of the disturbed shock front is

$$x = x_0 \sin(\lambda_y y + \omega t),$$ (4.29)

And speed of disturbed shock front is

$$s = \frac{\partial x}{\partial t} = x_0 \omega \cos(\lambda_y y + \omega t),$$ (4.30)

And then unit normal is

$$N = \left(1 - x_0 \lambda_y \cos(\lambda_y y + \omega t),0\right),$$ (4.31)

Using conservation laws on the disturbed shock front we get

$$[\rho \theta] = 0,$$ (4.32)

$$\theta = s - u.N,$$ (4.33)

$$[P]N - \rho \theta [u] = -\frac{1}{8\pi} \left[B^2 \right]N + \frac{1}{4\pi} (B.N)[B]$$ (4.34)

$$\rho \theta \left[\frac{\gamma}{\gamma - 1} \cdot \frac{P}{\rho} + \frac{1}{2} u^2 \right] = -\frac{1}{4\pi} (B.N)[u.B] + \frac{1}{4\pi} \left[B^2.(u.N)\right],$$ (4.35)

$$[u \times B]_r = 0, [B]N = 0.$$ (4.36)
Where $T$ denotes the tangent direction and the symbol $[ ]$ denote the difference of quantity in front of and behind the shock wave. Carrying out the linearization, we obtain a system of equation satisfied on the disturbed shockwave

$$\rho_1(u_{x_1} - s) + \rho_1 U_1 = \rho_2(u_{x_2} - s) + \rho_2 U_2,$$

(4.37)

$$P_1 + 2\rho_1 U_1 u_{x_1} - \rho_1 U_1^2 = P_2 + 2\rho_2 U_2 u_{x_2} - \rho_1 U_2 s + \rho_2 U_2^2,$$

(4.38)

$$P_1 N_y + \rho_1 U_1 u_y - \frac{B}{4\pi} B_{y_1} = P_2 N_y + \rho_1 U_1 u_{y_1} + \rho_1 U_1 u_y - \frac{B}{4\pi} B_{y_2},$$

(4.39)

$$\frac{\gamma}{\gamma - 1} P_1 U_1 \left( \frac{P_1 - \rho_1}{P_1 - \rho_1} \right) - \left( \frac{\gamma}{\gamma - 1} \frac{P_1 + \frac{1}{2} U_1^2}{\rho_1} \right) \left( \rho_1 (u_{x_1} - s) + \rho_1 U_1 + \rho_1 U_1^2 u_{x_1} \right)$$

$$= \frac{\gamma}{\gamma - 1} P_2 U_2 \left( \frac{P_2 - \rho_1}{P_2 - \rho_1} \right) - \left( \frac{\gamma}{\gamma - 1} \frac{P_2 + \frac{1}{2} U_2^2}{\rho_2} \right) \left( \rho_1 (u_{x_2} - s) + \rho_1 U_1 + \rho_1 U_2^2 u_{x_2} \right),$$

(4.40)

$$U_1 B_{y_1} - Bu_{y_1} = U_2 B_{y_2} - Bu_{y_2},$$

(4.41)

$$B_{x_1} = B_{x_2},$$

(4.42)

$$P_1 U_1 u_{z_1} - \frac{B}{4\pi} B_{z_1} = \rho_1 U_1 u_{z_2} - \frac{B}{4\pi} B_{z_2},$$

(4.43)

$$Bu_{z_1} - U_1 B_{z_2} = Bu_{z_2} - U_2 B_{z_2}.$$  

(4.44)

Since $\lambda_y$ and $\omega$ are taken positive and identical, so in each disturbed quantity a common factor $\cos(\lambda_y y + \omega t)$ is involved. By eliminating the common factor, we get eight algebraic equations which can be used to determined unknown amplitudes of small disturbances.

### 4.4 STABILITY OF SHOCK WAVE AND ALFVEN WAVES

We assume that monochromatic small disturbance waves in front of and behind the shock front are incident upon the discontinuity. When the amplitudes of incident waves are given, the shock wave is taken to be stable. The amplitudes of diverging waves are uniquely determined by the conditions on the disturbed shock front. Otherwise the shock wave is said to be unstable or non-evolutionary. The incident and
diverging waves can be defined according to the direction of phase velocity of small disturbances or the direction of group velocity. To define the incident and diverging waves according to the direction of phase velocity of small disturbances wave towards or away from the shock front, the condition for diverging waves in front of the shock wave is

$$-\frac{\lambda_x}{\omega} < 0, \text{ Or } \lambda_x > 0;$$

(4.45)

The condition for diverging wave behind the shock wave:

$$-\frac{\lambda_x}{\omega} > 0, \text{ Or } \lambda_x < 0.$$ (4.46)

To define the incident and diverging waves according to the direction of group velocity of small disturbances wave towards or away from the shock front, the condition for diverging waves in front of the shock wave is

$$-\frac{\partial \omega}{\partial \lambda_x} < 0 \text{ or } \frac{\partial \omega}{\partial \lambda_x} > 0;$$ (4.47)

The condition for diverging wave behind the shock wave:

$$-\frac{\partial \omega}{\partial \lambda_x} > 0, \text{ or } \frac{\partial \omega}{\partial \lambda_x} < 0.$$ (4.48)

These conditions on disturbed shock wave can be divided into two groups. First group contains first six equations with thirteen parameters $P, \rho, u_x, u_y, B_x, B_y$ in front of and behind the shock wave and $x_0$. The other group contains the last two equations with four parameters of $B_z, u_z$ in front of and behind the shock wave.

For the second group, define

$$\varpi = \frac{\omega}{a_1 \lambda_y}, \quad M_1 = \frac{U_1}{a_1}, \quad M_2 = \frac{U_2}{a_2},$$

(4.49)

$$\bar{X} = \frac{\lambda_x}{\lambda_y}, \quad A_1 = \frac{B}{\sqrt{4\pi \rho_1} a_1}, \quad A_2 = \frac{B}{\sqrt{4\pi \rho_2} a_1}.$$ (4.50)

Then in the Alfvén wave the dispersion relation is

$$\varpi + M_1 \bar{X} = \pm A_1 \bar{X}$$

(4.51)
and \( \sigma + M_2 \frac{a_2}{a_1} \mathcal{X} = \pm A_2 \mathcal{X} \) \hspace{1cm} (4.52)

Where

\[
M_2 = \sqrt{1 + \left( \frac{\gamma - 1}{2} \right) \frac{M_i^2}{\gamma M_i^2 - \left( \frac{\gamma - 1}{2} \right)}}
\] \hspace{1cm} (4.53)

\[
a_2 = \frac{\sqrt{2(\gamma - 1)}}{(\gamma + 1)M_1} \sqrt{\frac{2\gamma}{\gamma - 1} \left( \frac{M_i^2}{\gamma - 1} \right) - \frac{2\gamma}{\gamma - 1} \left( \frac{M_i^2}{\gamma - 1} \right) + 1}
\] \hspace{1cm} (4.54)

Two equations are on the disturbed shock. The stability condition of shock wave with respect to the Alfven disturbances is that among the four Alfven waves in front of and behind the shock wave, two waves are diverging. The stability conditions in case of normal disturbances are

\[ A_1 > M_1, \quad A_2 > M_2 \quad \text{Or} \quad A_1 < M_1, \quad A_2 < M_2 \]

For oblique disturbances we have

\[ \mathcal{X}_1 = \frac{\sigma}{-M_1 \pm A_1} < 0, \] \hspace{1cm} (4.55)

\[ \mathcal{X}_2 = \frac{-\sigma}{-M_2 \frac{a_2}{a_1} \pm A_2} < 0, \] \hspace{1cm} (4.56)

These two Alfven waves behind the shock wave are diverging, so the second stability condition is unchanged and the first stability condition becomes

\[ A_1 > M_1, \quad \text{and} \quad A_2 > M_2 \frac{a_2}{a_1} \] \hspace{1cm} (4.57)

In this case in front of and behind the shock there is only one diverging wave. We conclude that the stability condition of the shock with respect to the Alfven disturbances is

\[ A_1 > M_1, \quad A_2 > M_2 \frac{a_2}{a_1} \] \hspace{1cm} (4.58)

\[ \text{Or} \quad A_1 < M_1, \quad A_2 < M_2. \] \hspace{1cm} (4.59)
4.5 SLOW SHOCK, ENTROPY AND MAGNETOACOUSTIC WAVE

We assume that the small disturbances are entropy wave, fast and magnetoacoustic waves. There are six number of equations. We get five independent equations after eliminating $x_0$. The entropy waves ahead of the shock front are incident, while behind the shock front are diverging. So the shock is stable when there are four diverging waves among the eight fast and slow magnetoacoustic waves in front of and behind the shock wave. We get following condition under diverging waves.

\[
\frac{B^2}{8\pi} \gg p_1, \quad \frac{B^2}{8\pi} \gg p_2, \quad (4.60)
\]

\[
\text{Or} \quad A_i^2 \gg 1, \quad A_2^2 \gg \frac{a_2^2}{a_1^2}, \quad (4.61)
\]

Then in front of and behind the shock wave, the dispersion relations in fast magnetoacoustic wave are

\[
\left(\sigma + M_1 \bar{X}\right)^2 = A_i^2 \left(\bar{X}^2 + 1\right), \quad (4.62)
\]

\[
\left(\sigma + M_2 \frac{a_2}{a_1} \bar{X}\right)^2 = \frac{a_2^2}{a_1^2} \bar{X}^2, \quad (4.63)
\]

We assume that the stability condition of the slow shock in the interaction of MHD shock with normal disturbance

\[
1 < M_1 < A_i, \quad M_2 < 1, \quad (4.64)
\]

is satisfied. If this condition for the interaction of slow shock with oblique small disturbances is true. Every dimensionless parameter in undisturbed shock wave is determined by $\gamma, M_1, A_i$. The expression

\[
A_2 = M_2 \frac{a_2}{a_1}, \quad (4.65)
\]
\[ A_i = \frac{\gamma}{(\gamma+1)} \left( M_1^2 + \frac{2}{\gamma-1} \right) \] (4.66)

For \( \gamma > 1 \), the condition satisfying \( A_i > M_1, A_2 > M_2 \cdot \frac{a_2}{a_1} \) does not exist.

(a) For the fast magnetoacoustic waves, the dispersion relation in front of and behind shock wave, when \( 1 < M_1 < A_1 \). For the fast magnetoacoustic waves, the dispersion relation in front of the shock wave is a hyperbolic function. Taking \( \sigma > 0 \). The curve above p corresponds to one fast magnetoacoustic wave and the curve under p corresponds to another fast magnetoacoustic wave.
The dispersion relation for the magnetoacoustic wave behind the shock wave is shown

According to the direction of phase velocity, Incident and diverging waves:

(i) If $0 < \omega < \sqrt{A_1^2 - M_1^2}$, the incident and diverging wave has no fast magnetoacoustic wave in front of the shock wave.

(ii) If $\sqrt{A_1^2 - M_1^2} < \omega < A_1$, the incident and diverging wave has two fast magnetoacoustic diverging waves in front of the shock wave.

(iii) If $A_1 < \omega$, the incident and diverging wave has one fast magnetoacoustic diverging wave in front of the shock wave.

(iv) if $0 < \omega < A_2$, the incident and diverging wave has no fast magnetoacoustic wave behind the shock wave.

(v) If $A_2 < \omega$, the incident and diverging wave has one fast magnetoacoustic diverging wave behind the shock wave.

According to the direction of group velocity, Incident and diverging waves:

(i) If $0 < \omega < \sqrt{A_1^2 - M_1^2}$, the incident and diverging wave has no fast magnetoacoustic wave in front of the shock wave.
(ii) If \( \sqrt{A_1^2 - M_1^2} < \sigma \), the incident and diverging wave has one fast magnetoacoustic diverging waves in front of the shock wave.

(iii) If \( \sigma < \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \), the incident and diverging wave has no fast magnetoacoustic wave behind the shock wave.

(iv) If \( \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} < \sigma \), the incident and diverging wave has one fast magnetoacoustic diverging wave behind the shock wave.

Incident and diverging waves can be defined according to the direction of phase velocity.

(1) If \( A_2 < \sqrt{A_1^2 - M_1^2} \) then

(i) \( 0 < \sigma < A_2 \), has no fast magnetoacoustic diverging wave,

(ii) \( A_2 < \sigma < \sqrt{A_1^2 - M_1^2} \) has one fast magnetoacoustic diverging wave,

(iii) \( \sqrt{A_1^2 - M_1^2} < \sigma < A_1 \) has three fast magnetoacoustic diverging waves,

(iv) \( A_1 < \sigma \) has two fast magnetoacoustic diverging waves.

(2) If \( \sqrt{A_1^2 - M_1^2} < A_2 < A_1 \) then

(i) \( 0 < \sigma < \sqrt{A_2^2 - M_2^2} \), has no fast magnetoacoustic diverging wave,

(ii) \( \sqrt{A_1^2 - M_1^2} < \sigma < A_2 \) has two fast magnetoacoustic diverging waves,

(iii) \( A_2 < \sigma < A_1 \) has three fast magnetoacoustic diverging waves,

(iv) \( A_1 < \sigma \) has two fast magnetoacoustic diverging waves.

Incident and diverging waves can be defined according to the direction of group velocity.

(i) \( 0 < \sigma < \min \left( \sqrt{A_1^2 - M_1^2}, \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \right) \) has no fast magnetoacoustic wave
(ii) \( \min \left( \sqrt{A_1^2 - M_1^2}, \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \right) < \sigma < \max \left( \sqrt{A_1^2 - M_1^2}, \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \right) \)

has one fast magnetoacoustic diverging wave.

(iii) \( \max \left( \sqrt{A_1^2 - M_1^2}, \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \right) < \sigma \)

has two fast magnetoacoustic diverging waves.

(b) For the slow magnetoacoustic waves in front of and behind the shock wave.

The dispersion relations in front of and behind the shock wave are as

(i) \( \sigma = (-M_1 \pm 1)X \) and

(ii) \( \sigma = (-M_2 \pm 1) \frac{a_2}{a_1} X \).

The condition \( M_1 > 1 \) and \( M_2 < 1 \) are satisfied. So, there is only one slow magnetoacoustic diverging wave behind the shock wave for both the relation whether the incident and diverging waves are defined by the direction of the phase velocity or by the direction of the group velocity. The stability condition for slow wave where four fast and slow magnetoacoustic diverging waves in front of and behind the shock wave is same as the condition under which exist three fast magnetoacoustic diverging waves. When the incident and diverging waves defined by the direction of the group velocity do not exist three diverging fast magnetoacoustic waves in the frequency domain \( \sigma > 0 \). So, in the case of normal disturbances we conclude that the slow shock satisfies the stability condition \( 1 < M_1 < A_1 \) and \( M_2 < 1 \). In the case of oblique disturbances, there exist some frequency domain of magnetoacoustic waves which makes the slow shock unstable.

If the frequency satisfies condition \( \sigma > \sqrt{A_1^2 - M_1^2} \) in front of the shock

\( \sigma > \sqrt{A_2^2 - M_2^2 \frac{a_2^2}{a_1^2}} \) behind the shock, the variation of \( \sigma \) is the variation of the incidence angle.
If $\omega \to \infty$ then the small disturbances are normal. So, the slow shock is unstable whether the incident and diverging waves are defined by the direction of the phase velocity or by the direction of group velocity.

4.6 FAST SHOCK, ENTROPY AND MAGNETOACOUSTIC WAVES

We assume
\[
\frac{B^2}{4\pi \rho_1} \ll a_i^2 < U_1^2, \quad (4.67)
\]
and
\[
\frac{B^2}{4\pi \rho_2} \ll U_2^2 < a_2^2, \quad (4.68)
\]

For the fast shock wave the stability condition with respect to oblique disturbance is that there is two diverging waves. Two diverging waves do not exist whether the incident and diverging waves defined by the direction of the phase velocity or by the direction of group velocity. If $\lambda_y \to 0$ and $\omega \to \infty$ then the small disturbances are normal. So the fast shock wave is also unstable.
REFERENCES


