CHAPTER 3
MULTISTAGE OPTIMAL EXPANSION PLANNING
3.1 Introduction

Generation expansion planning in the electric utility industry, has been formulated as a single stage expansion planning problem in the earlier chapter. While formulating the above model, the total increase in system demand, is assumed to be lumped during the planning period and the expansion planned accordingly. Due to the long periods involved in generation expansion planning the above model determines expansions requiring large capital investments. In practice major portion of this is not utilized in the initial stages of the planning period. A planning engineer, who envisions an expansion plan, has to consider a future, extending to several years and should organize the planning period into stages so as to meet the demand for electrical energy as well as the optimal utilization of the funds available for the expansion of the system capacity. This multi objective task necessitates the subdivision of planning horizon into sub-periods or stages. This leads to what is called the multistage expansion planning of the system, which determines the optimal expansion strategy for the entire planning period.

Multistage expansion planning determines the expansion schedule of the generation and transmission facilities of the utility. Decisions have to be taken by the planning engineer which transform a system from one state to another state. Decisions
regarding the future states of the system is influenced by the present state and vice-versa. Dynamic programming approach is advantageous to consider this present future relationship. This involves the evaluation of a large number of alternative policies before deciding upon the optimal expansions strategy.

A number of models are available in literature for the long-term planning of power systems. A discrete dynamic optimization method, for the long term planning of power networks, has been proposed by Dusonchet and El Abiad [9]. This examines a large number of policies using a deterministic search procedure coupled with a heuristic stopping criterion. A linear programming model has been proposed by Kaltenbach et al [6], for the long term planning of transmission network for given nodal power injections. The optimal capacity expansions, in each right of way at every stage of the planning horizon is determined in this model. Using these independent decisions, the expansion of each branch is optimized separately via a forward dynamic program. The disadvantage of this model is that it determines only a sub optimal expansion policy. Puntel et al. [10] have proposed a non linear programming model which determines long term expansion planning by minimizing a performance index representing the overloads in the circuits. In all these models for a given generation planning the transmission
expansions are found out. A combined planning of generation and transmission facilities has been proposed by Sawey and Zinn [13] and is developed as a mixed integer linear programming model.

Here in this chapter a model has been developed to determine the long term planning of transmission and generation facilities in a power system. The mathematical formulation used in the model renders it computationally very simple to handle. Skipping rules have been incorporated in the solution technique to reduce the number of policy evaluations thereby reducing computing time and also memory requirements.

In the model developed in this chapter generation plant expansions are considered to be discrete valued and the transmission expansions as continuous valued. Dynamic programming technique is used to formulate the problem of the multistage expansion planning of the system [57, 63, 71]. The planning period is divided into a number of stages and a set of variables are used to define the state of the system at each stage. Preference ordering is then applied to construct the optimal partial policy associated with each element of the state variable set. This policy consists of predecessor state, transmission network expansion and operating schedule for the plants and network. At each stage, associated with very state variable and its predecessor
a linear programming problem is formulated and the solution of this problem determines the increase in transmission capacity in each right of way to meet the demand. Then the optimal expansion policy is selected from the completed partial policies. The number of alternatives to be evaluated is considerably reduced by incorporating skipping algorithms in the solution technique.

Representation of the transmission network expansion by continuous variables and the rounding off of the fractional values to the nearest available discrete capacity of the transmission lines may lead to more investment costs than that required if network expansions are represented by discrete variables. This drawback is compensated by the simplicity of the approach and ease of formulation of the technique developed.

3.2 Preference order Dynamic Programming

3.2.1 Preference Structure

A set \( N = (1, \ldots, n) \) of stages, a set \( S \) of states and a collection of decisions \( D \), constitute the three elements of a \( N \) stage decision problem. The set \( S \) of states is assumed to be partitioned into a collection of subsets \((s_1, \ldots, s_n)\) with the elements of \( S_m \) representing the possible levels of the process at stage \( m \). When referring to power system planning the possible generation levels of the system represents the state variables. The sub sets \((s_1, \ldots, s_n)\)
represent the possible generation additions to the system. Eg. \( \bar{x} \in s_m \) represents a possible generation expansion at stage \( m \). The decisions to be made are the possible expansions of the transmission facilities so that demand requirements and system security constraints are satisfied.

Let the generation level at the beginning of the planning period be represented as \( S^0 \). Then set \( S^m \) is defined recursively as

\[
S^m = s_m \cup S^{m-1}
\]  

(3.1)

The elements of \( S^m \) represent the possible generation level of the system, at stage \( m \). For every \( \bar{x} \in s_m \) one is interested in the set \( S_x = \bar{x} \cup S^{m-1} \) consisting of state \( \bar{x} \) plus all the states in the prior stages. Associated with state \( \bar{x} \) there is a set of decisions \( D_x \), whose elements represent the predecessor state, the transmission line capacity expansions, the generator power outputs and the branch power flows.

At each stage, since there is a decision set associated with every state variable, the problem faced by the planner is to select a policy from the set. The strategy evolved is to construct policies recursively by adding decisions to partial policies [ 38 ]. The recursive construction of the policy involves the addition of a
decision $\delta(\bar{x})$ from $D(\bar{x})$ associated with $\bar{x} \in s_m$ to the partial policy $\delta(S^{m-1})$ to obtain the partial policy

$$\delta(S_x) = \delta(\bar{x}) \times \delta(S^{m-1})$$  \hfill (3.2)

Combining the policies for all $x \in s_m$ the partial policy for next stage $\delta(S^m)$ is obtained.

The decisions associated with the state $\bar{x} \in s_m$ and its predecessor states are obtained from the solution of a linear programming problem. The costs due to transmission network expansions and the operating costs with all the capital investments represented by the equivalent annual costs constitute the objective function of the linear programming problem. The problem formulation treats the variables representing the transmission capacity increase as continuous variables. The constraints of the problem are based on power flow equations, the limits on the power output of the power plants and the limits on the maximum permissible capacity addition in each right of way etc.

The optimal partial policy associated with $\bar{x}$ at stage $m$ gives information regarding the predecessor state, the transmission line capacities and the operating schedule for the plants and the network. From the completed partial policies the optimal expansion planning strategy for the power system is selected and this
corresponds to the policy with the minimum revenue requirements. By proper discounting of the future costs one gets the expansion policy requiring minimum present worth of revenue requirements.

3.3 Problem Formulation

The multistage power system expansion planning problem consists of determining a suitable generation and transmission expansion of the system at every stage of the planning period and the operating schedule for the power plants such that the present worth of revenue required is minimum while meeting the constraints based on the system security demand and other system requirements. The problem essentially consists of selecting a proper generation expansion at each stage and then to determine the best value for the total generation level considering the preceding stages. Along with this, one also has to consider the required expansion of the transmission network in the system.

3.3.1 Objective Function

If \( \bar{x} \) represents a combination of the power plant sizes chosen for expansion of the system at stage \( m \) (an element of the state variable set \( s_m \)). Then the \( j^{th} \) component \( x_j \) of \( \bar{x} \) represents the capacity of the \( j^{th} \) plant associated with \( \bar{x} \). If \( G_n^m \), \( (j = 1, .., NG) \) be the capacity and \( C'_r(x_j), j = (1, ..... NG) \) be the capital
investments associated with \( x_j \). Then the annual cost of the capital investment is given by

\[
C_f(x_j) = (CC_j) C'_f(x_j)
\]  

(3.3)

The operating costs of the power plants are assumed to be proportional to the power outputs of each plant. The capital cost required for transmission network expansion is expressed in terms of money units required for unit capacity increase in each right of way. This is obtained by dividing the total cost of construction of the line including the cost of terminal apparatus at each end by the maximum capacity of the line [ 13 ].

The formulation also takes into account the costs due to transmission line losses. A linear approximation is made for the line losses and the transmission loss coefficient is calculated by multiplying the line loss for a specified transmission voltage by the length of each section of the line. This assumption is made so that the resulting model is simple and easy for computerization requiring less computation time [ 13 ].

In this model all the future costs are discounted to their present worth or present value of money as basis. Thus if at a
future period the investment is \( f_t \) then the present worth is given by

\[ f'_t = \left( \frac{1+i}{1+r} \right)^r f_t \]

(3.4)

\[ r = \text{rate of interest} \]
\[ i = \text{rate of inflation} \]
\[ \tau = \text{a m} \]
\[ a = \text{length of period between the decisions} \]
\[ m = \text{stage at which investment decision is made} \]

The decisions associated with \( \bar{x} \), are obtained by formulating and solving linear programming problems, for every possible generation level in the preceding stage. Let a possible generation level in the preceding stage be represented by \( \bar{y} \). The objective function of the linear programming problem consists of the capital costs associated with the network expansions and the operating costs and the constraints of the problem are based on the security and other system requirements.

The objective function of the linear programming problem can be stated as

\[ Z(\bar{x}, \bar{y}) = \left[ \sum_{i=1}^{M_l} a_{ol} P_{ol}^m(\bar{x}, \bar{y}) + \sum_{k=1}^{K} C_k T_k^m(\bar{x}, \bar{y}) \right] \]

(3.5)
Where

\[ T_k^m(x, \overline{r}) = \text{transmission expansion associated with } x \]

and \( \overline{r} \)

\[ P_{i}^{m} = \text{output of the } i^{th} \text{ power plant} \]

\[ a_{oi} = \text{operating cost coefficient of the } i^{th} \text{ power plant} \]

\[ C_k = \text{annual cost of expansion per unit capacity increase in the } k^{th} \text{ right of way} \]

The present worth of costs is given by

\[ Z(x, \overline{r}) = \left( \frac{1+i}{1+r} \right)^r \left[ \sum_{i=1}^{M} a_{oi} P_{i}^{m}(x, \overline{r}) + \sum_{k=1}^{K} C_k T_k^m(x, \overline{r}) \right] \] (3.6)

### 3.3.2 Constraints

The constraints of the problem are based on the security and other system requirements. Every power system will have their own physical constraints. Thus the transmission networks will have their carrying capacity limitation. The different generating plants in the system will also have limits on their output capacities. The load demand on the system also has to be considered. Any decommissioning of the existing plants in the system during the planning period will lead to problems. In this study it is assumed that there is no retirement or decommissioning of any existing plants.
3.3.2.1 Conservation of power flow

Conservation of power flow at every bus in the system can be stated as

\[ p_i^m(\bar{x}, \bar{Y}) = \sum_{j \in v(i)} T_{ij}^m - \eta_{ij} T_{ij}^m, \quad i = 1, \ldots, \text{NB.} \]  

(3.7)

Where

- \( p_i^m(\bar{x}, \bar{Y}) \) = injected power at the \( i^{th} \) bus
- \( v(i) \) = set of buses connected to the \( i^{th} \) bus
- \( T_{ij}^m \) = power flow through the circuit \( k = (i, j) \), connected buses between buses \( i \) and \( j \) in the direction from bus \( i \) to \( j \),
- \( \eta_{ij} \) = transmission loss coefficient of circuit \( (i, j) \).

3.3.2.2 Transmission network capacity constraint

At any instant the total power flowing through any circuit \( k = (i, j) \) must not exceed the capacity of the circuit and can be stated as

\[ T_{ij}^m + T_{ji}^m \leq T_k^{m-1}(\bar{Y}) + T_k^{m}(\bar{x}, \bar{Y}), \quad k = 1, \ldots, \text{K} \]  

(3.8)

where \( T_k^{m-1}(\bar{Y}) \) is the capacity of circuit \( k \) corresponding to optimal partial policy associated with \( \bar{Y} \).
3.3.2.3 Maximum capacity constraint of the transmission network

The transmission networks are associated with right of ways and in each right of way the maximum capacity that can be installed is limited. The constraint can be stated mathematically as

$$ T^m_k(x, y) + T^{m-1}_k(y) \leq T^\text{max}_k, \quad k = 1, \ldots, K \quad (3.9) $$

where $T^\text{max}_k$ is the maximum permissible capacity in the $k^{th}$ right of way.

3.3.2.4 Plant output constraints

Any power plant is limited in its maximum power output by the total plant capacity associated with $x$ and $y$. If $G^{m-1}_j$ is the capacity of the $j^{th}$ power plant in the $(m-1)^{th}$ stage and associated with $y$. Then the power output constraint can be stated as

$$ P_{O,j}^m(x, y) \leq G^m_j + G^{m-1}_j, \quad j = 1, \ldots, \text{NG.} \quad (3.10) $$

3.3.2.5 Demand constraints

At the demand nodes in a power system the net injection power at the bus must be equal to the negative of the demand requirements. The constraint can be stated as

$$ P_{O,j}^m(x, y) = -D_{mj}, \quad j = \text{NG} + 1, \ldots, \text{NB.} \quad (3.11) $$

where $D_{mj}$ is the demand at the $j^{th}$ bus at stage $m$
3.3.2.6 Non negativity constraints

The formulation of the problem assumes that there is no retirement or decommissioning of existing facilities during the planning period. This can be stated as

\[ T_k(x, \overline{Y}) \geq 0 \quad k = 1, \ldots K \] (3.12)

The power plant outputs are always greater than zero and hence non-negative and can be stated as

\[ P_{o_j}^m(x, \overline{Y}) \geq 0, \quad j = 1, \ldots \text{NG} \] (3.13)

Thus the linear programming problem associated with the generation expansion represented by \( x \) at stage \( m \) and generation level given by \( \overline{Y} \) at stage \( (m-1) \) can be stated as

\[
\text{Minimize } Z^m(x, \overline{Y}) = \left( \frac{1+i}{1+r} \right)^T \left[ \sum_{i=1}^{NB} a_{oi} P_{oi}^m(x, \overline{Y}) + \sum_{i=1}^{K} C_k T_k^m(x, \overline{Y}) \right]
\] (3.14)

Subject to

\[ P_{oi}^m(x, \overline{Y}) = \sum_{j \in \pi(i)} T_{o_j}^m - \eta_{oi} T_{o_j}^m \quad i = 1, \ldots \text{NB} \]

\[ T_{o_j}^m + T_{o_j}^m \leq T_{k}^{m-1}(\overline{Y}) + T_{k}^m(x, \overline{Y}) \quad k = 1, \ldots K \]

\[ T_{k}^m(x, \overline{Y}) + T_{k}^{m-1}(\overline{Y}) \leq T_k^{\text{max}} \quad k = 1, \ldots K \]

\[ P_{o_j}^m(x, \overline{Y}) \leq G_{o_j}^m + G_{\overline{Y}}^{m-1} \quad j = 1, \ldots \text{NG} + 1, \ldots \text{NB} \]

\[ P_{o_j}^m(x, \overline{Y}) = -D_{o_j} \quad j = 1, \ldots \text{NG} \]

\[ T_k(x, \overline{Y}) \geq 0 \quad k = 1, \ldots K \]
\[ P_0^j(\bar{x}, \bar{Y}) \geq 0, \quad j = 1, \ldots, \text{NG} \]

The above linear programming problem can be expressed in the standard matrix form as

\[
\text{Minimize} \quad \bar{d}'\bar{\theta} \\
\text{Subject to} \quad \begin{align*}
B \bar{\theta} &= \bar{b} \\
\theta_j &\geq 0
\end{align*}
\]

where

\[ \bar{\theta} = [P_0^m(\bar{x}, \bar{Y}), T^m(\bar{x}, \bar{Y}), T^o^m] \]

\[ P_0^m(\bar{x}, \bar{Y}) = \text{vector of generator bus net injected power} \]

\[ T^m(\bar{x}, \bar{Y}) = \text{vector of transmission expansions} \]

\[ T^o^m = \text{vector of network power flows} \]

\[ \bar{d} = \text{cost coefficient column vector} \]

\[ B = \text{constraint coefficient matrix} \]

\[ \bar{b} = \text{right hand side column vector} \]

The solution of the linear programming problem is got by the simplex method [57 - 67]. The solution gives the optimal costs, the transmission network expansions and the operating schedules for network and power plants for a given combination of \( \bar{x} \) and \( \bar{Y} \).
A set of decisions associated with every $\bar{y} \in S_{x}^{m-1}$ is obtained by evaluating the optimal solution of the associated linear program where $S_{x}^{m-1}$ is the feasible set of generation levels in the previous stage for a given $\bar{x}$. The optimal partial policy $\delta(\bar{x})$ associated with $\bar{x}$ is given by

$$\delta(\bar{x}) = \text{Minimize } \{Z_{0}^{m}(\bar{x}, \bar{y}) + J_{m-1}(\bar{y})\}$$

$$\bar{y} \in S_{x}^{m-1}$$

Where

$J_{m-1}(\bar{y})$ is the cost of the policy associated with $\bar{y}$

$Z_{0}^{m}(\bar{x}, \bar{y})$ is the optimal value of (3.14)

The cost of the partial policy associated with $\bar{x}$ is obtained by combining the minimum value of (3.16) and the capital cost of equation (3.3).

The optimal partial policies with $S_{m}$ at stage $m$ is obtained from the cartesian product of the decisions associated with every $\bar{x} \in S_{m}$ and can be stated as

$$\delta(S^{m}) = X\delta(\bar{x})$$

$$\bar{x} \in S_{m}$$
The optimal expansion strategy is obtained from the completed partial policies i.e., \( m = n \), as

\[
\text{Min \ } [ J_n \ Y ] \\
Y \in S^n
\]  

(3.18)

3.4 Solution Technique

3.4.1 Formulation of state variable set \( s_m \) at stage \( m \)

At every stage of the planning period the dynamic programming formulation uses state variables. The decisions associated with each element of the state variable set are then determined. In the power system generation expansion problem the state variables represent the possible generation expansions of the system. The requirement of computer memory and time for the problem depend mainly on the number of elements of the state variables and the number of linear programming problem evaluation at every stage. Hence it is highly desirable that at every stage a proper choice of the state variable set is made

\( \bar{x} \in s_m \) may be defined from the discrete generator sizes available for expansion of the different power plants in the system. The system variable \( \bar{x} \) is therefore a vector whose number of components is equal to the number of open plant sites (i.e: available for expansion of the system) Any vector \( \bar{x} \) whose components
represent the expansion of the different power plants in the system belongs to $s_m$ if and only if it satisfies the following postulate.

3.4.2 Postulate

For every $\bar{x}$ if there exists at least one predecessor state in the $(m-1)$th stage, so that the total generation at stage $m$ due to $\bar{x}$ plus the predecessor state satisfies the demand and reserve requirements at stage $m$ and the power plant site capacity limitations then $\bar{x}$ belongs to the state variable set $s_m$.

The elements of the state variable set are selected from the Cartesian product space generated by the discrete plant sizes available for expansion at the different power plant sites in the system. Let the discrete plant sizes available at the $i^{th}$ power plant site be denoted as $S_{zi}$. Then the Cartesian product space is defined as

$$S_Z = S_{z1} \times S_{z2} \times \ldots \times S_{zNG} \quad (3.19)$$

The Cartesian product space for a system with two power plant sites and each site having three and two discrete sizes respectively is shown in Figure 3.1. The space is bounded by the two co-ordinate axes and the lines corresponding to the maximum number of units available at sites 1 and 2. The intersections of the lines parallel to
Cartesian Product Space $S_Z$

FIG. 3.1
the co-ordinate axis give the possible combinations of the power plant sizes.

\( \bar{x} \in S_m \) should satisfy postulate 1

Let the capacity associated with the \( i^{th} \) component of \( \bar{x} \) be given by \( G_{x_i}^m \) then

\[ G_{x_i}^m = G_{k_i} \] (3.20)

Where

\( G_{k_i} = k^{th} \) discrete size available at the \( i^{th} \) plant site

Let \( G_{m}^{m-1} \) be the \( i^{th} \) plant capacity associated with the element \( \bar{y}_i \in S^{m-1} \) then

\[ G_i^m = G_{m}^m + G_{m}^{m-1} \] (3.21)

According to postulate 1, \( \bar{x} \in S_m \), if and only if there exists a \( \bar{y}_i \in S^{m-1} \) such that

\[ \sum_{i=1}^{NG} G_i^m \geq \alpha \ D_m \] (3.22)

\[ G_i^m \leq G_i^{max}, \ i = 1, \ldots, NG \] (3.23)

where

\( D_m = \) total system demand at stage \( m \)

\( G_i^{max} = \) is the maximum plant capacity available at the \( i^{th} \) site.
\[ \alpha = \text{a constant which depends on the reserve requirements of the system.} \]

3.4.3 Solution Algorithm

The general steps involved in the solution of the problem formulated in section (3.4) is as follows.

1. Organize the planning horizon into stages and define the number of stages and set the stage counter \( m \) to 1.

2. Define the elements of the state variable set at stage \( m \). They are determined as discussed in section (3.4.1). Arrange the elements of \( s_m \) in the ascending order of costs.

3. For every \( \bar{x} \in S_m \), determine the feasible set \( S_{x}^{m-1} \) from the set \( S^{m-1} \) of the preceding stage. Arrange the elements of \( S_{x}^{m-1} \) in the ascending order of costs.

4. For each \( \bar{x} \in S_m \) and for every \( \bar{y} \in S^{m-1} \) formulate the linear programming problem of (3.14). The coefficient matrix \( B \) is independent of the choice of \( \bar{x} \) and \( \bar{y} \). The cost coefficient vector \( \bar{a} \) is modified at every stage according to (3.4). The right hand side column vector \( \bar{b} \) is to be modified depending on the combination \( \bar{x} \) and \( \bar{y} \).

5. Check whether the linear program (3.14) has to be solved or not using skipping algorithm 2. If the linear program has to be solved
go to step 6. Otherwise choose the solution of the LP from the set of solution stored at each stage depending upon the generation level at each plant site associated $\bar{x}$ and $\bar{y}$.

6. Solve the linear program and find the optimal solution. Store the solution and identify the solutions in terms of the installed capacities and the power outputs of the plants.

7. The set of decisions associated with $\bar{x}$ are obtained for every $\bar{y} \in S_{\bar{x}}^{m-1}$.

8. The optimal partial policy associated with $\bar{x}$ is selected. This consists in choosing the best predecessor state $\bar{y} \in S_{\bar{x}}^{m-1}$ using (3.18). The optimal partial policy $\delta(\bar{x})$ is formed by combining the solution of LP associated with optimal $\bar{y}_0$ and $\bar{x}$ with the policy associated with $\bar{y}_0$.

9. If $m < n$ go to step 2. Otherwise from the completed partial policies, choose the optimal expansions strategy using (3.20). The sequence of generation and transmission expansion is obtained by back tracking the decision table.

3.4.4 Skipping Algorithm 2

In step 5 of the solution technique the skipping algorithm 2 is used. This can be stated as follows. In a linear programming problem (P) if the variables representing the plant outputs are less
than the installed capacities, a linear programming problem say \((P^1)\) with installed capacities of plants greater than or equal to the corresponding capacities in the problem \((P)\), the problem \((P^1)\), has the same optimal solution as that of \((P)\) and skip off the solution of the problem \((P^1)\).

The flow chart for the algorithm developed is shown in Figure 3.2.

3.5 Numerical Example

The model developed in section (3.3) is applied to a sample power system. The electric utility system used by Sawey and Zinn [13] is considered for expansion. The system is as shown in Figure 3.3. The existing system has one generation plant (bus 1) and two load or demand centers (buses 3 and 4). The planning period is assumed to be in 5 stages. The future system consists of an additional load center (bus 5). A new generation site is also available for expansion in the future (bus 2).

The existing system details are given in Table 3.1a. and 3.1b. The demand requirements for the future at different demand nodes are given in Table 3.2. Table 3.3 gives the existing and maximum capacities, cost per unit capacity expansion and transmission loss coefficient of the right ways in the network. There are seven right ways available for expansion.
START

READ SYSTEM DATA

m = 1

FORM THE COEFFICIENT MATRIX OF THE LP

GENERATE L

FOR \( \bar{x} \in L \) GENERATE \( S.*^{-1} \)

FORMULATE THE LP

SCREENING ALGORITHM

THE LP TO BE EVALUATED

YES

NO

ALL \( \bar{y} \) OVER

YES

DETERMINE THE OPTIMAL POLICY ASSOCIATED WITH \( \bar{y} \)

NO

All \( \bar{x} \) over

YES

m = m + 1

NO

m > n

MIN \( J_n(x) \) AT \( \bar{x} \)

\( \bar{x} \in S.* \)

PRINT OPTIMAL POLICY ASSOCIATED WITH \( x_0 \)

STOP

FLOW CHART - MULTISTAGE PLANNING

FIG. 3.2
FIG 3.3  SYSTEM STUDIED
### Table 3.1a

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Type of Bus</th>
<th>Generation/Demand (MW)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>Generation</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>Load</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>Load</td>
<td>30</td>
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### Table 3.1b

<table>
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<tr>
<th>Transmission circuit</th>
<th>Capacity (MW)</th>
</tr>
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<tr>
<td>(1,3)</td>
<td>80</td>
</tr>
<tr>
<td>(1,4)</td>
<td>30</td>
</tr>
<tr>
<td>(3,4)</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 3.2

<table>
<thead>
<tr>
<th>Stage m</th>
<th>Bus 3 (MW)</th>
<th>Bus 4 (MW)</th>
<th>Bus 5 (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>35</td>
<td>75</td>
</tr>
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<td>3</td>
<td>95</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>40</td>
<td>85</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>45</td>
<td>85</td>
</tr>
<tr>
<td>No.</td>
<td>Transmission Circuit</td>
<td>Existing Capacity (MW)</td>
<td>Maximum Capacity (MW)</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------</td>
<td>------------------------</td>
<td>-----------------------</td>
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<td>0.0</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>2,4</td>
<td>0.0</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>2,5</td>
<td>0.0</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>3,4</td>
<td>30.0</td>
<td>100</td>
</tr>
<tr>
<td>Plant site</td>
<td>Existing Capacity (MW)</td>
<td>Maximum Capacity (MW)</td>
<td>Unit sizes (MW)</td>
</tr>
<tr>
<td>------------</td>
<td>------------------------</td>
<td>-----------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>250</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50</td>
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<tr>
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<td>0</td>
<td>120</td>
<td>50</td>
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<tr>
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<td>55</td>
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</tr>
<tr>
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<td></td>
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<td>65</td>
</tr>
</tbody>
</table>
Table 3.5

<table>
<thead>
<tr>
<th>Stage</th>
<th>Capacity of generating plants</th>
<th>Transmission route capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G1</td>
<td>G2</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
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</tr>
<tr>
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<td>110</td>
</tr>
<tr>
<td>5</td>
<td>140</td>
<td>110</td>
</tr>
</tbody>
</table>
Table 3.4 gives the existing and maximum capacities of the power plants, discrete sizes available for expansion the capital costs and the operating costs of the generating plants. The operating costs of the power plants are assumed to be linear and linear cost coefficient are shown in Table 3.5.

The formulation of the linear programming problem at stage m for any $\bar{x} \in \Sigma_m$ is developed as follows. The objective function consists of

1. Generation expansion costs: This is the cost of installation of generator units represented by $\bar{x}$, e.g., associated with unit combination $x = (40, 50)$ capital cost $C_c = 62$ mu

2. Transmission expansion cost: The contribution to the objective function from network expansion at stage m is given by

$$C_T = 1 T_{12}^m + 2 T_{13}^m + 3 T_{14}^m + 2 T_{15}^m + 2 T_{24}^m + 2 T_{25}^m + 2 T_{34}^m$$ (3.24)

3. The Operating cost: The operating cost of the power plants is given by

$$C_o = 0.5 P_{01}^m + 0.2 P_{02}^m$$ (3.25)

Constraints

Let $G^m$ be the generation associated with $\bar{y} \in S_{\bar{x}}^{m-1}$ and $\bar{x}$. 

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The power production constraint for the power plants are given by

\[ P_{d}^{m}(\bar{x}, \bar{y}) \leq G^{m}_{i}, \quad i = 1,2 \quad (3.26) \]

The transmission capacity constraint for the circuit \((1,3)\) is given by

\[ T_{013}^{m} + T_{031}^{m} - T_{2}^{m}(\bar{x}, \bar{y}) \leq 80 + T_{2}^{m-1}(\bar{y}) \quad (3.27) \]

where \(T_{2}^{m-1}(\bar{y})\) capacity of line \((1,3)\) associated with \(\bar{y} \in S^{m-1}_{x}\).

Similar constraints exists for all other transmission lines.

Conservation of flow for the generation node 1 is given by

\[ P_{d}^{m}(\bar{x}, \bar{y}) - T_{12}^{m} + 0.97T_{21}^{m} - T_{13}^{m} + 0.97T_{31}^{m} - T_{14}^{m} + 0.95T_{41}^{m} - T_{15}^{m} + 0.90T_{51}^{m} = 0 \quad (3.28) \]

Similarly there would be a constraint of this type for the power plant node 2.

Conservation of power flow constraint for the demand node 3 is given by

\[ 0.97T_{013}^{m} - T_{031}^{m} + 0.97T_{043}^{m} - T_{034}^{m} = D_{m3} \quad (3.29) \]

Similar constraints exists for demand nodes 4 and 5.

Transmission line maximum capacity constraint for the line \((1,4)\) is given by

\[ T_{3}^{m}(\bar{x}, \bar{y}) \leq 20 - T_{3}^{m-1}(\bar{y}) \quad (3.30) \]

Similar constraints exist for all other lines in the system.
The non negativity constraints are

\[ P_i^m(x, \bar{y}) \geq 0 , \quad i = 1, 2 \] (3.31)

\[ T_{0ij}^m, T_{ij}^m \geq 0 , \quad v_{ij} \in K \] (3.32)

\[ T_k^m(x, \bar{y}) \geq 0 , \quad k=1, \ldots, K \] (3.33)

The coefficient matrix B is obtained from (3.27)-(3.29) and the objective function coefficients \( \bar{d} \) are determined by (3.24) and (3.25). The right hand side vector \( \bar{b} \) of (3.15) is calculated from (3.27)-(3.29). The problem has 23 variables and 12 constraints. Of the variables nine are upper bounded and the maximum values of these variables are calculated from the right hand side of inequalities (3.26) and (3.30).

As the problem consists of equality and inequality constraints, it is solved by the two phase simplex method incorporated with upper bound algorithm.

The optimal expansion policy of the system is given in Table 3.5. The sequences of expansions are shown in Fig 3.4 – 3.8.
Stage 1, \( m = 1 \)

FIG 3.4
Stage 2, \( m = 2 \)

FIG 3.5
Stage 3, $m = 3$

FIG. 3.6
Stage 4,  \( m = 4 \)

FIG. 3.7
Stage 5, \( m = 5 \)

FIG. 3.8
3.6 Conclusion

In multistage expansion planning the planning engineer has to take decisions which transform a power system from one state to another state in consonance with the increase in load demand on the system. The present state of the system influences the decisions taken regarding the future state of the system and vice versa. Dynamic programming approach is advantageous for considering this type of present future relationships. For these long term planning strategies it is preferable that the planning period is divided into stages or sub periods. In this study a formulation for multistage planning is presented suitable for determining the sequence of network and power plant expansions so that the present worth of total revenue requirement is minimum. The expansion planning study model developed considers the transmission system as part of a system consisting of generators and transmission lines. The formulation therefore considers simultaneous planning of the generation and transmission facilities. This approach is superior to the planning of transmission expansions taken up as a follow up to generation expansions, since the expansion of generation facilities and transmission networks are interdependent. Preference order dynamic programming approach has been used successfully for handling the multistage generation expansion planning problem in this chapter.
The mathematical formulation used in the model developed is computationally very simple. The solution technique incorporates skipping rules to reduce the number of policy evaluations required leading to higher computation efficiency and reduction in computation time.

The model developed has been applied to a model power system. The results of the example show the effectiveness of the approach developed.