SYNOPSIS

1. INTRODUCTION

The aim of solid mechanics is to study the behavior of solid matters under the influence of external forces. Research in the field of solid mechanics plays a significant role in many branches such as engineering sciences, material science and biomechanics etc. Therefore, interdisciplinary research in the area of solid mechanics is not limited to natural sciences but it has raised its demand from engineering to medical industries. Research in this field provide strength to engineers in order to assure safety, reliability, optimality and cost minimization for designing and manufacturing of goods, devices and structural components etc. Solid mechanics has large number of applications which begin under the earth and go beyond the sky in the different fields of science and technology for social needs i.e. geomechanics (earthquake prediction, modeling the shape of planet, components of rockets and missiles), biomechanics (designing medical devices and implants), civil engineering (designing soil foundations and structures), mechanical engineering (designing components for engines, turbines for energy generation and transmission etc.), microelectronics (designing micro-electronics circuits), material science (designing functionally graded material, composite and thin films etc.), nanotechnology (modeling sample interactions). The famous scientist and engineer Sir Henry G. Stott said: “Engineering is the art of organizing and directing men and controlling the forces and materials of nature for the benefit of the human race”.

Solid mechanics is the branch of engineering mechanics which is concerned with the mechanical properties, fracture patterns, displacement, stress, strain and deformation under external factors, i.e. pressure, temperature etc. The field of solid mechanics is subdivided into two major branches i.e. mechanics of rigid bodies and mechanics of deformed bodies. The mechanics of deformed bodies can be characterized as “Theory of Elasticity, Theory of Plasticity and Theory of Creep”. The theory of elasticity is the systematic investigation of displacement, stress and strain in elastic solid materials. A material which resume its original position after removal of external load is known as elastic material. Many structural materials, for example, metals, ceramics, wood, plastics etc. exhibit elastic behavior under small deformations. Theory of elasticity leads to the idea of plasticity in those materials which do not regain its original position after removal of external load. Theory of plasticity deals with the investigation of stresses and strains in a body, made up of ductile material, permanently
deformed under set of applied forces. In brittle materials, fracture occurs immediately after elastic deformation. For materials like concrete, elastic deformation is not remarkable while plastic deformation starts at very beginning of the deformation. However, most of solid materials show noticeable elastic and plastic deformation and are usually named as elasto-plastic materials. Stress-strain curve is available in the literature to describe the elastic-plastic behavior of the material [1-3]. Some materials shows more strain when it is loaded for a long time and for exploring such materials creep phenomenon is used. Creep is the time dependent phenomenon which occurs due to long term exposure of a material when it is subjected to a constant load and temperature. Creep phenomenon is more adequate to illustrate many physical problems under different temperature and climate conditions. Creep deformations are usually very important and must be taken into consideration at high-temperature for engineering problems such as gas turbine engines, furnaces and steam turbines etc. Creep curve is available in the literature to describe the time dependent behavior of the material [4, 5]. 

To explore infinitesimal and finite deformations in thin rotating disks and thick-walled circular cylinder, three principal theories namely, classical theory, transition theory [6] and micropolar theory [7] have been used in the present work. The classical and micropolar theories have been used for investigating infinitesimal deformations in the materials and transition theory has been used for investigating finite deformations in the materials.

Deformations in the classical theory of elasticity are named infinitesimal if the displacement gradients are so small such that their product and squares may be neglected. Therefore, the infinitesimal theory is also known as small deformation theory. According to classical theory, the elastic and plastic regions are two different regions, which can be joined by yield surface. In elastic region, deformations are completely recoverable but beyond the yield point deformations are not recoverable. A law defining the limit of elasticity and beginning of plastic deformation under any possible combination of stresses is known as yield criteria [8]. There are several yield criteria for the yielding of materials i.e. maximum strain theory, maximum stress theory, Tresca’s yield criterion [9], Von Mises’ yield criterion [10] etc. Among all these, Tresca’s and Von Mises’ are commonly used yield criterion. Elastic-plastic stresses in homogeneous thick-walled cylinders and thin rotating disks have been investigated by many researchers [11-21] using Tresca’s and Von Mises’ yield criterion. All these authors considered yield criteria, jump conditions, linear and nonlinear strain measures to calculate the stresses by using concept of infinitesimal strain theory [22].
Many applications of theory of plasticity involve large deformation for which infinitesimal theory is not adequate. Moreover, for large rotation problems, infinitesimal theory is insufficient, even if straining and stretching is small. The finite deformation theory [22] has been applied to solve different problems for which classical theory of infinitesimal deformation is not adequate. In finite deformation, displacements and its derivatives are not linear while in infinitesimal deformation, displacement and its derivatives are linear. The classical theory of elasticity and plasticity did not find any transition zone between elastic and plastic region and thus used yield condition to join elastic and plastic states. However, Seth [6, 23] recognized a transition zone in between elastic and plastic region. Seth investigated that yield conditions are completely unnecessary if we consider the transition zone between elastic and plastic regions. As explained by many authors that transition from one state to another is an asymptotic phenomena, Seth has argued that at transition the differential system governing the physical phenomena should attains some sort of criticality. Once these critical points or the transition points are identified, the solution at these points gives the solution at transition state. The ordinary measures used in classical mechanics are found insufficient to deal with transition state, so generalization of these measures is required to explain real physical phenomena in continuum mechanics. Seth has given generalized strain measures, which when combined with the transition point analysis of differential equation, not only eliminates the use of yield conditions and creep strain laws but also employs the same constitutive equations to give the solutions at elastic-plastic and creep states through some transition function. There are various ways to explain how transition may occur from one state to another state: (i) at transition, the differential system defining the elastic state should attain some criticality or singularity or (ii) if we consider the plastic state as an image of the elastic state under some transformation, then at transition, due to the relationship between elastic and plastic state the Jacobian of the transformation should be zero or infinite. This means that when transition occurs, one-to-one correspondence does not exist between elastic and plastic state. Once the 'transition points' are found, the asymptotic solution obtained at these points provides the solutions corresponding to the 'transition' states. Also, it has been shown [23, 24] that the transition field is a nonlinear field. Seth developed the 'transition theory' of elastic-plastic and creep deformation using the concept of generalized strain measure [24, 25]. Elastic-plastic and creep stresses in homogeneous and non-homogeneous thick-walled cylinders subjected to different loading using transition theory have been investigated by many researchers [26-32].
Due to mixed and complex behavior of materials such as granular materials, block structures etc., classical continuum theory is not adequate to describe the actual behavior of micro-materials i.e. micro strain or micro rotation. At the micro or nano scale, the roughness of surface and non-uniform shape of the particles affects the material behavior. Granular material shows high deformation at the point of failure (translational and rotational). The classical stress tensor cannot explain the actual kinematics such as micro strain or micro-rotation in granular materials and thus other alternative tensors are required because the rotational kinematics is more significant as compared to translational kinematics. To cope up with the strain non-uniformities, some additional degrees of freedoms are introduced. In addition to the displacement assumed in classical theory, the Cosserat theory [33] associates the local rotation of grain at a point. In classical continuum theory, force stress is the only type of stress, i.e. force per unit area while in Cosserat theory, due to particle rotation, couple stress also occurs in addition to force stress. Eringen [34] incorporated the micro-inertia to the Cosserat theory and renamed it as micropolar theory. The particle rotation at micro level is named as micro-rotation. Therefore, at every point of solid material there is six degree of freedom, which makes kinematic theory more complicated than the classical continuum theory. Therefore, in order to define a microstructure related to a material point, three additional degrees of freedom due to micro-rotation of the particles is required. Micropolar deformation can be subdivided into two parts. One is macro-level deformation due to displacement in the structures and other is micro level deformation due to rotation in particles.

2. OBJECTIVE

In manufacturing and designing industry, cylinders and disks made up of functionally graded materials [35, 36] and orthotropic micro material under internal and external pressure, thermal loading, rotation and twist have become a point of interest due to their wide range of applications, for example, in steam generator tubes, medical implants etc. The main objectives of the work reported in the thesis is to investigate elastic-plastic and creep stresses in thick-walled cylinders and thin rotating disks made up of functionally graded isotropic materials and orthotropic micro-materials subjected to different loadings such as temperature, internal and external pressure and twist etc.
3. MAIN CONTRIBUTION

The work presented in the thesis is divided into eight chapters. Elastic-plastic and creep stresses have been investigated in cylinders and disks made up of functionally graded materials and orthotropic micro-materials in order to assure safety of the structural components. To investigate the stresses in materials, two types of theories have been considered: (i) for infinitesimal deformation, linear classical and micropolar theories have been used (ii) for finite deformation, transition theory has been used.

The chapter wise brief overview of the work done is given as follows:

**CHAPTER 1:** This chapter is introductory in nature and provides a brief introduction to the mechanics of deformation using three different theories, i.e. classical theory, transition theory and micropolar theory. In this chapter, elastic-plastic and creep behavior of materials, their basic constitutive equations and applications have been discussed. In addition to this a concise description of the finite difference method, shooting method and generalized strain measure has also been included.

**CHAPTER 2:** In this chapter, we investigate elastic-plastic stresses in thick-walled circular cylinder made up of functionally graded material under pressure. This chapter is divided into two sections. In the first section, we investigated thermal elastic-plastic stresses in functionally graded thick-walled circular cylinder under internal and external pressure with Swift’s hardening law [12] using the concept of infinitesimal deformation theory. The temperature distribution in the cylinder varies as logarithmic function of radius and Young’s modulus of the cylinder varies according to power law in the radial direction. The results are presented graphically for radial and circumferential stresses with various parameters such as temperature, pressure and strain hardening measure. Second section is an extension of first section in which we have investigated thermal stresses in a rotating cylinder made up of functionally graded material under internal and external pressure. In this section, thickness and density of the cylinder varies according to power law in radial direction. Results are discussed with the help of graphs for radial and circumferential stresses with various parameters of thickness, density, rotation, temperature, pressure and strain hardening measure. From the analysis, it has been concluded that circumferential stresses are less in cylinder made up of functionally graded material with or without thermal loading as compared to homogeneous cylinder. The results of
this investigation plays a vital role in designing and manufacturing of nuclear reactors, boilers etc.

**CHAPTER 3:** In this chapter, we investigate elastic-plastic stresses in a thin rotating disk whose thickness and density varying exponentially in the radial direction by using the concept of small deformation theory. The influence of geometric parameters of thickness and density has been examined on radial and circumferential stresses and results are compared with flat disk and the disk whose thickness and density is varying according to power law in the radial direction. From the analysis, it has been noticed that circumferential stresses are less in thin rotating disk whose thickness and density varying radially according to power law as compared to rotating disk whose thickness and density varying exponentially in radial direction. The results of this investigation plays a lead role in designing and manufacturing of turbine rotors, flywheel etc.

**CHAPTER 4:** In this chapter, we investigate elastic-plastic stresses in a thin rotating disk made up of functionally graded material under thermal loading by using infinitesimal deformation theory. A functionally graded thin rotating disk has been considered whose compressibility varying according to power law, thickness and density varying exponentially in the radial direction. The temperature function follows logarithmic profile in the radial direction. The influence of various parameters such as compressibility, thickness, density and strain hardening measure with rotation on radial and circumferential stresses and plastic strains have been studied and presented graphically. From the analysis, it has been concluded that circumferential stresses are less in thin rotating disk made up of functionally graded material as compared to disk made up of homogeneous material.

**CHAPTER 5:** In this chapter, elastic-plastic and creep stresses have been investigated in non-homogeneous thick-walled circular cylinder under external pressure with generalized strain measure using the concept of transition theory. The non-homogeneity in the cylinder is due to varying compressibility of the cylinder in the radial direction. The influence of physical parameters namely i.e. compressibility, pressure and strain measure have been studied on radial and circumferential plastic and creep stresses. It has been noticed from the analysis that highly functionally graded cylinder is better choice for the design as compared to homogeneous and less functionally graded cylinder.
CHAPTER 6: In this chapter, we investigate creep stresses in torsion of thick-walled circular cylinder made up of functionally graded material under pressure. This chapter consists of two sections. In the first section, creep stresses have been investigated in torsion of thick-walled functionally graded cylinder under internal and external pressure with generalized strain measure using transition theory. The compressibility of the cylinder varies in radial direction using power law. The effect of internal and external pressure, strain measure with fixed angle of twist has been studied on radial, circumferential and shear creep stresses. In addition to the first section, effect of temperature has also been included to study radial, circumferential and shear creep stresses using transition theory in second section. The temperature function follows logarithmic profile in the radial direction. It is examined that with or without thermal effects, cylinder made up of less functionally graded material is on the safer side of design in torsion as compared to cylinder made up of highly functionally graded material because shear stresses are maximum in case of less functionally graded cylinder as compared to cylinder made up of highly functionally graded material. The results of this study plays crucial role in aerospace and automobile industries.

CHAPTER 7: In this chapter, a mathematical model has been developed to study the torsion in micropolar orthotropic thick-walled circular cylinder. Micropolar theory has been used to study the deformation due to displacement and micro-rotation. The effect of material internal length and material constants on shear stresses and couple stresses has been examined. From the analysis, it has been concluded that effect of small internal length on shear stresses is negligible. For small internal length or when internal length is zero, materials behave like classical materials. The results of this work plays important role in block structure, biological tissues, granular materials etc.

CHAPTER 8: Conclusion and future scope of the study are presented in this chapter. Different types of thick-walled circular cylinders, i.e. isotropic cylinders (homogeneous and non-homogeneous), orthotropic cylinders and rotating disks are considered for our study in this thesis. The concept of infinitesimal deformation theory, transition theory and micropolar theory have been applied to investigate elastic-plastic and creep stresses in different types of cylinders and disks with external factors i.e. internal pressure, external pressure, temperature etc. As we know that the permissible stress of any material is some proportion of yield or ultimate strength.
of the material which incorporates the safety factor. This safety factor provides the margin against collapse in cylinders and disks under different loadings and reduces the possibility of fracture of cylinders and disks due to external factors. From the analysis of stresses, it is concluded that in general functionally graded cylinders are more suitable for designing as compared to homogeneous cylinders. In future, the concept of infinitesimal deformation theory, transition theory and micropolar theory can be extended for cylinders made up of orthotropic materials and nano-materials under different types of loadings.

REFERENCES


