Although benchmark results on standard databases in themselves are useful only to a limited extent and may result in excessive tuning of the system parameters to improve the system performance, they constitute a good starting point for comparison of the gross performance characteristics of the systems. This research work is an effort to compare the performance of the proposed system for personnel identification using the unimodal hand based biometrics namely palmprint, fingerprint and finger-knuckle print along with multiple biometric fusions used for personal identification or system authentication.

This Chapter of the thesis has explored application of multi-resolution and multi-scale transforms to extract various features from imprint images of hand based biometrics used for personnel identification and system authentication. The transforms used includes Radon transform, Discrete Dyadic wavelets and orientation field. The subsequent sections cover a brief analysis of theoretical part of all the transforms mentioned. It also includes the feature extraction methods and experimental results.

4.1 Personnel identification of unimodal biometrics using Radon transform

This section explains the proposed person identification system using the Radon transform for extracting the features. Since the local features extracted as from ROI represents the identification image in better way in the form of texture information, the identification image is viewed as texture image. In particular direction line integrals along parallel direction are computed using Radon transform. Here the line integrals of identification image are computed at 60 different directions. At the interval of these directions are ranging from 0-180 degrees. As feature map these feature vectors are used. PolyU database and FVC2000 has been used for evaluating the performance of the algorithm on the machine having 2.6 GHz Pentium-IV processor and 512 MB RAM beneath MATLAB environment.
4.1.1 Radon transform

Considering for a 2D image \( f(x, y) \), Radon transform is denoted as \( P_\theta(t) \), is defined as its line integral along a line inclined at an angle \( \theta \) from the \( y \)-axis and at a distance \( t \) from the origin (see Figure 4.1).

That is at an angle \( \theta \), \( P_\theta(t) \) represents 1D projection of function \( f(x, y) \). It is given as

\[
P_\theta(t) = \int \int f(x, y) \delta(x \cos \theta + y \sin \theta - t) dx \, dy \tag{4.1}
\]

where,

\( \delta \) is the dirac distribution.

Line integral an important property of Radon transform is used for rotation estimation of the shapes in an imprint image. Radon transform presume a function which has line modeled through a delta function.

\[
g(x, y) = \delta(y - p^*x - \tau^*) \tag{4.2}
\]

Hence, the function gives non-zero values only when \((x, y)\) lies on the line with certain fixed parameters \((p^*, \tau^*)\). In this case the Radon transform is given by

\[
g(p, \tau) = \int \int \delta(y - p^*x - \tau) \delta(y - px - \tau) dx \, dy \tag{4.3}
\]

\[
= \int \delta((p - p^*)x + \tau - \tau^*) dx
\]
\[
\begin{cases}
    1 & \text{for } p \neq p^* \\
    \left| p - p^* \right| & \text{for } p = p^* \text{ and } \tau \neq \tau^* \\
    \int_{-\infty}^{\infty} \delta(0) dx & \text{for } p = p^* \text{ and } \tau = \tau^*
\end{cases}
\]

Note that for \( p = p^* \) and \( \tau = \tau^* \), the result is written as infinite function integrated over an infinite interval, hence at that point result is infinite. If the finite terms are neglected, an infinite value peak is produced by Radon transform of line in the parameter domain and the position of the peak matches the line parameters. This property has the basis of edge feature detection in images, which is demonstrated in Figure 4.2.

![Figure 4.2 Line property](image)

Figure 4.2 Line property (a) A 2-D function \( g(x, y) \), (b) Radon transform \( g(p, \tau) \).

### 4.1.2 Feature extraction of palmprint as unimodal biometrics using Radon transform

In order to extract the features Radon transform have been applied on the ROI of the palmprint. The enhanced texture information can be represented by deriving the local features from palmprint ROI.

For a square integrable function \( f(x_1, x_2) \) the Radon transform is defined as follows.

\[
R_A(t, \theta) = \iint f(x_1, x_2) \delta(x_1 \cos \theta + x_2 \sin \theta - t) dx_1 dx_2
\]

where,

\( \delta \) is the Dirac distribution.

The line integrals are computed using Radon transform. These line integrals are along parallel paths in a definite direction. For a palmprint we have used 60 different directions for computing line integrals. Experimentally, having thee interval of 3 degree 60 orientations (directions) are selected. Thus we used 185×60 size feature vector as the feature map. A query image is represented and matched using these feature maps. Figure 4.3 illustrates the
significant variation in Radon projections of two different palmprint images at few sample orientations.

![Image of palmprint and Radon projections]

Figure 4.3 Radon projections at 0, 22.5, 45, 67.5 and 90 degree obtained for two different palmprints and their respective hotplot.

### 4.1.3 Features extraction of fingerprint as unimodal biometrics using Radon transform

In order to extract the features Radon transform have been applied on the ROI of the fingerprint.

For a fingerprint we have used 60 different directions for computing line integrals. Experimentally, having thee interval of 3 degree 60 orientations (directions) are selected. Thus we used 380×60 size feature vector as the feature map. A query image is represented and matched using theses feature maps.

Figure 4.4 shows the significant variation in Radon projections of two different fingerprint images at few sample orientations.
4.1.4 Results and discussions

The proposed algorithm of imprint feature extraction is implemented using Radon transform and tested on 2.6 GHz Pentium-IV processor, 512 MB RAM with MATLAB environment. The imprint identification system performance using Radon transform has been tested using the standard databases FVC2000 and PolyU available on the website. From these localized images of ROI the Radon transformed images are obtained. Since the local features extracted as from ROI represents the identification image in better way in the form of texture information, the identification image is viewed as texture image. The feature maps obtained from all imprint images are stored in database. For a particular person when query imprint is applied, the feature vector is generated. This feature vector is matched with the feature vectors that are available in the database. The Euclidian distance as a classifier is used to get the minimum distance for best match of the query image with stored template in database. Figure 4.5 shows percentage FAR and GAR plotted at various thresholds.

Figure 4.4 Radon projections at 0, 22.5, 45, 67.5 and 90 degree obtained for two different fingerprints and their respective hotplot.
Themaximum recognition accuracy of 97.73% and 94.46% has been achieved for fingerprint and palmprint images respectively using Radon transform on FVC2000 and PolyU databases. The 0.61 s time required for testing about proves that the algorithm is computationally efficient.

![FAR and GAR for Radon Transform](image)

Figure 4.5 Percentage FAR and GAR plotted at various thresholds.

### 4.2 Personnel identification of unimodal biometrics using discrete dyadic wavelet transform

Previously in image processing applications discrete dyadic wavelet transform have been successfully applied. High redundancy is observed for this transform and the same can be exploited by transform coefficients modification in some non-linear manner and reconstructing. Convolving the zero-crossing of the signal with the Laplacian of a Gaussian, one can get the location of multi scale sharp variations points. This procedure has been used in many pattern recognition applications. A basic issue is to recognize whether the zero-crossing describe a stable and complete representation of the original signal. Indeed, when using the signal in pattern recognition applications we eliminate some vital components, when it is represented with multi scale zero crossing. In this Section we have implemented method for feature extraction from preprocessed palmprint and fingerprint images as unimodal biometrics using discrete dyadic wavelet transform for personal recognition. The local features from the extracted ROI of an imprint of palm and finger represent the textural information there in imprint image in enhanced sense. Discrete dyadic
wavelet transform calculates the dominant textural information representing it with multi-scale zero-crossing. Here we have computed zero-crossing representation corresponding to the lowest four resolution levels of the dyadic wavelet transform. As the feature map the feature vectors are of size $4 \times 256$ are used. The algorithm performance is evaluated using FVC2000 database and PolyU database on the machine with 2.6GHz Pentium-IV processor GHz with 512 MB RAM and MATLAB environment. Maximum recognition accuracy of 98.77% and 96.27% has been achieved for fingerprint and palmprint images respectively using dyadic wavelet transform on FVC2000 and PolyU databases. The required testing time of 0.76s proves that the algorithm is computationally efficient.

4.2.1 Discrete dyadic wavelet transform

Discrete non-redundant wavelet transforms have been successfully applied previously in image processing applications. However, the lack of translation invariance and aliasing present after the decomposition stage may introduce undesirable artifacts for the analysis of the signals and images hence use of a redundant wavelet representation can be justified. The discrete dyadic wavelet transform is one example of a redundant representation. The wavelet was a first derivative of a smoothing function and was used as a multi-scale edge detector to obtain a translation invariant parsimonious representation consisting of edges [214]. A reconstruction algorithm to approximate an original signal from its multi-scale edge coefficients alone was devised in [214, 215]. High redundancy is observed for this transform and the same can be exploited by transform coefficients modification in some non-linear manner and reconstructing. It is frequently useful to rearrange the signal data into an arrangement of subtle element segments of differing size for a signal including vital structures that have a place with distinctive scales. It is realized that one can acquire the position of multi-scale sharp varieties focuses from the zero-crossing of the signal convolved with the Laplacian of a Gaussian [216]. This method has been utilized as a part of numerous pattern recognition applications. A fundamental issue is to understand whether the zero-crossing define a complete and stable representation of the original signal. Indeed, when using the signal in pattern recognition applications we eliminate some vital components, when it is represented with multi-scale zero crossing.

The discrete dyadic wavelet transform was initially projected in one and two dimensions. However, medical imaging more generally requires for signal processing in more than 2D. I.Koreny et al [217] expand the discrete dyadic wavelet transform to multiple dimensions and explain as an efficient accomplishment within fast hierarchical digital
filtering methods. When digital filtering of a finite period discrete signal is carried out using circular convolution the filter will act equally on ends of a signal at the same time. It may guide to artifacts near both ends of the result. Mirror addition of an input signal is an accepted method in image processing, for alleviating these boundary effects. Hence they present a fast filter bank implementation of the discrete dyadic wavelet transform which takes benefit of mirror extended input to the filter bank.

A continuous representation through discretizing dilation and translation parameters is used to obtain discrete wavelet transform. A frame is formed from the resulting set of wavelets. With fixed dilation steps of exponential sampling the dilation parameter is characteristically discretized and the translation parameter through integer multiples of dilation-reliant step [218]. A variant under conversions property makes the transform less attractive intended for the study of non-stationary signals. The transform resulting into translation-invariance and redundant representation can be achieved by using the same sampling period for the input function and the translation parameter. Mallat and Zhong [214] proposed a Dyadic wavelet transform is one such illustration. A brief overview of the dyadic wavelet transform as described in [214] is as follows.

The dyadic wavelet transform of a function \( s(x) \in L^2(R) \) is defined as a sequence of functions

\[
\{W_m s(x)\}_{m \in \mathbb{Z}}
\]  

(4.6)

where, \( W_m s(x) = s \ast \psi_m(x) = \int_{-\infty}^{+\infty} s(t) \psi_m(x - t) dt \), and \( \psi_m(x) = 2^{-m} \psi(2^{-m}x) \) is a wavelet \( \psi(x) \) expanded by a dilation parameter/scale \( 2^m \).

The condition that needs to be satisfied everywhere to make sure coverage of the frequency axis condition on Fourier transform of \( \varphi_m(x) \) with existence of \( A_1 > 0 \) and \( B_1 < \infty \) is

\[
A_1 \leq \sum_{m=-\infty}^{\infty} |\hat{\psi}(2^m \omega)|^2 \leq B_1
\]

(4.7)

The limitation on Fourier transform of reconstructing function \( \varphi(x) \) is

\[
\sum_{m=-\infty}^{\infty} \hat{\varphi}(2^m \omega) \hat{\varphi}(2^m \omega) = 1
\]

(4.8)

A function \( s(x) \) can be completely recreated from its dyadic wavelet transform using the individuality

\[
s(x) = W_m s \ast \chi_m(x)
\]

(4.9)

Where \( \chi_m(x) = 2^{-m} \chi(2^{-m}x) \).

Instead of processing on continuous functions, processing is performed on discrete functions in numerical applications. When transforming the function in discrete form,
The scale $2^m$ no longer varies over all $m \in \mathbb{Z}$. Large scale restricts computational resources whereas small one prohibits finite sampling rate.

Consider that we have to normalize the finest scale to 1 and the coarsest scale is to set to $2^M$, where analysis levels are denoted by $M \in \mathbb{Z}$. The smoothing of a function $s(x) \in L^2(\mathbb{R})$ is defined as

$$S_m s(x) = s * \phi_m(x)$$  \hspace{1cm} (4.10)

where, $\phi_m(x) = 2^{-m} \phi(2^{-m} x)$ with $m \in \mathbb{Z}$ and $\phi(x)$ is a smoothing function. It has integral equal to 1 and $\phi(x) \to 0$ as $|x| \to \infty$.

A real smoothing function $\phi(x)$ has been selected of which Fourier transform satisfies the condition

$$|\hat{\phi}(\omega)|^2 = \sum_{m=1}^{\infty} |\hat{\psi}(2^m \omega)|^2$$  \hspace{1cm} (4.11)

Any discrete function of finite energy ($s(n) \in l^2(\mathbb{Z})$) can be rewritten as the uniform sampling of some function smoothed at scale 1, i.e., $s(n) = S_0 t(n)$, where $t(x) \in L^2(\mathbb{R})$ is not unique. Hence, the discrete dyadic wavelet transform of $S_0 t(n)$ for any coarse scale $2^M$ is defined as a sequence of discrete functions

$$\{S_M t(n + s), \{W_m t(n + s)\}_{m \in [1,M]}\}_{n \in \mathbb{Z}}$$  \hspace{1cm} (4.12)

where, $s$ denotes sampling shift dependent on $\psi(x)$.

For a convinced option of wavelets the discrete dyadic wavelet transform can be implemented inside a speedy hierarchical digital filtering scheme. Let

$$F_S(\omega) = e^{-jis} F(\omega)$$  \hspace{1cm} (4.13)

where, $F(\omega)$ takes up one of the forms $H(\omega)$, $G(\omega)$, or $K(\omega)$ defined as,

$$H(\omega) = e^{j\omega s} \left(\cos\left(\frac{\omega}{2}\right)\right)^{p+1}$$  \hspace{1cm} (4.14)

where, $p$ is a nonnegative integer and

$$s = \frac{p+1 \mod 2}{2}.$$

$$G(\omega) = e^{j\omega s} \left(4j \sin\left(\frac{\omega}{2}\right)\right)^{r}$$  \hspace{1cm} (4.15)

where, $r \in \{1, 2\}$ and

$$s = \frac{r \mod 2}{2},$$

determines $K(\omega)$.

$$K(\omega) = -\frac{1}{16} \left(e^{-j\omega G(\omega)}\right)^{r \mod 2} \sum_{l=0}^{p} \left(\cos\left(\frac{\omega}{2}\right)\right)^{2l}$$  \hspace{1cm} (4.16)
In this work $H(\omega)$ is a lowpass filter, $G(\omega)$ a highpass filter and $K(\omega)$ a highpass filter for $r = 1$ and a lowpass filter when $r = 2$ and $p > 0$. The filters described in Equations (4.14) through (4.16) are finite impulse response (FIR) filters. Filters referred in (4.14) through (4.16) at level $m + 1$ (i.e., filters applied to some scale $2^m$) become $F(2^m \omega)$, where $F(\omega)$ denotes some of the three filters at level 1. Inside the spatial domain it is equal to upsampling the filter impulse response by $2^m$ (i.e., inserting $2^m - 1$ zeros between subsequent filter coefficients at level 1). Noninteger shifts at level 1 is rounded to the nearest integer. Holschneider et al [219] proposed implementation with upsampling of filter impulse response with linear increase in complexity with number of levels. Figure 4.6 shows the construction of a filter bank implementation of the discrete dyadic wavelet transform.

Figure 4.6 One-dimensional discrete dyadic wavelet transform decomposition-filter bank implementation (left) and reconstruction for three levels of analysis (right). $H_s^*(\omega)$ denote the complex conjugate of $H_s(\omega)$.

### 4.2.2 Feature extraction of palmprint as unimodal biometrics using discrete dyadic wavelet transform

To achieve the rotation and translational invariance for the algorithm, a ROI of the palmprint image have been obtained before feature extraction. It is a five step algorithm as explained in Section 3.3. In order to extract the features discrete dyadic wavelet transform
has been applied on the ROI of the palmprint. The steps for extracting the features using discrete dyadic wavelet transform are as follows:

**Step 1 Compute the palmprint signature (PS) by plotting gray level values from ROI**

In this step of the preprocessing block we have plotted the signature of the palmprint image from the extracted ROI. The gray level values are recorded for the horizontal central line of the extracted ROI of the palmprint image. The signature of the palmprint is generated by plotting the gray level values of each pixel on that line against the pixel numbers. The sample signatures of three different palmprints are shown in Figure 4.7.

**Step 2 Compute zero-crossing illustration of the signature of palmprint by applying discrete dyadic wavelet transform to generate the feature vectors.**

The dyadic wavelet uses the quadratic spline of compact support which is given in [214]. A finite support of the function is the major advantage of using this function and it also has lesser coefficientsthan those of the second derivative in the smoothing function [216].

The dyadic wavelet transform of a function \( f(x) \in L^2(R) \)is defined as a sequence of functions \( W_{2^j} f(x) \) whose abscissae are respectively \( \{Z_{n-1}, Z_n\} \), werecord the value of the integral \( e_n = \int_{Z_{n-1}}^{Z_n} W_{2^j} f \, dx \). For any function \( W_{2^j} f \), the position of the zero-crossings \( Z_{n} \) can be represented by a piecewise constant function \( Z_{2^j} f = \frac{e_n}{z_n - z_{n-1}} \).

For obtaining a stable and complete representation, zero-crossing of thedyadic wavelet transform of the palmprint signature (PS) has been considered; rather than considering the zero-crossing of a wavelet transform on a continuum of scales. Dyadic scales have been restricted to \( 2^j, j \in Z \), and recorded the value of the wavelet transform between two consecutive zero-crossings.

![Signature of Palmprint](image-url)
Figure 4.7 Sample signatures of palmprints (a) extracted ROI of palmprint, (b) corresponding PS for the horizontal central line of the extracted ROI.
Figure 4.8 Resolution levels of the dyadic wavelet transform of palmprint signatures
(a) extracted ROI of palmprint, (b) corresponding PS along with six resolution levels of the
dyadic wavelet transform.
Figure 4.9 Zero-crossing representation of PS(a) extracted ROI of palmprint, (b) zero-crossing representation corresponding to the lowest four resolution levels of the dyadic wavelet transform.

The PS has been measured with a limited determination that forces a better scale when figuring the dyadic wavelet change instead of computing the wavelet transform at all scales $2^j$ for $j$ varying from $-\infty$ to $+\infty$. A nonzero finer scale and finite larger scale limit the resolution. For implementation purpose the finer scale is equal to 1 and the largest scale is $2^J$. The discretedyadic wavelet transform of the PS can be obtained using

$$\{S_{2^j}(PS), (W_{2^j}(PS))_{1 \leq j \leq J}\}$$

(4.17)

where $S_{2^j}(PS)$ is the coarse signal and $(W_{2^j}(PS))_{1 \leq j \leq J}$ denotes discrete details available when smoothing PS at scale 1 [220].

Sign changes of samples estimates zero crossing of $(W_{2^j}(PS))_{1 \leq j \leq J}$. Linear interpolation between two samples of different sign estimates the position of each zero crossing. As discrete wavelet representation contains at most $Nlog(N)$ samples and if PS has $N$ non-zero samples, the number of operations to obtain the position of the zero crossing is
$O(N \log(N))$. From a discreted dyadic wavelet transform the zero-crossing positions along the scales $2^j$ for $1 \leq j \leq J$ can only be computed. Thus the discrete zero crossing representation of $PS$ has been considered as the set of signals $\{(Z_{2^j}(PS))_{1 \leq j \leq J}\}$.

The data at fine resolution levels is emphatically influenced by quantization errors and noise that happen because of the utilization of a rectangular grid in digital pictures [221]. Hence, the coarsest levels have been excluded in order to obtain a robust representation in a noisy environment and reduce the number of computations required. For reducing the aforesaid effects on the zero-crossing representation, coarsest levels are excluded and a few low resolution levels have been used.

Lastly, for obtaining the robust demonstration in noisy environments and to lessen the computations required, only a reduced number of resolutions levels have been used. Figure 4.8 shows six resolution levels of the dyadic wavelet transform of various palmprint signatures. Figure 4.9 shows the zero-crossing representation corresponding to the lowest four resolution levels i.e. $3 \leq j \leq 6$ of the wavelet transform previously calculated.

### 4.2.3 Feature extraction of fingerprint as unimodal biometrics using discrete dyadic wavelet transform

In order to extract the features discrete dyadic wavelet transform has been applied on the ROI of the fingerprint. It is a two step algorithm as explained in Section 4.2.2. The signature of fingerprint has been computed by plotting gray level values from the extracted ROI.
Figure 4.10 Sample signatures of fingerprints (a) original fingerprint, (b) corresponding signature for the horizontal central line of the fingerprint.
Figure 4.11 Six resolution levels of the dyadic wavelet transform of fingerprint signatures.
The gray level values are recorded for the horizontal central line of the extracted ROI of the fingerprint image against the pixel numbers. The sample signatures of three different fingerprints are shown in Figure 4.10. In order to obtain a complete and stablerepresentation, zero-crossing of thedyadic wavelet transform of the fingerprint signature has been considered; instead of considering the zero-crossing of a wavelet transform on a continuum of scales. Dyadic scales are restricted and valueof the wavelet transform between two consecutive zero-crossings has been recorded. Sign changes of samples estimates zero-crossing. Linear interpolation between two samples of different sign estimates the position of each zero-crossing. Thus the discrete zero-crossing representation of the signature has been considered as the set of signals. Finally, in order to obtain a robust representation in noisy environments and to reduce the amount of computations required, only a reduced number of resolutions levels have been used. Figure
4.11 shows six resolution levels of the dyadic wavelet transform of various fingerprint signatures. Figure 4.12 shows the zero-crossing representation corresponding to the lowest four resolution levels i.e. $3 \leq j \leq 6$ of the wavelet transform previously calculated.

4.2.4 Results and discussions

The machine with 2.6 GHz processor and 512 MB RAM with MATLAB environment has been used for implementing and testing the algorithm. The performance of the imprint identification system for palmprint and fingerprint as unimodal biometrics using discrete dyadic wavelet transform has been tested using the standard databases FVC2000 and PolyU available on the internet.

![FAR and GAR using Dyadic wavelet](image)

Figure 4.13 Percentage FAR and GAR plotted at various thresholds.

To evaluate the performance of the proposed algorithm discrete dyadic wavelet transform based features have been computed. Each of the images in the database is matched with all other imprint images in the same database. A correct match is counted if two imprint images from the same palm are collected; otherwise it is an incorrect matching. The minimum distance gives the best match of query image with the stored templates. The Euclidian distance classifier is used to find this distance. Figure 4.13 shows percentage FAR and GAR plotted at various thresholds for palmprint and fingerprint. The maximum recognition accuracy of 98.77% and 96.27% has been achieved for fingerprint and palmprint images respectively using dyadic wavelet transform on FVC2000 and PolyU databases. The algorithm is computationally efficient as the testing time required is 0.76 s.
4.3 Personnel identification of unimodal biometrics using orientation field

Orientations describe the textural properties in the images and have been used extensively in image processing and texture analysis. The local features from the extracted ROI of a palmprint and fingerprint represent the texture information present in the imprint image in better sense. In this section person identification carried out efficiently using unimodal biometrics. Here the orientation features of fingerprint and palmprint images have been used as unimodal biometric. Initially the ROI is extracted from the fingerprint and palmprint images. From the orientation field the variance feature vector is computed. It is considered as the template (feature vector) of the imprint for the individual and stored in database in enrolment process. When the query image is applied, its variance feature vector (of orientation field) is computed and it is matched with the stored templates in the database. For the matching Euclidian distance is used. For performance evolution of the algorithm FVC2000 and PolyU databases are used on machine with 2.6GHz Pentium-IV processor, 512MB RAM under MATLAB environment. The algorithm performed well on the standard database images for various threshold values with maximum recognition accuracy of 93.81 percent and 96.62 percent. The time 0.53 sec require for testing indicates that the algorithm is computationally efficient.

4.3.1 Orientation field

The local orientation of ridges in palmprint image is defined by the orientation field of palmprint image. The orientation at pixel \((i, j)\)can be calculated with the following steps:

\textit{Step 1}: A block of size of \(W \times W\) is considered which is centered at pixel \((i, j)\) in the normalized palmprint image.

\textit{Step 2}: Find out the gradients \(\partial_x(i, j)\) and \(\partial_y(i, j)\) for each pixel in the block. These are the gradient magnitudes. To compute \(\partial_x(i, j)\), horizontal Sobel operator has been used which is defined as,

\[
\begin{bmatrix}
1 & 0 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1
\end{bmatrix}
\]

(4.18)

To compute \(\partial_y(i, j)\), the vertical Sobel operator has been used which defined as,

\[
\begin{bmatrix}
1 & 2 & 1 \\
0 & 0 & 0 \\
-1 & -2 & -1
\end{bmatrix}
\]

(4.19)

\textit{Step 3}: The estimation of local orientation at pixel \((i, j)\) done by using,

\[
\theta(i, j) = \frac{1}{2}\tan^{-1}\frac{\partial_y(i, j)}{\partial_x(i, j)}
\]

(4.20)
where,
\[ V_x(i, j) = \sum_{u=-w/2}^{i+w/2} \sum_{v=-w/2}^{j+w/2} 2 \partial_x(u,v) \partial_y(u,v) \]
\[ V_y(i, j) = \sum_{u=-w/2}^{i+w/2} \sum_{v=-w/2}^{j+w/2} \partial_x^2(u,v) \partial_y^2(u,v) \]

\( \theta(i,j) \) denotes the least square estimate of local orientation of the block centered at pixel \((i,j)\).

**Step 4:** The orientation field is smoothened in a local neighborhood by using a Gaussian filter. Initial orientation image conversion to a continuous vector field is defined as,
\[ \phi_x(i,j) = \cos[2\theta(i,j)] \]
\[ \phi_y(i,j) = \sin[2\theta(i,j)] \]

where, \( \phi_x \) and \( \phi_y \) are the vector field components in \( x \) and \( y \) direction respectively.

Gaussian smoothing is then performed after computing the vector field as follows,
\[ \phi_x'(i,j) = \sum_{u=-w_\phi/2}^{w_\phi/2} \sum_{v=-w_\phi/2}^{w_\phi/2} G(u,v) \phi_x(i-uw, j-vw) \]
\[ \phi_y'(i,j) = \sum_{u=-w_\phi/2}^{w_\phi/2} \sum_{v=-w_\phi/2}^{w_\phi/2} G(u,v) \phi_y(i-uw, j-vw) \] (4.22)

where, \( G \) denotes Gaussian low-pass filter of size \( W_\phi \times W_\phi \).

The final smoothed orientation field \( O \) at pixel \((i,j)\) is defined as,
\[ O(i,j) = \frac{1}{2} \tan^{-1} \frac{\phi_y'(i,j)}{\phi_x'(i,j)} \] (4.23)

### 4.3.2 Feature extraction of palmprint as unimodal biometrics using orientation field

In order to make the algorithm rotation and translation invariant, a ROI have been obtained from the palmprint image prior to feature extraction. It is a five step algorithm as explained in Section 3.3. The orientation field variance feature vector is computed from the ROI of the palmprint and it is used as a feature map (template).

\[ \sigma_k^2 = \sum_{i=1}^{n}[O(i,k) - \mu_k]^2 \text{ for } k = 1, ..., m \] (4.24)

Where \( \mu_k \) is a mean of \( k^{th} \) column of \( O(i,j) \), \( m \) and \( n \) are number of columns and rows of orientation field \( O \), respectively.

The feature vector is given by
\[ \vec{\sigma} = [\sigma_1^2 \sigma_2^2 ... \sigma_n^2]^T \] (4.25)
The query palmprint image variance feature vector of orientation field has been calculated by using steps (4.18)-(4.25). The $L^2$ norm is used for matching the variance feature vector of orientation field of query and the template image.

$$d = \| \tilde{\theta}^q - \tilde{\theta}^t \|^2 = \sum_{k=1}^{m} \left( \sigma_k^2 - \sigma_k^t \right)^2$$  \hspace{1cm} (4.26)

Where $\tilde{\theta}$ and $\tilde{\theta}'$ are feature vectors of template and query palmprint images, respectively. Figure 4.14 shows sample of hand image from PolyU database, the extracted palmprint and the orientation field has been shown in Figure 4.14.

![Sample image from PolyU database](image1)

(a) (b) (c)

Figure 4.14 Sample image from PolyU database (a) hand image, (b) extracted palmprint, (c) orientation field on the extracted palmprint.

### 4.3.3 Feature extraction of fingerprint as unimodal biometrics using orientation field

In order to make the algorithm rotation and translation invariant, a ROI have been obtained from the fingerprint image prior to feature extraction. The feature map or template is formed by computing the variance feature vector of orientation field from the ROI of the fingerprint. The steps (4.18)-(4.25) are used to compute the variance feature vector of orientation field for query image. The $L^2$ norm is used for matching the variance feature vector of orientation field of query and the template image. Figure 4.15 shows sample of FVC2000 fingerprint image and the orientation field.

![Sample fingerprint image](image2)

(a) (b)
4.3.4 Results and discussions

The proposed algorithm of imprint feature extraction using orientation field has been implemented on a machine with 2.6GHz Pentium-IV processor and 512 MB RAM beneath MATLAB environment. The system performance of the imprint identification using orientation field has been tested for databases of PolyU and FVC2000 available on the website. To conduct the experiments, the database is classified as training set and testing set. For training purposes, randomly selected six images per palm are considered. Testing set is formed using the remaining images. Initially from the database and captured imprint images, the ROI of size $128 \times 128$ is extracted. Then these localized imprint images are used to get the orientation field images. Since these local features symbolize the textural information there in the imprint image in better sense, these orientation fields of the imprint image are used as feature map. All the imprint image feature maps are stored in the database. On application of query image of the person, feature vectors are generated. The generated feature vectors are then compared with the feature vectors which are in the database. The Euclidian distance classifier is used to get the minimum distance for the best match.

For various thresholds false acceptance rate (FAR) and false rejection rate (FRR) is computed to measure the performance of algorithm. For calculation of FAR and FRR consider that $N$ is the number of subjects (with 17 imprints each). Which gives $T = 17 \times N$ total number of imprint images in the database.

For experimentation a single template for each subject has been considered. For finding true claims and imposter claims a total trials carried out are $(T - 6) \times N$, out of which total true claims are $11 \times N$ and imposter claims are equal to $(total\ trials - true\ claims)$. From this we get,
FRR = (true claims rejected/total true claims) × 100%,
FAR = (imposter claims accepted/total imposter claims) × 100% and
GAR = 100 – FRR in percentage

At different threshold values FAR and FRR has been calculated for every possible combination. It is plotted as shown in Figure 4.16. It is found that algorithm performed well for different threshold values on the FVC2000 and PolyU database. The maximum recognition accuracy of 96.62% and 93.81% has been achieved for fingerprint and palmprint images respectively using orientation field on FVC2000 and PolyU databases. Computationally this algorithm is efficient since the required testing time is very less of the order of 0.53s.

4.4 Personnel identification of palmprint as unimodal biometrics based on principal lines using Histogram of Radon transform

Two dimensional Radon transform gives projection of image intensity along the oriented line at specific angle. Radon value at any arbitrary pixel location is obtained by integrals along the lines of all direction passing from that pixel. Thus Radon transform maps linear signal components into pronounced extreme which can be detected very robustly. The coordinates of these peaks are reliable estimates of the geometrical parameters of collinear structures. In this Section these peaks of principal lines of palmprint are identified and used as feature maps for matching to identify the palmprint of a person.
Radon transform maps Cartesian coordinates of the palmprint pixels into polar pixel coordinates and replace the complex search for aligned topologically connected clusters of pixels by the search of relative maxima in the transform image. Each pixel of the transformed domain will be associated with a unique line in the original image. The larger the transformed image, the finer will be the granularity in the representation of lines in the original image. These two properties of Radon transform have been used in this proposed model to extract the principal line features from the palmprint. The various steps in the proposed model to extract the principal line features from the palmprint are as follows:

**Step 1** Calculation of the Radon coefficients.

**Step 2** Mapping of Radon coefficients in spatial domain palmprint image and formation of polygon.

**Step 3** Computation of area and periphery of the polygon.

**Step 4** Obtain the logarithm of Radon coefficients.

**Step 5** Calculation of histogram of Radon coefficients.

**Step 6** Calculation of correlation function of histogram of Radon coefficients.

**Step 7** Matching of the correlation function using predefined threshold.

### 4.4.1 Feature extraction based on principal lines

In the extracted ROI of palmprint image the principal lines can be considered as straight lines. Therefore Radon transform can be directly applied to the ROI of palmprint to give the number of peaks in Radon domain. The advantage of Radon transform is its ability to generate individual peak even for crossing lines. Therefore crossing of two palm lines do not make any problem in separation of two peaks that can be easily observed in Figure 4.17. Instead of using all the detected peak points from the palmprint image, a set of peak points in descending order have been utilized to reduce dimensionality of the feature vector. Once these reduced peak points are identified based on the location of the peak points in Radon domain, corresponding pixels have been identified in the palmprint spatial domain.

![Radon transform of principal lines](image)

Figure 4.17 Radon transform of principal lines (a) Original image having cross lines, (b) Corresponding Radon transform.
Let $g$ be an array consisting of Radon projection coefficients obtained with $360^0$ rotation as defined in (4.5). The size of $g$ is $n \times 360$, where $n$ depends on the size of an image $M \times N$. The maximum value coefficients $R_g(\theta)$ obtained from Radon transform $R_g(t, \theta)$ along each projection are selected in order to reduce the feature vector size and defined as,

$$R_g(\theta) = \max_t R_g(t, \theta)$$  \hspace{1cm} (4.27)

Thus, we have adopted a limited set of Radon coefficients for each palmprint image. In this work we have used 33 Radon coefficients from the available coefficients $R_g(t, \theta)$. These Radon coefficients are mapped as the corresponding pixels on the spatial domain palmprint image. Once these spatial positions are obtained, geometric relationship is established between these pixels to form the feature point. These pixels are joined together to form a polygon from the available spatial points and corresponding area and periphery has been computed to form the feature vector used for the purpose of matching. The mapping of the Radon coefficients on the spatial domain palmprint image and formation of polygon from the outlier points is shown in Figure 4.18. The mapping of Radon coefficients to spatial domain points on the palmprint image is computed using,

$$x = \frac{M}{2} + x_p \cos \left( \frac{R_g \pi}{180} \right)$$  \hspace{1cm} (4.28)

$$y = \frac{N}{2} + x_p \cos \left( \frac{R_g \pi}{180} \right)$$  \hspace{1cm} (4.29)

![Figure 4.18](image)

Figure 4.18 Polygon formation for two different palmprint images (a) Original palmprint, (b) Maximum value Radon coefficients, (c) Polygon formation from outlier points
In order to measure the similarity between points, geometric attributes can be established between these points. In this work, closed polygon is formed using outlier pixel related Radon peaks and rests of the pixels which lie inside this polygon are discarded for calculating its area and periphery.

For all the images from the database, we have calculated the area and peripheral matrix corresponding to the image within database forming the feature vectors. To match the query image, the polygon area of query image is compared with database features matrix. Thus, overall method requires calculation of outlier points of the polygon along with its area and periphery. In Gabor based method, the size of feature points depends on number of orientation selected to design the Gabor filter and therefore the dimensionality of the feature vector increases. In order to reduce the dimensionality of feature vector, it requires application of PCA or ICA. While in proposed model, it needs calculation of points based on Radon projection and to form the single polygon. Thus, proposed model is computationally more efficient as compared to Gabor based model. The proposed model has less computational complexity as compared to the method proposed by J. Dai et al [222] that need calculation of the Delaunay triangle.

4.4.2 Palmprint matching based on Histogram of Radon transform

Histogram of Radon transform plays vital role in shape analysis as introduced by S. Tabbone et al [223]. The histogram of Radon transform represents the shape length at each orientation. It is also translation and rotation invariant. Thus histogram of Radon transform provides similar response to the palm having either rotation or translation. This is the major advantage in comparison with finite Radon or modified Radon transforms presented in [224, 225]. In order to achieve the scaling invariance, normalization of Radon image and histogram is utilized by S. Tabbone et al [223] which is highly sensitive to the noise. Instead of normalization process, A. Kumar et al [226] use logarithm conversion and phase correlation which is invariant to the noise. In this work, we have used logarithmic histogram of Radon transform to extract the features of principal lines of palmprint. The application of logarithm on (4.5) gives,
\[ R_j(\theta) = \ln R_x(\theta) \]  \hspace{1cm} (4.30)

Corresponding histogram of Radon transform is calculated as,
\[ LHRT(\theta, y) = H(R_j(\theta))(y) \]  \hspace{1cm} (4.31)

where,
\[ H \] is the histogram of Radon in direction \( \theta \).
The features of palm are obtained by calculating the phase correlation of the obtained logarithmic histogram of Radon transform using (4.31). In this work, the Fourier transform of logarithmic histogram of Radon transform and its corresponding correlation function is calculated for the identification process. The correlation function is defined as,

\[ C = \sum \sqrt{G(u,v)} \ast G(u,v) \]

(4.32)

where,

\[ G(u,v) \] is the inverse Fourier transform of logarithmic histogram of Radon transform of query palmprint image,

\[ G(u,v) \] is the inverse Fourier transform of logarithmic histogram of Radon transform of template palmprint image from database.

In order to match the two palmprints, the correlation function is compared with specified threshold in such a way that if C is higher than the predefined threshold then the two palmprints are matched.

4.4.3 Results and Discussions

The proposed algorithm of palmprint feature extraction using histogram of Radon transform has been implemented and tested on Pentium-IV processor with 2.8 GHz, 1 GB RAM under MATLAB environment. The performance of the palmprint identification system using histogram of Radon transform has been tested on the standard database available (PolyU) on the website.

![Figure 4.19](image_url)

Figure 4.19Four palmprint images at different intensities, noise and rotations of two different persons from PolyU database.

Figure 4.19 shows palmprint images of two persons having variation in intensities, containing noise and small rotation from the standard PolyU database. In this section we
have performed three experiments. In the first experiment we have computed correlation coefficients of four different sub-images of a palmprint of the same person captured at different times with different intensities, noise and rotations. Table 4.1 shows the calculation of correlation coefficients of four different palmprint images of the same person. From Table 4.1, it can be seen that the autocorrelation coefficients are more than 98% while cross-correlation coefficients are less than 97%.

Table 4.1 Palm image correlation coefficient values

<table>
<thead>
<tr>
<th>Palm Image</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>94.86</td>
<td>95.25</td>
<td>95.85</td>
</tr>
<tr>
<td>2</td>
<td>95.24</td>
<td>98.24</td>
<td>95.27</td>
<td>95.26</td>
</tr>
<tr>
<td>3</td>
<td>96.38</td>
<td>95.77</td>
<td>98.43</td>
<td>95.28</td>
</tr>
<tr>
<td>4</td>
<td>96.30</td>
<td>95.91</td>
<td>95.85</td>
<td>98.29</td>
</tr>
</tbody>
</table>

In the second experiment, the algorithm for computation of histogram of Radon transform is applied on the extracted ROI of the palmprint to obtain the Radon coefficients. In order to measure the similarity between two palmprints, the area and periphery of the polygon obtained from the query image is compared with the stored values in the database. Figure 4.20 and Figure 4.21 show the area and periphery of 130 images from polyU database. Table 4.2 and Table 4.3 shows the similarity measures based on the area and periphery respectively, obtained from sample palmprints. It is seen that each palmprint has unique feature vector in the form of polygon area and periphery. The resultant difference of area and periphery is zero, when the two palmprints match. The proposed model requires polygon calculation only, as compared to the other models [222, 227 and 228]. Also the feature vector size required to compare the palmprints is of the order of only $1 \times 2$ (i.e. area and periphery) and hence the time complexity of the algorithm is very less.

Figure 4.20 Area of polygons for various palmprints from PolyU database.
The proposed model only requires calculation of histogram that is utilized as feature vector for comparison. The performance of model is tested on $30 \times 4$ palmprint images, where four images are of the same person. The performance of the algorithm is evaluated using region of convergence curve. This consists of False Rejection Rate and False Acceptance Rate. These FAR and FRR can be measured at different thresholds. For testing the model, FAR and FRR has been computed for every possible combination as shown in Figure 4.22.
Table 4.3 Similarity measure based on periphery of the polygon

<table>
<thead>
<tr>
<th></th>
<th>208.4</th>
<th>234.5</th>
<th>200.9</th>
<th>217.0</th>
<th>209.3</th>
<th>182.8</th>
<th>148.0</th>
<th>230.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>208.4</td>
<td>0</td>
<td>-26</td>
<td>7.48</td>
<td>-8.57</td>
<td>-0.88</td>
<td>25.5</td>
<td>60.4</td>
<td>-22.0</td>
</tr>
<tr>
<td>234.5</td>
<td>26</td>
<td>0.00</td>
<td>33.53</td>
<td>17.4</td>
<td>25.1</td>
<td>51.6</td>
<td>86.4</td>
<td>3.98</td>
</tr>
<tr>
<td>200.9</td>
<td>-7.48</td>
<td>-33.5</td>
<td>0</td>
<td>-16</td>
<td>-8.36</td>
<td>18.1</td>
<td>52.9</td>
<td>-29.5</td>
</tr>
<tr>
<td>217.0</td>
<td>8.57</td>
<td>-17.4</td>
<td>16</td>
<td>0</td>
<td>7.69</td>
<td>34.1</td>
<td>69</td>
<td>-13.5</td>
</tr>
<tr>
<td>209.3</td>
<td>0.88</td>
<td>-25.1</td>
<td>8.36</td>
<td>-7.69</td>
<td>0</td>
<td>26.4</td>
<td>61.3</td>
<td>-21.1</td>
</tr>
<tr>
<td>182.8</td>
<td>-25.5</td>
<td>-51.6</td>
<td>-18.1</td>
<td>-34.1</td>
<td>-26.4</td>
<td>0</td>
<td>34.8</td>
<td>-47.6</td>
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<tr>
<td>148.0</td>
<td>-60.4</td>
<td>-86.4</td>
<td>-52.9</td>
<td>-69</td>
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<td>-34.8</td>
<td>0</td>
<td>-82.5</td>
</tr>
<tr>
<td>230.5</td>
<td>22</td>
<td>-3.9</td>
<td>29.5</td>
<td>13.5</td>
<td>21.1</td>
<td>47.6</td>
<td>82.5</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.22FAR and FRR plotted against various thresholds

4.5 Personnel identification using multimodal biometrics

This Section proposes the use of multiple modalities for personal identification. Due to availability of features having great distinction and easy in acquisition of fingerprint and palmprint modalities, they are used in the proposed model. The main distinctive feature of palmprint is the principal lines and fingerprint has orientations of valley and ridge structures as distinctive features. The Radon Transform is utilized to extract these features in the proposed work. A major drawback of Radon transform is sensitivity to the orientation. Generally features are normalized to make them invariant to the rotation and noise insensitive. In the proposed model, logarithm operation is used for normalization. Euclidean distance which is rotation invariant is calculated in matching process. Thus
proposed model is capable to match low resolution images and with characteristics of rotation invariance, noise insensitive. The computational complexity is also less in comparison with other models.

Various modalities like finger and palm, finger and iris, finger and face, face and palm etc are used as modalities in the multimodal biometric based identification system. Many literatures previously propose multimodal biometric. All modality except fingerprint and hand geometry (palmprint), have weakness in their characteristic. This work proposed the fingerprint and palmprint as a multiple modalities to develop a system of multimodal biometric. Fusion of these modalities is the fundamental requirement for these biometric systems.

4.5.1 Fusion levels for multiple biometrics

Fusion of these modalities can be achieved at four different levels namely sensor level, feature level, match level and decision level. A brief discussion about these levels is as follows:

1) Sensor-level fusion

In multi-sample method, a single sample is formed by combining the multiple samples. For example, the series of fingerprints from the individual identities are scanned by the fingerprint scanner and these series are combined to form a large and single "rolled" image.

2) Template-level fusion

Template-level fusion is also known as feature-level fusion technique. Feature extractor software converts samples (images) into simplified computer representations known as templates or feature sets. In template-level fusion, multiple templates are combined to form a single template.

3) Match-level fusion

In these methods, multiple modalities also referred as samples or instances are compared and similarity scores are calculated. A single fused score is formed by combining these scores. In algorithms where a single sample is searched, the results of multiple algorithms are combined.

4) Decision-level fusion

It is applicable to cases listed for match-level fusion, but match / non-match decision is declared form the scores before fusion.
The comparison of the match-level fusion and feature-level fusion with the decision-level fusion shows that the decision level fusion has least complexity in terms of utmost interoperability across different templates formats, biometric features, recognition algorithms and template safety, and score rules’ comparison. The only drawback of this fusion is that amount availability of information at this level is limited and hence it is rigid while fusing at the decision level.

4.5.2 Proposed Radon based feature extraction

Radon transform holds distinguishable place in the field of biometric. Various biometric recognitions like iris recognition face recognition and fingerprint recognition have been proposed using Radon transform. Using Radon transform, the core of fingerprint is extracted by calculating Radon transform at 0° and 90°. Radon transform is most suitable technique to make the recognition algorithm independent of image acquisition device. Here we have proposed algorithm for extracting the features of modalities using Radon transform.

Modality selection:
Face, fingerprint, iris, signature, voice and hand geometry are normally employed or studied as Biometrics. The face recognition needs camera capturing image. This method avoids the direct contact between human and machine. 3D facial recognition technique is computationally expensive since it needs to process large amount of data. Iris based biometric systems are highly accurate, but they need appropriate alignment and positioning. The recognition system based on voice modality is a contactless system however its recognition rate is poor when large databases are used. Other difficulties involved in such systems are channel variances and to control sensor which impact significantly to the capabilities. In signature based identification systems, individual’s consistency is required to signature which is difficult to enroll and verifying in signature verification.

Table 4.4 shows that except hand geometry and fingerprint, remaining all the modalities have weakness in at least one characteristic. Therefore combination of hand geometry and fingerprint can be a preferable choice in comparison with other biometric pairs. Fingerprint feature of every individual is unique. The valleys and ridges are the feature which extracted from fingerprint for authentication. The detailed research on fingerprint based recognition system shows that the fingerprints can be characterized by minutia instead of their ridges and valleys. Compared to other modalities, palm possesses more features. The palmprint matching approach can be used for the features having low resolutions like principal lines,
wrinkles and texture; and features with high resolutions such as minutiae, ridges and singular points.

Table 4.4 Comparison of various biometric technologies based on the perception of the authors of [3], here H indicates High, M indicates Medium and L indicate Low

<table>
<thead>
<tr>
<th>Biometric Identifier</th>
<th>Universality</th>
<th>Distinctiveness</th>
<th>Permanence</th>
<th>Collectability</th>
<th>Performance</th>
<th>Acceptability</th>
<th>Circumvention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face</td>
<td>H</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Facial Thermogram</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>M</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>Fingerprint</td>
<td>M</td>
<td>H</td>
<td>H</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>Gait</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>M</td>
</tr>
<tr>
<td>Hand Geometry</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
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<tr>
<td>Iris</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>M</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Palmprint</td>
<td>M</td>
<td>H</td>
<td>H</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>Voice</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Retina</td>
<td>H</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>DNA</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>H</td>
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<tr>
<td>Keystroke</td>
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<td>L</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

Radon based feature extraction:

Radon transform provides energy with respect to the spatial distribution. The Radon transform represents the lines \(ax + by = c\) to a set of oriented lines with radial parameters as shown in Figure 4.23. Considering the line \(ax + by = c\), where \(a\), \(b\) and \(c\) are constants. Using this, the points on unit circle using equation can be defined as

\[
\frac{a}{\sqrt{a^2+b^2}} x + \frac{b}{\sqrt{a^2+b^2}} y = \frac{c}{\sqrt{a^2+b^2}}
\]

(4.33)

where,

two coefficients describing these points are given as, \(\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}\).
Let $\theta$ is the angle corresponding to the points defined using (4.33), then

$$\theta = \cos^{-1} \frac{a}{\sqrt{a^2 + b^2}}, \quad \cos \theta = \frac{a}{\sqrt{a^2 + b^2}} \quad \text{and} \quad \sin \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

(4.34)

The angle of points ranges in $[0, \pi]$. Considering the distance of lines as $t$ along the angle $\theta$ from the origin, then lines can be represented using this distance as solution of equation

$$t = \langle (x, y), (\cos \theta, \sin \theta) \rangle = \langle (x, y), \omega \rangle$$

(4.35)

Let vector $\omega = \langle (\cos \theta, \sin \theta) \rangle$ perpendicular to line $ax + by = c$. Corresponding orthogonal vector $\hat{\omega} = \langle (\cos \theta, \sin \theta) \rangle$ is parallel to line. Using these, a vector equation in terms of $t$ and $\theta$ can be created for the lines

$$l_{t, \theta} = t \omega + s \hat{\omega} = \langle t \cos \theta, \sin \theta \rangle + s \langle -\sin \theta, \cos \theta \rangle$$

(4.36)

Let some function $f$ is parameterized over the lines $l_{t, \theta}$. Radon transform is defined as,

$$Rf(t, \theta) = \int_{l_{t, \theta}} f(s) ds = \int_{-\infty}^{\infty} f(t \omega + s \hat{\omega}) ds = \int_{-\infty}^{\infty} f(t \cos \theta - s \sin \theta, t \sin \theta + s \sin \theta) ds$$

(4.37)

Since Radon transform computes line integral of an image, for all of the points and directions Radon transform of noise is constant and which is equal to the mean of the noise. Generally the mean of noise is considered to be zero. The palmprint holds features in terms of principle lines. Access of these features is easy even in the low resolution images.
Therefore, use of Radon transform is proposed to characterization of palmprint principle lines. The fingerprints are identified using characteristics like orientation of ridge and valley, and Radon transform have potential to extract these characteristics precisely. It is found that the Radon profile from various acquisition of the same finger is similar while it generates different Radon profile for the distinct figures. Figure 4.24 and Figure 4.25 represents the Radon profile for three fingerprints and palmprints of same individual and another individual respectively. Graph proves that correlation is high amongst same individuals in comparison with that of different individuals. Based on this concept, the proposed model uses the Radon for the person identification.

Palmprint features are invariant to translation, rotation and scales. Since image captured by the web cam have low resolution, image has slow variation and hence it has low sharpness. It causes difficulties in extracting the ridges. The principle lines of captured image are illumination invariant. As Radon has capability to extract these principle lines from palmprint successfully, the Radon transform based algorithm is proposed to extract principle lines of palm as features. Numbers of orientation are used for calculation of the Radon transform. The orientations of valleys and ridges are used to characterize fingerprints. Thus Radon transform can easily extract the ridges and valleys features from fingerprint. The features are integrated at each orientation in Radon transform, the obtained profile noise using Radon is always constant and equal to mean of noise, where mean is considered as zero. Hence from the fingerprint, noise insensitive features can be extracted using Radon transform.

To validate that the Radon features are invariant to the noise, the comparison of histogram of Radon profiles for fingerprint and palmprint images are plotted in Figure 4.26. The noisy images are generated using additive white Gaussian noise, where mean is considered to be zero and by adding variance of 0.15 to the original fingerprint image. It is found that the correlation is higher between the histogram of Radon profile of the original fingerprint / palmprint (blue color), noisy fingerprint / palmprint (red color) and fingerprint / palmprint of same individual (black color) is higher than the correlation between the two different individual (i.e. original fingerprint / palmprint and fingerprint / palmprint of another individual (green color).
Figure 4.24 Fingerprint test images and its Radon profiles (a) (b) Fingerprints of same individuals and (c) Fingerprint of different individuals, (d) Radon profiles for fingerprints in (a) and (b), (e) Radon profiles for fingerprints in (a) and (c).

The regular databases, PolyU database [229] and FVC-2000 database [230] are used for palmprint and fingerprint respectively. The palmprint and fingerprints in these databases are in gray color map. Palmprints and fingerprints have $128 \times 128$ (75 ppi) and $640 \times 480$ (96 ppi) spatial resolution respectively. As discussed earlier the Radon is utilized in the proposed model for line detection in the images. To process the image and to extract the features from individual’s modality using Radon, each modality is pre-processed using the gradient based edge detection. Various filters like canny, Prewitt, Laplacian etc can be used for edge extraction. Therefore for the single user identification, the query image is needed to be compared with these entire databases. Hence, the proposed model utilizes the separable Laplacian filter. This filter reduces the pre-processing time in comparison with above said edge detection filters.

For finger identification, Radon profile of finger is processed using Singular value decomposition [208, 231], gradient is used to extract the features of palmprint and knuckle image. To achieve invariance to the rotation and translation, histogram of Radon is used [232].
Figure 4.25 Palmprint test images and its Radon profiles (a) (b) Palmprints of same individuals and (c) Palmprint of different individuals, (d) Radon profiles for palmprints in (a) and (b), (e) Radon profiles for palmprints in (a) and (c).

Figure 4.26 Noise invariance validation of Radon features (a) Comparison of Histogram of Radon coefficients of fingerprint images with noisy fingerprint image, (b) Comparison of Histogram of Radon coefficients of palmprint images with noisy palmprint image.

Histogram of Radon can be computed using equation,

$$HR^\theta(k) = \frac{R^\theta_k}{R(t, \theta)}$$  \hspace{1cm} (4.38)
where,
\[ |R(t, \theta)| \] denotes the Radon feature vector’s total length at orientation \( \theta \).

Thus the features at each orientation are represented by the histogram. Invariance to rotation, scaling and translation is the main prerequisite of every biometric identification system. Shifting operation indicates variation along radial parameter. In the proposed model to satisfy this condition histogram is computed at each orientation. The Radon transform is sensitive to the image rotation. The angular parameter of Radon profile changes with the change in the orientation, i.e.
\[
HR^\theta(k) = HR^{\theta+\theta}(k), \ \theta \in [0, \pi)
\]

(4.39)

This change in the rotation shifts the values of histogram to another orientation which causes the problem during the matching process. To avoid this, the Euclidean distance which is rotation invariant is employed during the matching process. In the past literature, the features are normalized to make them invariant to the scaling. However, this process produces error and is highly noise sensitive. Hence to avoid the feature normalization and to make them scale invariant, logarithm is calculated in normalization process. Thus reformulation of histogram of Radon using proposed logarithm based normalization can be written as
\[
HR^\theta(k) = \ln(HR^\theta(k))
\]

(4.40)

For database generation, the fusion is the last process in multimodal biometric system. Because of the simplicity of decision level fusion, it is used in the proposed model. In the literatures, several approaches are presented at the decision level i.e. by assigning weightage to the features of every modality [233], using different logical and/or arithmetic operations like XOR or AND, addition, min, max between the features of both modalities. The proposed model uses the concatenation of the features of these modalities at every orientation. The Radon extracts the features of palmprints and fingerprints at different orientation of image. Use of maximum orientations is desirable to increase the matching accuracy. The proposed model uses, 180 orientations for calculating the Radon transform.

To identify the person, the query image is matched with the stored database. Here the stored database contains the palmprint and fingerprint histogram fused at various orientations. The correlation amongst features from the query image and stored features is calculated for person identification. The correlation attains maximum values when the query image is identical with the stored template. As stated previously, the modality rotation shifts the histogram. To obtain the rotation invariant matching, the query image histogram is rotated.
in steps and correlation is calculated at each rotation. The matching point is defined at the maximum value of calculated correlation as.

\[ C = \max \{ \text{corr}(\text{HR}_{\text{query},l} - \text{rotate}_n(\text{HR}_{l}^{\theta})), n = 1, 2, ..., \text{length}(\theta) \} \] (4.41)

4.5.3 Results and discussions

The proposed model uses, 60 images each having 4 subsets per individual for both modalities. The performance of the proposed model is checked by using PolyU [229] and FVC2004 [230]. The following steps are used for features extraction and matching:

Step 1: Enhance the features by preprocess the modalities.
Step 2: For both the modalities compute the Radon transform.
Step 3: Obtain the histogram of Radon and calculate the logarithm of it to obtain the scale invariant features.
Step 4: Create the database by fusion of the fingerprint and palmprint features.

The proposed model uses 180 angles in Radon transform. Histogram of 10 bins is computed from the Radon coefficients to obtain the feature vector.

Following steps are used to match the query images:
Step 1: Extract the feature using step 1 to step 3.
Step 2: Fuse the features of query images and calculate its correlation with the database features. To obtain rotation invariant matching, features of query images are rotated and correlation is calculated.
Step 3: The maximum correlation provides the matching of query image.

Thus total 240 correlation calculation is required to match one query template within the database. The average matching time for the given database is 0.081 second.

Figure 4.27 Correlation amongst fused feature set of two individuals for 28 templates from used dataset
The inverse correlations amongst 28 templates from the considered datasets are shown in Figure 4.27, i.e. the inverse correlation of first template with other 27 templates and so on. It can be seen that when the identity is compared its self, the inverse correlation is zero. However when it is compared with other identities, the inverse correlation exists. As stated before, in the proposed model each individual has 4 subsets. When identity is compared with its own subsets, the inverse correlation should be less. To exemplify it, the inverse correlation of forth identity with its remaining three subsets and with other identities are shown in Figure 4.28. This figure shows that the inverse correlation amongst the subsets is less than when it is compared with other individuals.

![Figure 4.28 Correlation between the features set of same individual and another individual.](image)

Figure 4.29 FAR vs FRR graph to see the accuracy of the proposed model.
Well accepted parameters from various literatures are calculated to calculate the accuracy of the proposed model. First is false acceptation ratio (FAR) and false rejection ratio (FRR). FAR represents the number of times the query image matches inaccurately with stored database. FRR describes the rate of rejection of query image matching with one of the template from database. FAR and FRR comparison is shown in figure 4.29. The EER value of 12.1% is achieved at normalized threshold value of 0.27. It proves the capability of the proposed model to identify the individual in terms of better accuracy using appropriate selection of threshold value.

Table 4.5 Performance summary of different fusion based algorithms

<table>
<thead>
<tr>
<th>Model</th>
<th>Biometric Identity</th>
<th>EER(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[233]</td>
<td>Fingerprint and Palmprint without F Ratio</td>
<td>14.54</td>
</tr>
<tr>
<td></td>
<td>Fingerprint and Palmprint without F-Ratio</td>
<td>13.80</td>
</tr>
<tr>
<td>[234]</td>
<td>Fingerprint, Face and Signature (extracting Significant Coefficients)</td>
<td>22.35</td>
</tr>
<tr>
<td></td>
<td>Fingerprint, Face and Signature (PCA)</td>
<td>5.32</td>
</tr>
<tr>
<td>[154]</td>
<td>Fingerprint and palmprint (Random tiling + 2N discretization)</td>
<td>11.09</td>
</tr>
<tr>
<td>Proposed model</td>
<td>Fingerprint and palmprint</td>
<td>12.10</td>
</tr>
</tbody>
</table>

The performance of the proposed model is compared with different models and summarized in Table 4.5. The EER of proposed model is comparable. The proposed model uses Radon computation and histogram for database generation. The computational time is less in comparison to other model. Again Radon based model works on low resolution images. Hence computational complexity reduces further by processing low resolution images.