CHAPTER-6
SHOCKS WAVES PROPAGATION IN VISCOSITY GAS

Magneto-hydrodynamics flows have stimulated increasing attention in physics and engineering sciences during the last decades and developing rapidly in magnetic materials processing magnetic induction heating, liquid metal flow control, plasma control in high speed aerodynamics and magnetic breaking technologies.

Astrophysical geophysical problem and in technological developments for example propagation of shocks in stellar atmosphere propagation of waves and discontinuities in magneto plasma of solar wind, novae and supernovae out burst, generation of gas imaging shock waves by magnetic compressive to produce high temperature plasma samples in laboratory exploration of detonation. The propagation of shock wave under the influence of weak and strong magnetic field constitute a problem of great interest in physics and astrophysics. Evolution of disturbances is essential in a viscous MHD to obtain method for controlling the turbulent transition in the flow around the hypersonic objects at lower Mach number. A study of unsteady MHD free convection flow from a porous vertical flat plate immersed in a porous medium in presence of magnetic field with radiation was carried out by Khan et al., [1], [2]. analyzed the MHD transient flows in a channel of rectangular cross section containing a porous medium. Radiation effects on MHD flow past an impulsively stared infinite isothermal vertical late were studied by Chandrakala and Antony [3]. Ahmed [4] investigated the effect of periodic heat transfer on unsteady mixed convection MHD flow past a vertical porous plate with constant suction and heat sink when the free stream velocity oscillates in time. The above studies have generally been confined to very small magnetic Reynolds numbers, allowing magnetic induction effects to be neglected. Such effects must be considered for larger values of magnetic Reynolds number.

Glauert [5] presented a seminal analysis for hydro-magnetic flat plate boundary layers along a magnetized plate with uniform magnetic field in the stream direction at the plate. He obtained series expansion solutions (for both large and small values of the electrical conductivity parameter) for the velocity and magnetic fields, indicating that for a critical value of applied magnetic field, boundary-layer separation arises.
Dorrepaal and Moosavizadeh [6] studied the oblique stagnation-point magnetohydrodynamic flow in the vicinity of a location where a separation vortex reattaches itself to a rigid boundary, obtaining a similarity solution for the velocity and induced magnetic field distributions near the point of reattachment. Rankine [7] published his dissertation on the structure of shock waves in which he gave an explicit solution for the case of heat conduction only. Hugoniot [8] obtained the equations for shock jumps in particle velocity, stress, and specific internal energy that is known as the Rankine-Hugoniot conditions. To study the complex gas motion behind the shock waves, one has to solve two points’ boundary value problems for a set of non-linear partial differential equation. One of the boundaries, e.g., the point of explosion is fixed in space and has boundary conditions arising out of special symmetry about it. The other boundary is the shock waves across which the unknown flow variable satisfies certain condition laws.

An explicit solution for viscosity alone was given by Taylor [9] for weak shock waves with both viscosity and heat conduction present. The solutions were derived from the conservation laws of mass, momentum and energy for the single steady shock wave of finite thickness. Hoffmann and Teller [10] extended the Rankine-Hugoniot conditions of hydrodynamics to shock waves in an infinitely conducting fluid with magnetic field. The mathematical discontinuity given by the Rankine-Hugoniot conditions at a shock front is not physically possible, and considerations of dissipation of energy by viscosity and heat conductivity enable the physical quantities to vary continuously in a finite width of the shock front. Sen [11] described the structure of a magneto-hydrodynamic shock wave in infinitely conducting plasma. Richtmyer and Von Neumann [12] suggested a method for numerical calculation. They observed viscosity-like term into gas dynamics equations could lead to the continuous shock flow in which the finite thickness of discontinuities at the shock wave was removed and replaced by a region in which physical parameters changed rapidly and smoothly. The thickness of a thin transition layer representing the shock across which the gas undergoes transition from the initial to the final state, and this layer is generally known as a shock front. In this layer the density, the pressure and the velocity of fluid change as entropy increases. The increase in entropy indicates that there is a dissipation of mechanical energy and thus an irreversible conversion of mechanical energy into heat energy takes place in the transition layer. The dissipative
processes of viscosity (internal friction) and heat conduction are attributable to the molecular structure of a fluid. Such processes create an additional, non-hydrodynamic transfer of momentum and energy, and result in non-adiabatic flow and in the thermodynamically irreversible transformation of mechanical energy into heat. Viscosity and heat conduction appear only when there are large gradients in the flow variables within a shock front.

Landau and Lifshitz [13] investigated the weak viscous shock waves with respect to the small changes in the flow variables. Painter [14] studied a viscous shock wave in an elastic tube. Maslov [15] studied the wave processes in a viscous shock layer and control of fluctuations. Yadav and Anand [16] investigated the propagation of planar, cylindrically and spherically viscous shock waves in an ideal gas. Anand [17] formulated the shock jump relations for MHD shock waves in non-ideal gas and discussed the change-in-entropy across the shock front. The main purpose of this paper is to present an exact solution for one dimensional MHD shock wave in an ideal gas, by assuming that the viscosity is present, and that the heat conductivity is neglected, to find the effect of viscosity on the shock wave profile. For this purpose, a model was developed to provide a simplified, complete treatment of the structure of planar MHD shock waves in ideal gas. The general non-dimensional forms of the analytical expressions for the distribution of particle velocity, temperature, pressure and change-in-entropy within the shock transition region are derived, assuming the medium to be viscous, non-heat conducting, electrically infinitely conducting, initially uniform and at rest. The magnetic field is assumed having only constant axial component which is perpendicular to the shock front. The effects of viscosity are investigated on the MHD shock transition region due to an increase in the propagation distance from the centre of front, the strength of magnetic field, the strength of shock wave i.e., Mach number, Coefficient of viscosity, and the adiabatic index, on the particle velocity, temperature, pressure and change-in-entropy within the shock transition region.

6.1 EQUATIONS GOVERNING THE FLOW

In presence of magnetic field the conservation equation governing the flow of one dimensional, viscous, ideal gas under equilibrium condition can be expressed as conveniently in Eulerian coordinate as follows:
\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} = 0.\quad (6.1)
\]
\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + P - q)}{\partial r} + \frac{\mu^2}{2} \frac{\partial B^2}{\partial r} = 0.\quad (6.2)
\]
\[
\frac{\partial \left( \rho E + \rho \frac{u^2}{2} \right)}{\partial t} + \frac{\partial \left[ \rho u \left( E + \frac{u^2}{2} \right) + p u - q u \right]}{\partial r} = 0.\quad (6.3)
\]
\[
\frac{\partial B}{\partial t} + u \frac{\partial B}{\partial r} + B \frac{\partial u}{\partial r} = 0.\quad (6.4)
\]

Where \(\rho(r, t), u(r, t), q(r, t), B(r, t), p(r, t)\), \(t\) and \(r\) are density, particle velocity, viscous stress tensor, axial magnetic field, pressure, time coordinate and position coordinate with respect to the origin in the direction normal to the shock front, respectively, and \(\mu\) is the constant magnetic permeability of the gas taken to be unity throughout the problem. The diffusion term is omitted in the Equation (6.3) by virtue of the assumed perfect electrical conductively. The viscous stress tensor \(q\) is given by
\[
q = \frac{4u}{3} \frac{du}{dr},\quad (6.5)
\]

Where \(\mu\) is the coefficient of viscosity. It is assumed that \(\mu\) is independent of temperature. Landau and Lifshitz (1958) gave the equation of state for non ideal gas as
\[
p = k \rho t \left[ 1 + \rho C_1(t) + \rho^2 C_2(t) + \ldots \right],\quad (6.6)
\]

Where \(k, p, \rho\) and \(t\) are the gas constant, pressure, density, and temperature for the non ideal gas respectively. In the expansion \(C_1(t), C_2(t)\) are virial coefficients. It is remarkable that with conditions \(B_\theta = B_r = 0\) and \(B_z = B_r = 0\) Equation (6.4) can be written as
\[
\frac{\partial B}{\partial t} + \nabla \times (B \times u) = 0,\quad (6.7)
\]

And further \(\frac{\partial (\nabla \cdot B)}{\partial t} = 0\).

Thus, the Maxwell equation \(\nabla \cdot B = 0\) is included in equation (6.4). In a coordinate system the shock strength with stationary shock front remains practically unchanged during the small time interval required to travel a distance of the order of the shock front thickness, as a result the term containing the partial derivative with respect to
time is dropped in the Equations (6.1) to (6.4) and further the partial derivative with respect to \( r \) is replaced by the total derivative \( d/dr \). Thus, the Equations (6.1) to (6.4) governing the one dimensional plane symmetrical flow of a viscous gas under the influence of an axial magnetic field are written as

\[
\frac{u \, d\rho}{dr} + \rho \frac{du}{dr} = 0 , \quad (6.8)
\]

\[
\frac{\partial}{\partial r} \left( \rho u^2 + P - q \right) + \frac{\mu^2}{2} \frac{\partial B^2}{\partial r} = 0 , \quad (6.9)
\]

\[
\frac{\partial}{\partial r} \left[ \rho u \left( E + \frac{u^2}{2} + pu - qu \right) \right] = 0 , \quad (6.10)
\]

\[
u \frac{\partial B}{\partial r} + B \frac{\partial u}{\partial r} = 0 , \quad (6.11)
\]

Where \( \gamma = \frac{C_p}{C_v} \) is the adiabatic index. The boundary condition on the solution of these differential Equations (6.6) to (6.9) requires that the gradient of flow variables must vanish ahead of the shock front (at \( r = +\infty \)) as well as behind the shock front (at \( r = -\infty \)). With these limits, the initial flow variables designated by the subscript ‘\( o \)’ are \( p_o, \rho_o, u_o, B_o \) and the final flow variables with no subscript are \( p, \rho, u, B \). If the shock front is moving with velocity \( U \), then in the coordinate system fixed with the shock front, the initial particle velocity \( u_0 \) will be

\[
u_0 = U \quad (6.12)
\]

**6.2 EXACT SOLUTIONS FOR THE FLOW VARIABLES**

In order to obtain the exact solutions for the flow variables, we need to solve the flow Equations (6.8) to (6.11) using the boundary condition given by Equation (6.10) in the equilibrium state. For this, we integrate the Equations (6.6) to (6.9) which yields,

\[
\rho = \rho_o \frac{U}{u} , \quad (6.13)
\]

\[
p = p_o + q + \rho_o U^2 - \rho u^2 - \frac{B^2}{2} + \frac{B_o^2}{2} , \quad (6.14)
\]
\[
pu \frac{\gamma}{(\gamma - 1)} + \rho \frac{u^3}{2} - qu = p_0 U \frac{\gamma}{(\gamma - 1)} + \rho_0 \frac{U^3}{2}, \quad \tag{6.15}
\]

\[
B = B_0 \frac{U}{u}, \quad \tag{6.16}
\]

Using equations (6.13), (6.14) and (6.15), the equation (6.11) becomes

\[
\gamma p_0 u + \gamma qu + \gamma \rho_0 U^2 u + \gamma B_0^2 \frac{u}{2} - \gamma \rho_0 U u^2 - \gamma B_0^2 \frac{U^2}{2u} + 
\]

\[
(\gamma - 1) \rho_0 U \frac{u^2}{2} - (\gamma - 1) qu = \gamma p_0 U + (\gamma - 1) \rho_0 \frac{U^3}{2}, \quad \tag{6.17}
\]

Let particle velocity \( \eta \) and shock strength \( M \) are two new dimensionless quantities as

\[
\eta = \frac{u}{U} \quad \text{and} \quad M = \frac{U}{a_0} \quad \tag{6.18}
\]

Where \( a_0 \) is the speed sound in the unperturbed state. Using the equations (6.5) and (6.18) the equation (6.17) becomes

\[
(\gamma + 1) \eta^3 / 2 - \left( \gamma + M^{-2} + \frac{B_0^2}{2 p_0 M^2} \right) \eta^2 + \left[ M^{-2} + \frac{(\gamma - 1)}{2} \right] \eta +
\]

\[
\frac{H_0^2}{2 p_0 M^2} = \frac{4u}{3M \left( \gamma p_0 \rho_0 \right)^{1/2}} \eta^2 \frac{d\eta}{dr}, \quad \tag{6.19}
\]

The equation (6.19) can be written as

\[
a \eta^3 + 3b \eta^2 + 3c \eta + d = e \eta^2 \frac{d\eta}{dr} \quad \tag{6.20}
\]

Where

\[
a = \frac{(\gamma + 1)}{2}, \quad b = -\left( \frac{\gamma M^2 + 1 + \frac{H_0^2}{2 p_0}}{3M^2} \right), \quad c = \left( M^{-2} + \frac{(\gamma - 1)}{2} \right) / 3
\]

\[
d = \frac{B_0^2}{2 p_0 M^2} \quad \text{and} \quad e = \frac{4u}{3M \left( \gamma p_0 \rho_0 \right)}
\]

There is no gradient in the flow variable in the equilibrium state outside the transition regions we can write \( \frac{d\eta}{dr} = 0 \) with \( \eta = \eta_e \). The equation (6.8) becomes a cubic
equation in equilibrium state with respect to the particle velocity in equilibrium state \( \eta_c \). For finding real solution we write cubic equation as

\[
X^3 + 3fX + g = 0
\]  

(6.21)

Where \( X = a\eta_c + b \), \( f = ac - b^2 \) and \( g = a^2d - 3abc + 2b^3 \).

Now, we define \( \tan \phi = -k/g \), where \( g^2 + 4f^3 = -k^2 \). The algebraic solution of equation (6.21) using the Cordom’s method are

\[
\eta_c = \eta_1 = 2(-f)^{1/2} \cos(\phi/3) \quad \text{and} \quad \eta_{2,3} = -2(-f)^{1/2} \cos[(\pi \pm \phi)/3],
\]  

(6.23)

If \( g^2 + 4f^3 < 0 \), then the equation (6.12) will have three real roots and the particle velocity can be numerically compute corresponding to the equilibrium state in which there is no gradients in the flow variables. The point of inflection is obtained by the condition \( d^2\eta/dr^2 = 0 \) and the equation (6.18). The cubic equation becomes

\[
\eta_c^3 + 3f'\eta_c + g' = 0
\]  

(6.24)

Where \( f' = -c/3a \), \( g' = 2d/a \).

Now, we define \( \tan \phi = -k'/g' \), where \( g'^2 + 4f'^3 = -k'^2 \). The algebraic solution of equation (6.23) using the Cordom’s method are

\[
\eta_c = \eta_1' = 2(-f'^{1/2} \cos(\phi'/3) \quad \text{and} \quad \eta_{2,3}' = -2(-f'^{1/2} \cos[(\pi \pm \phi')/3],
\]  

(6.25)

(6.26)

If \( g'^2 + 4f'^3 < 0 \), the equation (6.24) will have three real roots and the point of inflection is obtained. The equation (6.22) gives an analytic solution s as

\[
r = \left[ A \log \frac{\eta - \eta_1}{\eta_c - \eta_1} + B_i \log \frac{\eta - \eta_2}{\eta_c - \eta_2} + C \log \frac{\eta - \eta_3}{\eta_c - \eta_3} \right] \left( \frac{e'}{a} \right)
\]  

(6.27)

Where

\[
A = \frac{\eta_1^2}{(\eta_1 - \eta_2)(\eta_1 - \eta_3)},
\]  

(6.28)

\[
B_i = \frac{\eta_i^2}{(\eta_i - \eta_2)(\eta_i - \eta_3)},
\]  

(6.29)

and \( C = \frac{\eta_3^2}{(\eta_3 - \eta_1)(\eta_3 - \eta_2)} \).
By the equation (6.25) we get a relation between the particle velocity and the distances. The particle velocity depends on the distance s within the shock transition region. From the equations (6.15) and (6.20), the temperature across the shock front can be written as

$$\frac{t}{t_0} = 1 + (\gamma - 1)M^2 \left[ \frac{\eta^2}{2} - \left( \gamma + \frac{1}{M^2} + \frac{B_0^2}{2p_0M^2} \right) \eta + \left( \frac{1}{M^2} + \gamma \right) + \frac{B_0^2}{2p_0M^2\eta} \right]$$

(6.30)

From the equation (6.27) we get a relation between the particle velocity and the temperature and also variations of the temperature with respect to the distance s within the shock transition region. From the equations (6.5), (6.12), (6.16) and (6.20), the pressure across the shock front can be written as

$$\frac{P}{P_0} = 1 + (1 - \eta)\gamma M^2 + \left( 1 - \frac{1}{\eta^2} \right) \frac{B_0^2}{2p_0} + \gamma M^2 \left( a\eta^3 + 3b\eta^2 + 3c\eta + d \right) \frac{1}{\eta^2}$$

(6.31)

The equation (6.31) gives a relation between the particle velocity and the pressure and also variations of the pressure with respect to the distance s within the shock transition region.

The change in entropy across the shock front is given by

$$\left( \frac{\Delta S}{R} \right)_\eta = \gamma \log \left( \frac{t}{t_0} \right) - \log \left( \frac{P}{P_0} \right)$$

(6.32)

The entropy production across the MHD shock front is obtained by putting the equation (6.24) and the equation (6.25) into the equation (6.26).

6.3 RESULT

An exact equation for MHD shocks waves in a viscous gas are obtained. Within shocks transition region the general analytical expressions for distribution of the flow variables (the particle velocity $\eta$, the temperature $\frac{t}{t_0}$, the pressure $\frac{P}{P_0}$, and the change in entropy $\frac{\Delta S}{R}$) are given by equations (6.25), (6.26), (6.27) and (6.28) respectively. By assuming the disturbance due to reflections these analytical expression were formed. The non-dimensional analytical expression for particle pressure, velocity, temperature...
and change in entropy are the function of distance \( r \), strength of magnetic field \( B_0 \), coefficient of viscosity \( \mu \), strength of shock \( M \) and adiabatic index \( \gamma \) of the gas. So the value of the constant parameter are taken to be \( \mu = 15 \times 10^{-6}, 17.2 \times 10^{-6}, 20 \times 10^{-6} \) pascal sec, \( B_0 = 0.0, 2.0, 4.0, 6.0, 0.8 \), \( M = 1.0, 1.5, 2.0 \), \( \gamma = 1.33, 1.40, 1.66 \), initial pressure \( P_0 = 0.9, 1.0, 1.1 \) bar and initial density \( \rho_0 = 1.2, 0.1, 2.1 \) kg/m\(^3\) for numerical computations.

**Fig 1** Non-dimensional particle velocity, temperature, pressure and change in entropy distribution with distance for various values of \( P_0 \) and the constant value of \( M = 2 \), \( \gamma = 1.33 \), \( P_0 = 0.9 \) bar, \( \rho_0 = 1.2 \) kg/m\(^3\) and \( \mu = 15 \times 10^{-6} \) pascal sec.

Fig 1 shows the variations of the particle velocity \( \eta \), the temperature \( t/t_0 \), the pressure \( P/P_0 \) and the change in entropy \( \Delta S/R \) with distance for the different values
of initial pressure $P_0=0.9, 1.1$ bar and axial magnetic field $B_0=0, 0.4, 0.8$ Tesla and constant values of $M=2$, $\gamma=1.33$, $\rho_0=1.2 kg/m^3$, $\mu=15 \times 10^{-6}$ pascal.sec.

The spreading of the flow variables decreases with increase in the value of initial pressure. However, the decrease in the thickness of shock front with increase in the initial pressure is more noticeable for large values of the strength of magnetic field than that for small values of the strength of magnetic field. Thus, the thickness of MHD shock front increases with decrease in the initial pressure.

Fig 2 shows the variations of the particle velocity $\eta$, the temperature $t/t_0$, the pressure $P/P_0$ and the change in entropy $\Delta S/R$ with distance for the different values of initial density $\rho_0=1.2, 1.29, 1.4 kg/m^3$, 0.9, 1, 1.1 bar and axial magnetic field.
$B_0=0, 0.4, 0.8$ Tesla and constant values of $M = 2$, $\gamma = 1.33$, $P_0 = 1.2 \text{kg/m}^3$, $\mu = 15 \times 10^{-6} \text{ pascal.sec}$. 

The spreading of the flow variables decreases with increase in the value of initial density. However, the decrease in the thickness of shock front with increase in the initial density is independent of the strength of magnetic field. Thus, the thickness of MHD shock front increases with decrease in the initial density.

![Graphs](image)

**Fig 3** Non-dimensional particle velocity, temperature, pressure and change in entropy distribution with distance for various values of $\mu$ and $B_0$ and the constant value of $M = 2$, $\gamma = 1.33$, $P_0 = 0.9 \text{bar}$ and $\rho_0 = 1.2 \text{kg/m}^3$.

Fig 3 shows the variations of the particle velocity $\eta$, the temperature $t/t_0$, the pressure $P/P_0$ and the change in entropy $\Delta S/R$ with distance $r$ for the different values of initial pressure $B_0 = 0.9, 0.8$ Tesla bar and $\mu = 15 \times 10^{-6}, 17.2 \times 10^{-6}, 20 \times 10^{-6} \text{ pascal.sec}$.

The spreading of the flow variables increases with increase in the coefficient of viscosity. However, the increase in the thickness of shock front with increase in the coefficient of viscosity is more noticeable for large values of the strength of magnetic
field than that for small values of the strength of magnetic field. Thus, the thickness of MHD shock front increases with increase in the coefficient of viscosity.

REFERENCES

problems).


