CHAPTER 8
CONCLUSION

In this thesis, the differential effects of the magnetohydrodynamics shocks have been discussed in astrophysical medium. We have derived jump conditions across the shock. We have obtained the density, pressure strength of the MHD shock and derived the well-known particular cases.

We now summarized our main conclusions.

In chapter 2, there are three distinct wave propagation modes which can be classified according to the magnitude of their propagation speed as fast, intermediate and slow. The velocity changes over the three waves are commonly opposite. For fast and slow waves, both the velocity and the magnetic field remain in the plane defined by the magnetic field ahead of the wave and the wave normal. While, for the intermediate wave the velocity and magnetic field changes are purely in the direction perpendicular to this plane. When the density increases, the magnetic pressure increases for the fast mode. An increase in magnetic pressure for slow mode corresponds to a decrease in density. While neither the magnetic pressure nor the density change across an intermediate wave. The study when extended to waves in arbitrary direction, results in 3 different waves. Figures 1 and 2 represents, variations in wave speeds, with varying angle between wave vector and equilibrium magnetic field. The three different wave speeds relate to fast magnetosonic, slow magnetosonic and Alfven waves. Different waves exist with Alfven wave speed larger or smaller compared to isentropic sound speed. In both cases, when equilibrium magnetic field is perpendicular to wave propagation, only fast magnetosonic wave exists as stated earlier. For cases, when $B_0$ is parallel to wave vector, two wave speeds, fast and slow magnetosonic exists, as shown earlier. Alfven wave separately exist only when $B_0$ is neither parallel nor perpendicular to wave propagation. It merges with fast magnetosonic and slow magnetosonic wave speeds, respectively for cases with Alfven wave speed larger and smaller compared to sound speed.
The same results also can be seen in terms of wave normal surfaces. Wave normal surfaces for MHD waves are presented in figures 3 and 4. Wave vector drawn with respect to $B_0$ will give wave speeds for different modes.
In chapter 3, a shock propagating through an MHD fluid creates a significant difference in plasma properties on the either side of the shocks front. For a parallel MHD shock in the shock frame, the upstream plasma velocity \( v_1 \) must be supersonic. In other word in a stationary plasma, a parallel, or hydrodynamic, shock propagates along the magnetic field with a supersonic velocity. A perpendicular shock compresses the magnetic field by the factor \( \left( \gamma + 1 \right) / \left( \gamma - 1 \right) \). For a Oblique shock, fast shocks refract the magnetic field and plasma flow away from the normal to the shock front. The shocks with planar transverse magnetic fields are either fast or slow shocks, the former of which amplifies the magnitude of transverse magnetic fields though the later decreases it. The shocks that change the direction of transverse magnetic fields are referred to as intermediate shocks. The terminology employed in the literature is rather confusing. It is remarked that the intermediate shock in this paper is different from the intermediate discontinuity defined in Jeffrey & Taniuti (1964). In fact, the latter is used for the discontinuities that satisfy the evolutionary conditions whereas the intermediate shocks in this paper are non-evolutionary as will be evident shortly. Slow shocks refract these quantities toward the normal, whereas, the tangential magnetic field and plasma flow generally reverse across an intermediate shock front. MHD shocks have been observed in a large variety of situations e.g. by supernova explosions, by strong stellar winds, by solar flares and by solar wind upstream of planetary magnetospheres.
In chapter 4, the solution of interaction problem of one dimension MHD shocks wave with normal shock wave is that only fast and slow shocks are stable and intermediate shocks are unstable. When the disturbances are the entropy wave, the fast and slow magneto acoustic waves. The stability condition is related to the frequency of small disturbances.

The useful forms of generalized MHD shock jump relation for weak and strong shock derived in chapter 5 provide simpler and more accurate boundary condition to determine the MHD shock structure and propagation of MHD shock wave in non-ideal gases. An important direct application of generalized shock jump relation is the study of propagation of shock waves produced due to supernova explosions etc. The shock jump relations derived for strong shock in weak magnetic field and strong shock in strong magnetic field can be applied in certain problems of steady supersonic flow in interstellar media.

The following conclusions can be drawn from the determining of the current analysis.

i) The shock velocity, pressure and particle velocity behind the MHD shock wave in non-ideal gas flow increase with the strength of magnetic field and nonidealness parameter. A large increase in the flow quantities behind shock is seen in the presence of magnetic field as compared to the absence of magnetic field. Thus, in the presence of magnetic field the shock propagates more rapidly in real gases comparatively in
an ideal gas atmosphere. The rate of rise in the flow variable increases with the strength of magnetic field.

ii) In case of weak shock in weak magnetic field, the properties of the weak shock waves are not affected by the presence of weak magnetic fields. The shock velocity, pressure and particle velocity decrease with the non-idealness parameter.

iii) In case of weak shock in weak magnetic field, the shock velocity and particle velocity increase with the strength of magnetic field. The pressure (except $c\rho_0 = 0$) decreases with the strength of magnetic field. The shock velocity and pressure decrease with increase in the value of non-idealness parameter. It increases with the strength of magnetic field.

iv) In case of strong shock in weak magnetic field, the pressure and the speed of sound increase (except $c\rho_0$) with increase in the strength of magnetic field, whereas the pressure and speed decrease for $c\rho_0 = 0$. The pressure and speed of sound increase with non-idealness parameter. It remains unchanged with the strength of magnetic field. The pressure, speed of sound and particle increase with increase in the shock velocity.

v) In case of strong shock in strong magnetic field, the pressure, speed of sound and particle velocity increase with increase in the shock velocity. The pressure and particle velocity increase with the non-idealness parameter but remain unchanged with the strength of magnetic field. The particle velocity does not depend on the non-idealness parameter and adiabatic index. It remains unchanged with the strength of magnetic field. There are very large variations in pressure and speed of sound with the shock velocity in the absence of magnetic field as compared to the presence of strong magnetic field. The variations of flow quantities behind the shock front in real gases are similar to that of behind the shock front in an ideal gas. The pressure and speed of sound in presence of strong magnetic field increase with the strength of shock and shock velocity whereas the pressure and speed of sound decrease with the strength of shock and shock velocity.

The effect of non-idealness parameter does not change the trends of variation of flow variables behind the weak and strong shock waves. The generalized results can be reduced to the case of MHD shocks in ideal gas atmosphere. Thus, the generalized MHD shock jump relation have a key role in real gases, air, stars, interstellar medium,
etc. This work can be extended to the astrophysical application of MHD shock waves considering the effects of self-gravitation and rotation of fluids. The complex astrophysical phenomena such as supernova, explosions, accretion of material onto star, stellar pulsation are now accessible to simulation code.

In chapter 6, the following conclusions are drawn from determining MHD shock structure in a viscous gas.

i) The thickness of MHD shock front increases with increasing strength of the magnetic field. The magnetic field on the flow variables is ahead of the point of inflection.

ii) The thickness of MHD shock front increases with decreasing initial pressure for large values of the strength of magnetic field than for small values of the strength of magnetic field.

iii) The thickness of MHD shock front increases with decreasing initial density and the change in thickness with initial density is independent of the strength of magnetic field.

iv) The thickness of MHD shock front increases with increase in the viscosity and the change in thickness is more for large values of the strength of magnetic field.

v) The thickness of MHD shock front decreases with increasing strength of the shock wave and the change in thickness is more for large values of the strength of magnetic field. The effect of magnetic field is appear for small values of shock strength, while, it is small for large values of shock strength.

Thus, the results obtained are useful for the cases where the viscosity of fluid plays an important role and in the study of effects of magnetic fields on blood circulation, cardiovascular events, crude oil transportation etc.

The shock wave interstellar medium interaction problem in chapter 7 is calculated for a controversial case of Alfvén numbers close to unity. The flow structure in this case is completely different in the region around the symmetry axis compared with that for larger values of A. Besides, a high-entropy layer originates, which spreads over the helio pause up to 300 AU. The latter effect may be important in the interpretation of observational data. The non-evolutionary MHD shocks can be avoided by adding rotational perturbations. Such kind of shocks often originate if three-dimensional MHD equations are reduced to two-dimensional ones or even in 3D cases if the plasma is not ideal and/or if out-of-plane perturbation are forcefully
diminished. In this case non evolutionary shocks can exist for a long time depending on the value of molecular and magnetic viscosities (see Barmin et al. [22]). No need to say that perturbations of any kind can be encountered in the interstellar medium, since it can hardly be assumed absolutely uniform. The flow pattern which can be realized for other type of disturbances may be different, of course, quantitatively, though non evolutionary shocks will anyway be absent. The flow pattern obtained in the axisymmetric case can exist as a transient one. It is stationary only if three-dimensional disturbances do not act at all. This pattern, however, can be rather persistent to disturbances in the numerical treatment. This is due to the numerical viscosity and resistivity. In this case the disturbance must be either large enough or act for a sufficiently long time (have low frequency).

APPLICATIONS:
The generalized MHD shock jump relations derived in the chapter 5 may be used directly to a wide variety of astrophysical problems, aerodynamics, gas turbines, internal combustion engines, fusion reactors, and a host of other computational fluid dynamics applications and also in many other fields including oceanography, and atmospheric sciences. The following are few astrophysical examples where the role of magnetic fields is important. The presence of sunspots in the photosphere, and structures such as filaments, prominences, and flares in the solar corona, demonstrate the key role of the magnetic fields. In fact, the very existence of the hot corona is now interpreted as due to heating by MHD effects. It is thought that most of the magnetic activity of the Sun is driven by the combination of rotation and turbulent flows in the convection zone. Both the processes that produce sunspots, and the large-scale magnetic field of the Sun, are very active areas of research. The key role of magnetic fields is also observed in the interstellar medium of galaxies. The observation of polarized synchrotron emission from the interstellar medium of the Milky Way and other galaxies, produced by relativistic electrons spiraling around magnetic field lines, is direct proof of the presence of such magnetic fields. Thus, interpretation of the dynamics of the interstellar medium requires an understanding of highly compressible MHD turbulence.
To study of Magneto hydrodynamics shocks in Astrophysical medium

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Abstract
The aim of the study is to discuss the effect of Magneto hydrodynamics shocks on the dynamics of Astrophysical plasma. The solution of Riemann problems to compute numerical solutions of Ideal magneto hydrodynamics for an arbitrary initial condition is also obtained which is partly based on the algorithm proposed by Torrilhon (2002).

Key words-Ideal Magneto hydrodynamics -MHD, turbulence

1. Introduction
Shocks and discontinuities are transition layers where the plasma properties change from one equilibrium state to another. The plasma is a fluid composed by charged particles in addition to this, a moving charged particle creates a magnetic field which also interacts with the other charged particles. The relation between the plasma properties on both sides of a shock or a discontinuity can be obtained from the conservatives form of the magneto hydrodynamics equations, assuming conservative of mass, momentum energy and of in the study of compressible fluid through nozzle the continuity consideration led. In some case to the formation of a discontinuity surface across which these are jumps in pressure, density, temperature etc. such surface are called shock wave these may also result from their causes, for example detonation of explosives supersonic flights of projectile and so on.

Magneto hydrodynamic (MHD) flow is governed by classical fluid dynamics and electromagnetic. Examples of such fluids include liquid metals, plasmas, and salt water or electrolytes. Application to MHD can be derive and control flows in
astronomy geo-physics, network, electromagnetic casting of metal and MHD power generation. Many Astrophysical phenomena is based on MHD flows of plasma.

The system of ideal MHD is not strictly hyperbolic. Gogosov (1962) investigated the wave-pattern of the solution in MHD Riemann problems on the existence and uniqueness of solutions of ideal MHD Riemann, considering only the evolutionary waves and switch-off waves. Torrilhon (2002) investigated the uniqueness of the solution considering the intermediate shocks.

The Riemann problem is a kind of initial value problems for hyperbolic systems like system of MHD equations with discontinuous initial condition in the form

$$u(x)|_{x=0} = \begin{cases} u^1, & x < 0 \\ u^0, & x > 0 \end{cases},$$

Solving Riemann problems is one of main tasks in numerical schemes for MHD flow because there is no convincing criterion for the physically relevant solution. The entropy condition admits only the shocks across.

The objective of present work is to explain Ideal MHD equations and exact solution of Riemann solutions are used to obtain numerical fluxes and problems to compute numerical solutions of Ideal MHD equations and also discussed the internal rotation motion of stars and turbulence which is one of basic problems in astrophysics.

The paper is organized as follows. Sec. 2 deals Ideal MHD equations and simple waves in Ideal MHD and discontinuity. Sec. 3 deal a brief review of shock waves which are constituents of the solution of the Riemann problems. Sec. 4 discuss the importance of magnetic field in stars and turbulence. Finally conclusion that have been drawn from this work are presented in Sec. 5.

2. Ideal MHD equation

The ideal MHD equations consist of the continuity equation, the Cauchy momentum equation, Ampere's Law neglecting displacement current, and a temperature evolution equation. Ideal MHD is only strictly applicable when the plasma is strongly collision, so that the time scale of collisions is shorter than the other characteristic times in the system, and the particle distributions are therefore close to Maxwell’s equation.

In plane symmetry the MHD equation are given by
\[
\frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0, \tag{2.1}
\]
\[
\frac{\partial \rho u}{\partial t} + \nabla \left[ \rho uu + P + \frac{B^2}{2} \right] = 0 \tag{2.2}
\]
\[
\frac{\partial e}{\partial t} + \nabla \left[ \left( e + P + \frac{B^2}{2} \right) u - B(B(u)) \right] = 0 \tag{2.3}
\]
\[
\frac{\partial B}{\partial t} - \nabla \times (u \times B) = 0 \tag{2.4}
\]

Where \( \rho, u, P \) and \( B \) are density, flow velocity, pressure and the magnetic field respectively. Total energy density
\[
e = \frac{P}{\gamma - 1} + \frac{1}{2} \rho u^2 + \frac{B^2}{2} \tag{2.5}
\]

For 1D plane symmetric flow \( \nabla \cdot B = 0 \)

Equations (2.1)-(2.5) can be written in conservative form as
\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0
\]

The vector of conserved variables \( U = (\rho, \rho u_x, \rho u_y, \rho u_z, e, B_x, B_y) \)

And the flux vector function
\[
F = \left( \rho u_x, \rho u_y + \frac{P + \frac{B^2}{2}}{\gamma - 1} - B_x^2, \rho u_y u_x - B_y B_x, \rho u_z u_x - B_z B_x, \left( e + \frac{B^2}{2} \right) u_x - B_x \left( B_x u_x + B_y u_y + B_z u_z \right) \right)
\]

**2.1 Simple waves in Ideal MHD**

The jacobian matrix \( J = \frac{\partial F}{\partial U} \) has 7 real eigenvalues. Ideal MHD equations are hyperbolic one for each wave. The dynamics of magnetized plasmas can be interpreted using solution of the properties of linear and nonlinear wave. The properties of linear waves can be study using the dispersion relation \( e^{i(\omega t + \lambda \cdot x)} \), where \( \omega \) is the frequency and \( \lambda \) is the wave vector. The MHD wave families are fast, Alfven, slow and entropy.

1. Fast magnetosonic waves \( u \mp c_s \) are longitudinal waves with variations in pressure and density. Fast magnetosonic waves are correlated with magnetic field.
2. Alfven waves $u \mp c_a$ are transverse waves with no variation in pressure and density. Alfven waves can be polarized. The sum of linear polarizations can lead to circularly polarized alfven, slow and entropy waves.

3. Slow magnetosonic waves $u \pm c_s$ are longitudinal waves with variations in pressure and density but slow magnetosonic waves are anticorrelated with magnetic field.

4. Entropy wave $u$ is a contact discontinuity with no variation in pressure and velocity. In these waves, the minus (plus) sign is applied to left-going (right-going) waves. MHD waves involve transversrs motion.

The characteristic speeds are expressed as

$$c_{f,s} = \left[ \frac{1}{2} \left( \frac{B_x^2 + B_y^2 + B_z^2}{\rho} + a^2 \right) \pm \sqrt{\frac{1}{4} \left( \frac{B_x^2 + B_y^2 + B_z^2}{\rho} + a^2 \right)^2 - \frac{a^2 B_s^2}{\rho}} \right]^{1/2},$$

$$c_a = \sqrt{\frac{B_s^2}{\rho}}$$

Where $a$ is the sound speed given by $a = \sqrt{\gamma P / \rho}$. The slow or fast signal could be shocks or rarefaction.
Since \( c_s \leq c_a \leq c_f \), this reflects that eigen values may coincide at a special point. Hence, the system of Ideal MHD equations is not strictly hyperbolic. There are certain cases where two or more eigen values collapse to the same value.

1. If \( B_y = 0 = B_z \) and \( a = \sqrt{\frac{\gamma P}{\rho}} \neq \sqrt{\frac{B_z^2}{\rho}} = c_a \), in this case either both fast or slow eigen values collapse with Alfvén speed.

2. If \( B_y = 0 = B_z \) and \( a = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{B_z^2}{\rho}} = c_a \), in this case Slow and Fast eigen values collapse with Alfvén eigen values.

3. If \( B_z = 0 \). Both Slow and Alfvén waves collapse to entropy wave.

When Eigen values collapse, there exists possibility of over compressive and under compressible Shocks. This fact makes finding solutions to the equations to ideal MHD more complicated. Today, the most important method for solving the MHD equations are numerical methods. But the development of numerical technique to solve MHD has been slower due to complexity of the MHD flow. Grid based methods to solve MHD equations is based on explicity finite difference scheme called the total variation dimising (TVD) scheme. In this method the conserved variables are discretized on a grid, with volume averaged values stored at cell centers. One of grid code for MHD is Athena. Athena implements a higher-order Godunov scheme. In this method, the difference in cell average values at each grid interface define set of Riemann problems. Solution of Riemann problems averaged over cell give time-evolution of cell average value. Riemann solvers is available to compute the fluxes.

The electric currents transmitted in an electrolyte solution interact with the magnetic field to form Lorentz body forces that in turn, drive fluid motion. Lorentz force is the flow in the direction perpendicular to both magnetic and electric fields in conductive, solutions in the MHD system with initial velocity and initial magnetic field by keeping electric field as negligible. The current flow in a direction perpendicular to the direction of magnetic field causes the fluid to experience a force. The force is in a direction perpendicular to both the magnetic field and the current flow. However, in many situation the changes in pressure and temperature are sufficient small that the changes in density are negligible. That flow can be modelled as an incompressible flow.
For gases, to determine whether to use compressible or incompressible fluid dynamics, the Mach number of the flow is to be evaluated. Compressible effect can be ignored at Mach numbers below approximately 0.3.

For liquids, the incompressible flow depends on fluid properties and the flow condition how close to critical pressure the actual flow pressure becomes. The phenomenon of strong and weak discontinuities in a compressible fluid has been of great interest amongst the scientist and mathematician. Rankine Hugoniot developed the jump conditions for sudden changes in physical parameters such as temperature, pressure, and velocity etc. with discontinuity. This sudden change may occur on account of sudden explosions in compressible gases, collision of clouds, movements of supersonic jets with high mach numbers.

Since most of the problems occurring are non-linear in nature, the similarity condition plays an important role in solving such problems. Analytical similarity solutions as well as numerical solutions exist for electrically conducting compressible flows across such discontinuities. It is therefore proposed that study of such non-linear problems nil come out in presence of conducting and non-conducting medium where conditions may be adiabatic or isothermal or both.

### 2.2. Discontinuities in ideal MHD

In one-dimensional $u_x$ and $B$ are scalar normal component of vector variables $u$ and $B$ in the direction of the space variable and the two-dimensional transversal parts $u_t$ and $B_t$. If $x$ is the space direction. We have $B = (B_x, B_y, B_z) = (B_n, B_t)$ and following Torrilhon (2002) MHD discontinuity satisfy the Rankine-Hugoniot relations, which in ideal MHD are expressed as

\[ \dot{P} - 1 + \gamma M^2_0 (\dot{u} - 1) + \frac{1}{2}(\dot{B}^2 - A^2) = 0, \]

\[ \gamma M^2_0 (\dot{u} B_t - A) - B^2 (\dot{B}^2 - A^2) = 0, \]

\[ M_0 \left[ \frac{1}{\gamma - 1} (\dot{P} - 1) + 1/2 (\dot{u} - 1)(\dot{P} + 1) + 1/4 (\dot{u} - 1)(\dot{B}^2 - A)^2 \right] = 0, \]

where $u = \frac{1}{\rho}$ is the specific volume. Fixing the upstream normalize quantities

\[ A := \frac{B_0}{\sqrt{P_0}}, B := \frac{B_n}{\sqrt{P_0}}, M_0 = \frac{u_{x0}}{a_0} \]
Solving the equation (2.21)-(2.23), using the quantities
\[ \dot{P} := \frac{P_1}{P_0}, \dot{B}_i := \frac{B_{1i}}{\sqrt{P_0}}, \dot{u} := \frac{u}{u_0} \quad (2.2.5) \]
The other downstream quantities can be calculated as
\[ u_{n1} = \dot{u}u_{n0} \]
\[ u_{r1} = u_{r0} \pm \frac{a_{r0}B}{\gamma M_0} [\dot{B}_{r0}] \quad (2.2.6) \]
The plus and minus signs correspond to the left- and right-going discontinuities respectively.

3. SHOCKS WAVE

When macroscopic motion with supersonic speed occurs in an interplanetary atmosphere occurs, strong and weak discontinuities popularly known as shock wave come into picture. In other word, A shocks wave is a special kind of wave as a steep finite pressure wave. In some situations shocks are undesirable because they interfere with the normal flow behaviour as turbomachines Parker (1963) studied the flow produced in solar wind using similarity method. Lee and Chen (1969) attempted the only self consistent similarly variable model of flow generated in a conducting plasma. Verma et al. (1986) studied the effect of magnetic field on shock in a rotating interplanetary gases. The normal shock wave is perpendicular to one dimensional flow. Shock may occur on account of supersonic flow developed on account of local accelerations. These shocks may be normal or inclined to the direction of local flow. They may cause boundary layer separation and deviation of flow from its designed direction. To study the complex gas motion behind the shock waves, one has to solve two points boundary value problems for a set of non linear partial differential equation. Other undesirable forms of shock waves are the sonic boom created by supersonic aircrafts and the blast waves generated by an explosion. One of the boundaries e.g. the point of explosion is fixed in space and has boundary conditions arising out of special symmetry about it. The other boundary is the shock waves across which the unknown flow variable satisfies certain condition law. On account of high temperature that prevail in many phenomenon with shocks.

Since the conservation laws apply across the phenomenon of non-linear discontinuities in a conducting plasma the analytical solutions may be obtained in
some cases, which restricts the propagation of these discontinuities along some characteristics. Riemann gives the method of characteristics for analytic expressions for the velocity of shocks waves.

Some useful applications of shock waves are in the shock tubes and supersonic compressors. A strong moving shock wave is utilised to accelerate the flow to a high Mach number in shock tube where flow behaviour at high Mach numbers can be studied. On account of abrupt changes of pressure, density, etc. Across shock waves, they are profitably used in supersonic compressors to obtain considerably high pressure ratios in one stage; in such compressors the pressure ratio developed per stage may be as high as 10.0. The thickness of such waves is of the order of $10^{-3}$ mm which is comparable with mean free path of the gas molecules.

![Diagram of stationary normal shocks](image)

Figure shows a normal shock wave in a constant area frictionless duct, the shock wave is considered to be contained in a control volume.

As the mathematical expressions which derive the non linear problems of propagation of shocks in conducting radiative medium an analytical solution may be obtained as far as possible but in most cases it become very difficult to achieve a complete solutions due to its non-linearity.

In such case, solutions shall be derived numerically on a computer by many well known methods such as Runge-Kutta approximoters or Newton’s Rephoson and other iterative methods. These approximate solutions however still project a near-complete picture of patterns of pressure, temperature density and velocity distributions across these discontinuities.
However with the expansion of knowledge computer softwares. It is now a day more prominent to achieve the numerical approximations in case a analytical solution is not obtained various software such as RKGS etc. are available to cope up such difficulties and an approximate numerical solutions may easily be achieved in getting non-linear equations.

it is therefore ,proposed that the non-linear equations shall be integrated with the help these software whenever required. The respective physical parameter thus, will exhibit the pattern of flow and field distribution (of course with the help of graph theory) of problem. Jeffrey and Taniuti(1964), the solution of Riemann problems is generally not unique in the sense of the weak solution and other conditions should be imposed to single out the physically relevant one.

Entropy condition, which admits only the shocks across which the entropy increases. The entropy condition discards manifestly unphysical solutions which include expanding shocks, across which the entropy is decreased. Takashi and Yamada(2013), In ideal MHD, some initial conditions have more than one solutions that satisfy the entropy condition . Therefore the so-called evolutionary conditions are introduced, which define physically relevant shocks should be structurally stable. Structurally stable shocks just remain close to the initial discontinuity.

If $M_0 \neq 0$ and $\hat{u} > 1$ The solutions of (2.21)-(2.23) is compressed as it passes through the discontinuities, are called shock waves. In this case the magnetic fields are either planar or coplanar. If $\hat{u} > 1$ and $u_{n1} = \hat{u} u_{n0}$, then we get $\hat{B}_i = \frac{u_{n0}^2 - c_{Al}^2}{u_{n1}^2 - c_{Al}^2} A$, Transverse magnetic fields are coplanar if and only if the upstream flow velocity is super Alfvenic whereas the downstream speed is sub-Alfvenic. The shocks with planar transverse magnetic fields are either fast or slow shocks. The shocks that change the direction of transverse magnetic fields are referred to as intermediate shocks.

4. MHD in Astrophysical

The solar magnetic fields that wave in some may connected with sun’s rotation imply large scale magnetic fields may appear in a rapidly rotating planet. The rotation of planet in the presence of a magnetic field is controlled by laws of isorotation i.e. field is symmetric about the axes of rotation and each line of force lies wholly on a surface. The matter in the Sun is in a plasma state, that is, an ionised gas with enough abundance of free charges. One way to have a reasonable description of the plasma
under solar conditions, among other applications is magneto-hydrodynamics. The
effect of magnetic fields to the dynamics and evolution of astrophysical plasmas
comes from observations of the outer layers of the Sun. The dynamo activity involves
patterns of magnetohydrodynamics flows, the interaction of differential rotation and
convection needed to magnetic field in large scale. Both the presence of sunspots in
the photosphere, and structures such as filaments, prominences, and flares in the solar
corona, demonstrate the key role that magnetic fields play in shaping the dynamics. In
fact, the very existence of the hot corona is now interpreted as due to heating by MHD
effects.

It is thought that most of the magnetic activity of the Sun is driven by the combination
of rotation and turbulent flows in the convection zone. In fact, the properties of MHD
turbulence driven by convection was one of the problems that first interested Chandra
in plasma physics.

Understanding the origin and evolution of the Sun’s magnetic field via a dynamo
process has been a challenging problem for many decades. In addition to generation
of the dipole field due to differential rotation, a process first proposed by
Parker (1955), there are also small-scale multipoles fields thought to be generated by
the convective turbulence that play a role in shaping sunspots and coronal activity.
Both the processes that produce sunspots, and the large-scale magnetic field of the
Sun, are very active areas of research.

In the case of sunspots, direct numerical simulations of magneto-convection in the
outer layers, including realistic radiative transfer to capture the outer radiative zone,
can now reproduce details of observed sunspots, including the penumbral filaments; a
beautiful example is given in Rempel et al. (2009). In the case of the solar dynamo, the
dipole field is now thought to originate in the tachocline, a region of strong shear
between the radiative core (which is in solid body rotation, according to results from
helioseismology) and the outer convective zone (which is in differential rotation).
However, although the sophistication of modern global MHD simulations of
magnetoconvection in spherical and rotating stars is impressive, they still fail to
explain both the origin of the differential rotation in the convective zone, and the
origin of the cyclic dipole field. Solving the solar dynamo problem is important, as we
are unlikely to understand magnetic fields in other stars if we cannot first understand
the Sun. These phenomenon usually occur in stellar atmosphere where turbulence in
medium is more often, many more scientists discover that in stanleous energy release along a line cylindrical shocks or along point, spherical shocks may propagate with increasing strength without limit.

The non-dimensional similarly conditions define the pattern of propagation of such motions and hence, it will be a matter of great interest if it is analysed whether the motion of these discontinuities strong or weak really propagate with high energy yields in the problems solar photosphere rocket reentry fission fusion reactions etc. One method to investigate the properties of MHD turbulence is through direct numerical simulation. Lemaster and Stone (2009), Numerical simulations of highly compressible MHD turbulence with both strong and weak magnetic fields. The turbulence is driven with a forcing function whose power spectrum is highly peaked at a wave number corresponding to about 1/8 the size of the computational domain. The energy input rate of the driving is held constant, and the turbulence is driven so that the Alfvenic Mach number of the turbulence is about one in the strong field case, and 7 in the weak The spectrum of fluctuations, such simulations can be used to measure properties such as the decay rate of the turbulence, and how it depends on the magnetic field strength. Early predictions suggested the decay rate of strongly magnetized turbulence would be very low, since it would be dominated by incompressible Alfven waves. Stone, Ostriker & Gammie (1999),The simulations found the decay rate of supersonic MHD turbulence was very fast, with the decay time about equal to an eddy turn over time on.

5.Conclusion:-
Converging and diverging shocks generated by instantaneous energy release over a cylindrical and spherical surface in a conducting medium are of great interest amongst the scientists working in area of solar explosions, detonations, stellar turbulence and other astrophysical phenomenon. If the effect of such discontinuities are formulated and these pattern is observed it is a very helpful in designing the space crafts and other missiles entering in such turbulent medium where occurrence these discontinuities is quite regular.

It is therefore, our efforts to provide a complete scenarios of physical distributions which may cause severe hazards to well calculated missiles entering in such medium. Goldreich & Sridhar (1995),The theories of the power spectrum and
statistical properties of MHD turbulence can be tested and compared. It is quite clear from the images that in the weak field case, the density fluctuations are isotropic, and the magnetic field is highly tangled. Goldreich & Sridhar (1995), In the strong field case the density fluctuations are elongated along the field lines, and the field is more or less ordered. The suggests that the power spectrum of the turbulence will be anisotropic.

It is impossible to describe studies of Astrophysical MHD shocks without mentioning the important role that numerical method. The types of waves generated and their order are not known a priori in Magneto hydrodynamics Riemann solver. Solution of Riemann are used to obtain numerical fluxes.
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Abstract
Shocks and discontinuities are nonlinear phenomena that are commonly observed in space plasma. The plasma is a fluid composed by charged particles in addition to this, a moving charged particle creates a magnetic field which also interacts with the other charged particles. The MHD conservation conditions are derived for flow angles and downstream velocities for given upstream variables. The shock conditions are solved at different densities.

Keywords: shock waves, MHD, magnetic field,

I. Introduction
A shocks wave is a special kind of wave as a steep finite pressure wave. In some situations shocks are undesirable because they interfere with the normal flow behaviour as turbo machines Parker (1963) studied the flow produced in solar wind using similarity method. Lee and Chen (1969) attempted the only self consistent similarly variable model of flow generated in a conducting plasma verma etal (1986) studied the effect of magnetic field on shock in a rotating interplanetary gases. The normal shock wave is perpendicular to one dimensional flow.

Shock may occur on account of supersonic flow developed on account of local accelerations. These shocks may be normal or inclined to the direction of local flow. They may cause boundary layer separation and deviation of flow from its designed direction. Other undesirable forms of shock waves are the sonic boom created by
supersonic aircrafts and the blast waves generated by an explosion. One of the boundaries e.g. the point of explosion is fixed in space and has boundary conditions arising out of special symmetry about it. The other boundary is the shock waves across which the unknown flow variable satisfies certain condition laws. On account of high temperature that prevail in many phenomenon with shocks.

Shock waves occur when macroscopic motion with supersonic speed occur in an interplanetary atmosphere. Shocks are characterized by strong and weak discontinuous changes in the characteristics of the medium, like pressure, temperature, energy etc in some situations shocks are undesirable for example detonation of explosives supersonic flights of projectile and so on. Since the width of the discontinuity is quite infinitesimal in comparison to the actual system, so it is assumed to be a single propagating front. Verma et al (1986) studied the effect of magnetic field on shocks in a rotating interplanetary gases. The normal shock wave is perpendicular to one dimensional flow. Shocks may occur on account of supersonic flow developed on account of local accelerations. These shocks may be normal or inclined to the direction of local flow.

Magneto hydrodynamic (MHD) flow is governed by classical fluid dynamics and electromagnetic. Examples of such fluids include liquid metals, plasmas, and salt water or electrolytes. Application to MHD can be derive and control flows in astronomy geo-physics, network, electromagnetic casting of metal and MHD power generation Many Astrophysical phenomena is based on MHD flows of plasma. The paper is organized as follows. Sec. 2 discuss Rakine-Hugoniot equations for shock jump conditions, Ideal MHD equations and a brief review of shocks waves. Sec. 3 discuss discontinuities in MHD and different types of shocks. Finally conclusion that have been drawn from this work are presented in Sec. 4.

2. Rakine-Hugoniot equations for shock jump conditions

In any frame where the shock is stationary, the Rankine-Hugoniot condition become

Mass conservation \[ \rho_1 v_1 = \rho_2 v_2 \] (2.1)

Momentum conservation: \[ \rho_1 v_1^2 + p_1 = \rho_2 v_2^2 + p_2 \] (2.2)

Energy conservation:

\[ \rho_1 v_1 u_1 + p_1 v_1 + \frac{1}{2} \rho_1 v_1^3 = \rho_2 v_2 u_2 + p_2 v_2 + \frac{1}{2} \rho_2 v_2^3 \] (2.3)
Specific internal energy \( U = \frac{P}{(\gamma - 1)\rho} \)

Where \( V, \rho, u \) refer to material stream velocity, density, and the internal energy respectively.

### 2.1 Ideal MHD Equation

The MHD equations and Maxwell’s equations can be integrated across a shock to give a set of jump conditions which relate plasma properties on each side of the shock front. In plane symmetry, the MHD equations are given by

- **Conservation of mass**
  \[
  \frac{\partial \rho}{\partial t} + \nabla (\rho v) = 0 \quad (2.1.1)
  \]

- **Conservation of momentum**
  \[
  \frac{\partial \rho v}{\partial t} + \nabla \left[ \rho v v + P + \frac{B^2}{2} \right] = 0 \quad (2.1.2)
  \]

- **Conservation of total energy**
  \[
  \frac{\partial e}{\partial t} + \nabla \left[ \left( e + P + \frac{B^2}{2} \right) v - B \times (Bv) \right] = 0 \quad (2.1.3)
  \]

- **Conservation of magnetic flux**
  \[
  \frac{\partial B}{\partial t} - \nabla \times (v \times B) = 0 \quad (2.1.4)
  \]

Where \( \rho, v, P, B \) are density, flow velocity, pressure, and the magnetic field, respectively.

**Total energy density**
\[
e = \frac{P}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2} \quad (2.1.5)
\]

For 1D plane symmetric flow \( \nabla \cdot B = 0 \)

### 2.2 Shock Waves

Shock waves are thin transitions from supersonic to subsonic flow involving compression and dissipation. Shocks occur when waves move faster than the ambient medium, leading to steepening of front portion of the wave mode. Small
amplitude limit the profile of an MHD wave does not change it propagates but a small amplitude wave will eventually distort due to steepening.

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I) If flow speed < signal speed \( (\omega/k) \) then subsonic speed occur.

Ii) If flow speed > signal speed then supersonic speed occur.

3. Discontinuities in MHD

The plasma properties behind the shock referred as the downstream region and the plasma properties ahead of shock referred as the upstream region. The conservative conditions are solved for downstream velocities and flow angles for given upstream variables. The dynamic behaviour of plasma is described by magneto hydrodynamics equations, where central role is played by Alfven speed. Due to the magnetic field the anisotropy arises in downstream velocity for the space-like shocks. With respect to the shock normal the downstream velocity vector always points downward. The magnetic field has no effect on time-like shocks. In the absence of magnetic field in upstream region one independent variable Mech number \( M \) (ratio of flow speed to sound speed) is usually chosen. In the presence of magnetic field the independent parameter is the angle \( \theta \) which lies between flow velocity and magnetic field. For \( \theta = \pi/2 \) shocks is perpendicular and for \( \theta = 0 \) (or \( \theta = \pi \)) as parallel. Consider the changes in velocity and magnetic field across a shock front.

The upstream plasma with velocity \( v_1 \) is perpendicular to shock front and receding downstream plasma with velocity \( v_2 \) is at an oblique angle to the shocks normal. In the upstream region the magnetic field makes an angle \( \phi_1 \) to shock normal and \( \phi_2 \) in the downstream region and \( s_1 = \tan \phi_1 \)
The phenomenon of strong and weak discontinuities in a compressible fluid has been of great interest amongst the scientist and mathematician. Rankine Hugoniot developed the jump conditions for sudden charges in physical parameters such as temp pressure and velocity etc. with discontinuity. This sudden change may occur on account of sudden explosions in compressible gases, collision of clouds, movements of super sonic jets with high mach numbers.

3.1 Parallel MHD shock

When $s_1 = 0$, both the upstream and downstream plasma are parallel to the magnetic field as well as perpendicular to the shock front. The dimensionless number that characterizes the strength of a shock speed to the upstream sound speed:

$$M_1 = \frac{v_1}{c_{s1}}$$

(3.1.1)

In terms of the Mach number, shock jump conditions are

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2}$$

(3.1.2)

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

(3.1.3)
Thus the upstream flow is supersonic if \( M_1 > 1 \) or \( v_1 > c_{s1} \), flow is ahead the shock if \( M_1 < 1 \).

If \( v_2 < c_{s2} \), Flow is subsonic behind the shock. If \( P_2 > P_1 \) and \( \rho_2 > \rho_1 \). The shock is compressive. If \( 1 < \rho_2/\rho_1 < (\gamma + 1)/(\gamma - 1) \) the maximum density ratio is \((\gamma + 1)/(\gamma - 1)\), but the pressure increases \( \propto M_1^2 \). If \( v_2 < v_1 \) and \( T_2 > T_1 \), a parallel shock is unaffected by the presence of magnetic field and is, therefore, usually called a hydrodynamic shock.

When \( M_1 = 1 \), the upstream flow is exactly sonic. If \( M_1 \neq 1 \), the upstream and downstream plasma parameters become different.

### 3.2 Perpendicular MHD Shock

When a magnetic field is applied perpendicular to the shock front, \( B \) compressed as the mass density. In other words, perpendicular shocks is, in which both the upstream and downstream plasma flows are perpendicular to the magnetic field. The analogues to the Rankine-Hugoniot equations, then are

\[
\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{B_1}{B_2} = \frac{1}{X}, \quad \frac{P_2}{P_1} = \gamma M_1^2 \left( 1 - \frac{1}{X} \right) - \frac{1 - X^2}{\beta_i}
\]

Where plasma beta \( \beta = \frac{2 \mu_0 P}{B^2} \) and \( X = \frac{\rho_2}{\rho_1} \) is the positive root of

\[
2(2 - \gamma)X^2 + \left[ 2\beta_i + (\gamma - 1)\beta_i M_1^2 + 2 \right]X - \gamma(\gamma + 1)\beta_i M_1^2 = 0
\]

Thus the condition for the existence of a perpendicular shock is that the relative upstream plasma velocity must be greater than the upstream fast wave velocity. A perpendicular shock cannot compress the density by more than a factor \((\gamma + 1)/(\gamma - 1)\). However, a perpendicular shock compress the magnetic field by the same factor.

### 3.3 Oblique shocks

For the case, \( 0 \leq s_1 \leq \infty \), in which the plasma velocities and the magnetic fields on each side of the shock are neither parallel nor perpendicular to the shock front. The direction of the material streaming away from the shock front not being necessarily unique.
Oblique shocks is best treated in de Hoffmann-teller frame in which \( \mathbf{v}_1 \times \mathbf{B}_1 = 0 \)

Upstream field is parallel to flow. In case \( \mathbf{E} \) vanishes downstream field will be parallel to flow.

If \( \mathbf{v}_1 / \cos \theta < c \), jump condition

\[
\frac{v_{2x}}{v_{1x}} = \frac{\rho_1}{\rho_2} = \frac{1}{X}, \quad (3.3.1)
\]

\[
\frac{v_{2y}}{v_{1y}} = \frac{v_1^2 - v_{Al}^2}{v_1^2 - Xv_{Al}^2}, \quad (3.3.2)
\]

\[
\frac{B_{2x}}{B_{1x}} = 1, \quad \frac{B_{2y}}{B_{1y}} = \frac{(v_1^2 - v_{Al}^2)X}{v_1^2 - Xv_{Al}^2}, \quad (3.3.3)
\]

\[
\frac{P_2}{P_1} = X + \frac{(\gamma - 1)Xv_1^2}{2c_{s1}^2} \left(1 - \frac{v_2^2}{v_1^2}\right), \quad (3.3.4)
\]

\[
\left(v_1^2 - Xv_{Al}^2\right)^2 \left[ Xc_{s1}^2 + 1/2 v_1^2 \cos^2 \theta \{X(\gamma - 1) - (\gamma + 1)\} \right] + 1/2 v_{Al}^2 v_1^2 \sin^2 \theta X \left[ (\gamma + X(2 - \gamma))v_1^2 - Xv_{Al}^2 \{ (\gamma + 1) - X(\gamma - 1) \} \right] = 0
\]

(3.3.5)

A fast and a slow shock are compressive. A fast wave travels faster than an intermediate wave, which travels faster than a slow wave. For fast shock \( B_{2y} > B_{1y} \)

, whereas \( B_{2y} < B_{1y} \) for a slow shock. For an intermediate shock \( B_{2y} \to -B_{1y} \).
\(v_1\), exceed the slow/fast speed ahead the shock while \(v_{2x}\) is smaller than the slow/fast speed behind the shock.. At the limit \(B_x \to 0\), the fast shock become perpendicular shock whereas the slow shock becomes a tangential discontinuity( \(v_x \to 0\) ) with arbitrary jumps in \(v_y\) and \(B_y\) subject to total pressure balance over the shock.

5. Conclusion:

A shock propagating through an MHD fluid produces a significant difference in plasma properties on the either side of the shocks front. For a parallel MHD shock in the shock frame, the upstream plasma velocity \(v_1\) must be supersonic. In other word, in a stationary plasma, a parallel, or hydrodynamic, shock propagates along the magnetic field with a supersonic velocity. A perpendicular shock compresses the magnetic field by the factor \((\gamma + 1)/(\gamma - 1)\). For a Oblique shock, fast shocks refract the magnetic field and plasma flow away from the normal to the shock front. Slow shocks refract these quantities toward the normal, whereas, the tangential magnetic field and plasma flow generally reverse across an intermediate shock front. MHD shocks have been observed in a large variety of situations e.g. by supernova explosions, by strong stellar winds, by solar flares and by solar wind upstream of planetary magnetospheres.