CHAPTER 6

SECTION 6.1

CREEP TORSION IN THICK-WALLED FUNCTIONALLY GRADED CYLINDER UNDER INTERNAL AND EXTERNAL PRESSURE

6.1.1 INTRODUCTION

Functionally graded materials are new age advanced material which plays a vital role in the designing of composite structures. Functionally graded materials (FGMs) are those non-homogeneous materials whose properties and composition changes in the certain direction. Due to structural integrity, these materials are intentionally designed for specific application i.e. medical implants, thermal barriers, aerospace industry, structural components operating under high temperature etc.[8, 110, 111]. Gupta et al. [28] studied steady state creep for cylinder made up of compressible material in torsion with generalized principal strain measure using seth’s transition theory [22, 59]. They found that maximum shear stress in compressible cylinder is greater than that of incompressible cylinder and these shear stresses increases with increase in strain measure. Further, Gupta and Rana [30] studied transitional stresses in torsion of transversely isotropic cylinder and found that maximum shear stress is high in cylinder made up of transversely isotropic material as compared to isotropic incompressible material. It is thus concluded that shear stresses increases significantly with the increase in strain measure. The torsion problem for homogeneous and non-homogeneous elastic materials have been studied in numerous aspects by many authors [5, 28, 30, 46, 112-117]. Analysis of torsion is major topic of research in engineering applications [5]. Nazarov and Puchkov [112] constructed a class of functions in cylindrical coordinates for shear moduli and general solution has been investigated for displacement and stresses in hollow cylinder under torsion. Rooney and Ferrari [113] studied torsion in bars with inhomogeneous shear moduli.
in the direction of coordinates. Horgan and Chan [114] studied the effect of material inhomogeneity on linearly elastic bars and found that maximum shear stress does not exist on the boundary of the rod. Jiang and Henshall [115] developed a model for the analysis of prismatic bars subjected to torsional loading by dividing the bar into small slices. Alberto [116] presented analytical solution for hollow circular cylinder made up of isotropic micropolar material subjected to twist. Kobelev [117] discussed secondary phase torsion and bending creep of rods. Sharma et al. [46] presented analytical solution for torsion of thick-walled cylinder made up of functionally graded material for elastic-plastic region subjected to internal and external pressure using transition theory.

The present work investigates creep stresses in a thick-walled pressurized cylinder made up of isotropic functionally graded material subjected to torsion using transition theory. A parametric study of the influence of internal and external pressure, strain measure over stresses have been investigated. The radial, circumferential and shear stresses have been computed and presented with the help of tables and graphs.

6.1.2 MATHEMATICAL FORMULATION

Consider a thick-walled circular cylinder made up of functionally graded material with an inner radius \( a \) and outer radius \( b \), subjected to internal and external pressure \( p_i \) and \( p_0 \) respectively. Geometric parameters of the cylinder are shown in Figure 6.1.1.

The components of displacement in cylindrical polar coordinates [22] are given by

\[
\begin{align*}
\mathbf{u} &= r (1 - \beta), \\
\mathbf{v} &= \eta r z \\
\mathbf{w} &= d z,
\end{align*}
\]

where \( \beta \) is a function of \( r = \sqrt{x^2 + y^2} \), \( d \) is a constant and \( \eta \) is the angle of twist per unit length.

The compressibility of functionally graded cylinder is defined as

\[
C = C_0 r^k,
\]

where \( a \leq r \leq b \), \( C_0 \) and \( k \geq 0 \) are constants.

The generalized components of strain [65] are given as

\[
\begin{align*}
\varepsilon_{rr} &= -\frac{1}{n} \left[ 1 - (r \beta' + \beta)^n \right], \\
\varepsilon_{\theta \theta} &= -\frac{1}{n} \left[ 1 - \beta^n \right], \\
\varepsilon_{zz} &= -\frac{D}{n} \left[ 1 - \left( \frac{\eta r \beta}{D} \right)^n \right], \\
\varepsilon_{\theta z} &= \frac{1}{n_m} \left[ \eta^{n/2} r^{n/2} \beta^n \right], \\
\varepsilon_{r \theta} &= \varepsilon_{z r} = 0,
\end{align*}
\]

(6.1.3)
where \( D^n = \left[ 1 - (1 - d)^n \right] \), \( n \) is the measure of deformation and \( \beta' = \frac{d\beta}{dr} \).

**Figure 6.1.1:** A FGM pressurized thick-walled circular cylinder under torsion

The constitutive equations for isotropic materials in linear theory of elasticity is given by

\[
T_{ij} = \lambda \delta_{ij} I_i + 2\mu e_{ij}, \quad (i, j = 1, 2, 3) \tag{6.1.4}
\]

where \( T_{ij}, e_{ij} \) are stress and strain tensors respectively, \( I_i = e_{kk} \) are strain invariants, \( \lambda, \mu \) are Lame’s constants and \( \delta_{ij} \) is Kronecker’s delta.

Equation (6.1.4) using Equation (6.1.3) can be rewritten as

\[
T_{rr} = \left( \frac{\lambda + 2\mu}{n} \right) \left[ (r\beta' + \beta')^n \right] + \left( \frac{\lambda}{n} \right) \left[ (1 - \beta')^n \right] + \left( \frac{\lambda D^n}{n} \right) \left[ 1 - \left( \frac{\eta r \beta}{D} \right)^n \right],
\]

\[
T_{\theta\theta} = \left( \frac{\lambda}{n} \right) \left[ (1 - (r\beta' + \beta')^n \right] + \left( \frac{\lambda + 2\mu}{n} \right) \left[ (1 - \beta')^n \right] + \left( \frac{\lambda D^n}{n} \right) \left[ 1 - \left( \frac{\eta r \beta}{D} \right)^n \right],
\]

\[
T_{zz} = \left( \frac{\lambda}{n} \right) \left[ (1 - (r\beta' + \beta')^n \right] + \left( \frac{\lambda}{n} \right) \left[ (1 - \beta')^n \right] + \left( \frac{\lambda + 2\mu}{n} \right) \left[ D^n - (\eta r \beta)^n \right],
\]

\[
T_{0z} = \frac{2\mu}{n} \left[ \eta^{n/2} r^{n/2} \beta^n \right], \quad T_{rr} = T_{\theta\theta} = 0. \tag{6.1.5}
\]
Equation of equilibrium is expressed as
\[
\frac{d}{dr} (T_{rr}) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0 .
\] (6.1.6)

### 6.1.3 IDENTIFICATION OF TRANSITION POINT

As there exists an intermediate state in between elastic and creep states which is known as transition state. Thus, differential system defining the elastic state should reach a critical value in the transition state. The nonlinear differential equation at transition state is obtained by substituting Equation (6.1.6) in Equation (6.1.6) as,
\[
n \beta P (P + 1)^{n-1} \frac{dP}{d\beta} = r \left( \frac{\mu'}{\mu} - C' \right) \left[ (3 - 2C) - (1 - C)(1 - d)^n \right] \frac{1}{\beta^n} - \left( 1 - C \right) \left( 1 + \eta^n r^n \right) + (P + 1)^n \]
\[
C \left[ 1 - (P + 1)^n \right] + r C' \left[ (1 + \eta^n r^n) - \left( 2 - (1 - d)^n \right) \frac{1}{\beta^n} \right] - nP \left[ (1 - C) + (P + 1)^n \right] - n \eta^n r^n (1 - C) (P + 1) - n r^{\alpha+1} \eta^{\alpha+1} \eta (1 - C),
\] (6.1.7)

where \( r \beta' = \beta P \), \( C = \frac{2\mu}{\lambda + 2\mu} \), and \( \eta = \frac{2\eta_i (3 - 2C)}{(2 - C)} \), \( \eta_i \) is a constant.

The critical points of Equation (6.1.7) are \( P \rightarrow -1 \) and \( P \rightarrow -\infty \). As the transition zone is that state from which the elastic state switches to the creep state. At this physical point, difference between elastic and creep disappears.

The boundary conditions are
\[
T_{rr} = -p_i \text{ at } r = a \quad \text{and } T_{rr} = -p_o \text{ at } r = b .
\] (6.1.8)

The resultant axial force in the circular cylinder is given by
\[
\int_a^b r T_{rr} \, dr = L_a .
\] (6.1.9)

### 6.1.3.1 METHOD OF APPROACH

For finding the creep stresses at the transition point \( P \rightarrow -1 \), the transition function \( R \) [22, 28, 30] through the principal stress difference is defined as follows
\[
R_f = T_{rr} - T_{\theta\theta} = \frac{2\mu}{n} \beta^n \left[ 1 - (P + 1)^n \right].
\] (6.1.10)
Taking the logarithmic differentiation of the Equation (6.1.10) with respect to \( r \), we have

\[
\frac{d}{dr} \left( \log R_f \right) = \frac{nP}{r} + \frac{\mu}{\mu} - \frac{n\beta P(P+1)^{n-1} \frac{dp}{d\beta}}{r \left[ 1 - (P+1)^n \right]}.
\]  

(6.1.11)

Substituting the value of \( \frac{dp}{d\beta} \) from Equation (6.1.7) in Equation (6.1.11) and taking the asymptotic value \( P \to -1 \), we get

\[
\frac{d}{dr} \left( \log R_f \right) = -\frac{2n}{r} + \frac{2\mu}{\mu} = \frac{C'}{C} + X,
\]

(6.1.12)

where

\[
X = \frac{(n-1)C}{r} - C' \frac{\dot{\mu}}{\mu} + \frac{C'}{\beta^n} \left[ 2 - (1 - d)^n \right] - \left( \frac{\dot{\mu}}{\mu} - \frac{C'}{C} \right) \left[ (3 - 2C) - (1 - C)(1 - d)^n \right] \frac{1}{\beta^n} + (1 - C)n r^n \eta^{-1} \eta.
\]

Integration of Equation (6.1.12) yields

\[
R_f = A \frac{\mu^2}{Cr^{2n}} \exp f,
\]

(6.1.13)

where \( f = \int X \, dr \) and \( A \) is a constant of integration.

Using Equation (6.1.10) and Equation (6.1.13), we have

\[
R_f = T_{rr} - T_{\theta\theta} = A \frac{\mu^2}{Cr^{2n}} \exp f = A r F,
\]

(6.1.14)

where \( F = \frac{\mu^2}{Cr^{2n+1}} \exp f \).

The asymptotic value of \( \beta \) from Equations (6.1.10) and (6.1.14) is

\[
\beta^n = A \frac{n\mu}{2Cr^{2n}} \exp f.
\]

(6.1.15)

Substituting Equation (6.1.14) in Equation (6.1.6) and integrating, we get

\[
T_{rr} = B - A \int F \, dr,
\]

(6.1.16)

where \( B \) is a constant of integration.

The asymptotic value of \( \beta \) is \( \frac{D}{r} \) as \( P \to -1 \), where \( D \) is a constant.

The constants \( A \) and \( B \) are calculated by using the boundary conditions (6.1.8) in (6.1.16) as

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\[ A = \frac{p_0 - p_i}{b} \int_a^b F \, dr, \quad B = -p_0 + A \int_a^b F \, dr \mid_{r=b}. \]  

(6.1.17)

Substituting the value of \( B \) in Equation (6.1.16), we get

\[ T_{rr} = -p_0 + A \int_r^b F \, dr. \]  

(6.1.18)

The circumferential, axial and shearing stresses are obtained from Equations (6.1.14), (6.1.18) and Equation (6.1.5) as

\[ T_{\theta\theta} = -p_0 + A \left[ \int_r^b F \, dr - rF \right], \quad T_{zz} = \frac{(1-C)}{(2-C)} \left( T_{rr} + T_{\theta\theta} \right) + 2\mu \frac{(3-2C)}{(2-C)} e_{zz}, \]

\[ T_{\theta z} = A \left( \frac{\mu^2 \eta^{n/2} r^{-3n/2}}{C} \exp f \right), \]  

(6.1.19)

where \( e_{zz} = \frac{1}{\lambda} \int_a^b \frac{r C(3-2C)}{(1-C)(2-C)} \, dr \) and

\[ D^a = A \left( \frac{2\eta(3-2C)}{(2-C)} \right)^n \frac{n \mu}{2Cr^a} \exp f. \]

The twisting couple \( M \) is given by

\[ M = 2\pi \int_a^b r^2 T_{\theta z} \, dr = 2\pi \int_a^b A \left( \frac{\mu^2 \eta^{n/2} r^{(-3n+4)/2}}{C} \right) \exp f \mid_{r=b}. \]  

(6.1.20)

Now we introduce non-dimensional components as

\[ R = \frac{r}{b}, \quad R = \frac{a}{b}, \quad p_0 - p_i = \frac{p_0 - p_i}{E}, \quad \sigma_{rr} = \frac{T_{rr}}{E}, \quad \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{E}, \quad \sigma_{zz} = \frac{T_{zz}}{E}, \quad \sigma_{\theta z} = \frac{T_{\theta z}}{E}, \]

where \( E \) is the Young’s modulus of elasticity.

With the use of compressibility, the radial, circumferential and shear stresses under internal and external pressure defined in Equations (6.1.18)-(6.1.20) can be expressed in non-dimensional form as
\[
\begin{align*}
\sigma_{rr} & = -P_0 + A_2 \int_R r F_2 \, dR, \quad \sigma_{\theta\theta} = -P_0 + bA_2 \left[ \int_R r F_2 \, dR - RF_2 \right], \\
\sigma_{zz} & = \left( \frac{1-C_0 b^k R^k}{2-C_0 b^k R^k} \right) \left[ -2P_0 + bA_2 \left( \frac{1}{R} \int_R r F_2 \, dR - RF_2 \right) \right] + e_{zz}, \quad \sigma_{zz} = bRA_2 F_2 \left( 2\eta R \right)^n \left[ \frac{3-2C_0 b^k R^k}{2-C_0 b^k R^k} \right]^\frac{n}{2}, \\
M & = 2\pi \int_{k_0}^1 A_2 b^{k-2n+3} R^{k-2n+2} \frac{E^2 \left( 2-C_0 b^k R^k \right)^2}{4C_0 \left( 3-2C_0 b^k R^k \right)^2} \left[ \frac{2\eta_1 \left( 3-2C_0 b^k R^k \right) b R}{2-C_0 b^k R^k} \right]^{n/2} \exp f_2 \, dR,
\end{align*}
\]

where \( A_2 = \frac{P_0 - P_i}{\int_R r F_2 \, dR} \), \( F_2 = \frac{E^2 \left( 2-C_0 b^k R^k \right)^2}{4C_0 \left( 3-2C_0 b^k R^k \right)^2} b^{k-2n-1} R^{k-2n-1} \exp f_2 \),

\[
\begin{align*}
f_2 & = \left( \frac{n-1}{k} \right) C_0 b^k R^k + 2k C_0 b^{n+k} R^{n+k} - C_0 k \left[ \int b^{n+k} R^{n+k-1} \left( \frac{3-2C_0 b^k R^k}{1-C_0 b^k R^k} \right) dR \right] \\
& + \log \left( 1-C_0 b^k R^k \right) - 2\eta C_0 nk \left[ \int b^{n+k} R^{n+k-1} \left( \frac{3-2C_0 b^k R^k}{1-C_0 b^k R^k} \right) \left( \frac{2\eta_1 \left( 3-2C_0 b^k R^k \right)}{2-C_0 b^k R^k} \right)^{n-1} \right] \, dR,
\end{align*}
\]

and \( e_{zz} = -\left[ \frac{L_a}{2\pi} \int_{k_0}^1 \frac{b R \left( 1-C_0 b^k R^k \right)}{2-C_0 b^k R^k} \left[ \sigma_{rr} + \sigma_{\theta\theta} \right] \, dR \right] \cdot \int_{k_0}^1 b^2 R E dR.
\]

Equation (6.1.21) represents radial, circumferential, axial, shear stresses and twisting couple in a non-dimensional form for secondary state of creep.

### 6.1.4 RESULTS AND NUMERICAL DISCUSSION

The material properties of the cylinder made up of functionally graded material are defined as: compressibility coefficient \( C_0 = 0.5 \), Poisson’s ratio \( \nu = 0.3 \). The inner and outer radii of the cylinder are taken as \( a = 1 \) [m] and \( b = 2 \) [m] respectively. The parameters of the compressibility are considered as \( k = 0, 0.5, 1, 1.5 \).
To observe the influence of various parameters i.e. strain measure \( N \), and pressure \( P_i \) and \( P_o \), Figures 6.1.2-6.1.5 and Table 6.1.1 have been plotted between radii ratio and stresses.

The influence of strain measure and angle of twist has been examined on creep stresses in Figure 6.1.2 and Table 6.1.1 when internal pressure is greater than that of external pressure. It is found that circumferential stresses are tensile in nature and these stresses are maximum at internal surface for linear measure while these stresses are maximum at external surface for nonlinear measure. Circumferential stress decreases at the internal surface but increases towards outer surface with the increase in strain measure. Negative value of the stresses indicate that the stresses are compressive in nature. With the increase in internal and external pressure circumferential stress increases for nonlinear measure as can be seen in Figure 6.1.3. Figure 6.1.4 and Table 6.1.1 are drawn to see the influence of strain measure on stresses when internal pressure is less than that of external pressure. It is noticed that circumferential stresses are compressive in nature throughout the radii and are maximum at internal surface for linear and nonlinear measure excluding less functionally graded cylinder with strain measure \( N = 3 \). It is observed that these stresses increased with increase in pressure as can be seen in Figure 6.1.5.

A shear stress generated when a structural component is twisted. Due to torsion, shear stress generated in addition to principal stresses. The localized parallel shear forces will be highest when the normal forces are high. Due to combined loading i.e. axial and torsional loading, stresses generated in functionally graded material intersecting stresses generating in homogeneous material. As cylinder is subjected to different normal forces on each side and thus when internal pressure is greater than that of external pressure, forces on outer side are small and inner side are high. Figures 6.1.6-6.1.9 and Table 6.1.2 have been drawn between radii ratio and shear stresses when internal pressure is greater than that of external pressure to show the effect of strain measure and pressure on shear stresses.

It is observed that shear stresses are compressive in nature throughout the radii as can be seen from Figure 6.1.6 and Table 6.1.2. These shear stresses are maximum at external surface for highly functionally graded cylinder with nonlinear measure while these stresses are maximum at internal surface for all other values of strain measures. It is noticed that shear stresses are maximum for cylinder made up of less functionally graded material as compared to homogeneous and highly functionally graded material with linear and nonlinear measure. Also, it is noticed that with the
change in measure from linear to nonlinear, shear stress decreases. Shear stress increases with the increase in pressure as can be seen in Figure 6.1.7. Creep shear stresses have been investigated when internal pressure is less than that of external pressure as shown in Figure 6.1.8 and Table 6.1.2. It has been observed that shear stresses are maximum at internal surface and are tensile in nature for cylinder made up of homogeneous and less functionally graded material with linear and nonlinear measure while these stresses are maximum at external surface for cylinder made up of highly functionally graded material with nonlinear measure. It has been observed that shear stresses with linear measure are maximum for cylinder made up of less functionally graded material as compared to homogeneous material. With the increase in pressure, shear stress increases significantly as can be seen in Figure 6.1.9. It has been noticed that shear stresses are again tensile in nature and these shear stresses are maximum at internal surface for linear measure. It has also been noticed that shear stresses are maximum at external surface for highly functionally graded cylinder while these shear stresses are maximum at internal surface for cylinder made up of less functionally graded and homogeneous materials in case of nonlinear strain measures. The solution of the problem has been obtained using the Mathematica software version 9.

6.1.5 CONCLUSION

In this chapter, we have investigated the effect of strain measure and pressure on creep stresses in thick-walled cylinder made up of functionally graded material subjected to twist. Using transition theory, the closed form solution has been obtained for nonlinear differential equation. It has been observed that strain measure and pressure have significant effects on stresses. From the analysis, it has been concluded that cylinder made up of functionally graded material under torsion is better choice in manufacturing of cylinders under torsion as compared to cylinders made up of homogeneous material because shear stresses are maximum for cylinder made up of functionally graded material as compared to homogeneous material.
Table 6.1.1: Circumferential creep stresses in a thick-walled FGM cylinder for \( \eta_l = 100 \)

<table>
<thead>
<tr>
<th>( \sigma_{th} )</th>
<th>( N = 1 )</th>
<th>( N = 3 )</th>
<th>( N = 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_l = 1.5, \ P_0 = 0.5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k ) ( R )</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1.167</td>
<td>0.426</td>
<td>0.167</td>
</tr>
<tr>
<td>0.5</td>
<td>1.539</td>
<td>0.379</td>
<td>0.013</td>
</tr>
<tr>
<td>1</td>
<td>1.932</td>
<td>0.330</td>
<td>-0.130</td>
</tr>
<tr>
<td>1.5</td>
<td>2.199</td>
<td>0.317</td>
<td>-0.283</td>
</tr>
<tr>
<td></td>
<td>( P_l = 0.5, \ P_0 = 1.5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-3.539</td>
<td>-2.379</td>
<td>-2.013</td>
</tr>
</tbody>
</table>

Table 6.1.2: Creep shear stresses in a thick-walled FGM cylinder for \( \eta_l = 100 \)

<table>
<thead>
<tr>
<th>( \sigma_{gs} )</th>
<th>( N = 1 )</th>
<th>( N = 3 )</th>
<th>( N = 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_l = 1.5, \ P_0 = 0.5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k ) ( R )</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>-49.564</td>
<td>-21.531</td>
<td>-11.308</td>
</tr>
<tr>
<td>1.5</td>
<td>-60.328</td>
<td>-17.277</td>
<td>-2.097</td>
</tr>
<tr>
<td></td>
<td>( P_l = 0.5, \ P_0 = 1.5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>43.546</td>
<td>23.704</td>
<td>15.396</td>
</tr>
<tr>
<td>0.5</td>
<td>49.564</td>
<td>21.531</td>
<td>11.308</td>
</tr>
<tr>
<td>1</td>
<td>55.981</td>
<td>19.172</td>
<td>7.238</td>
</tr>
<tr>
<td>1.5</td>
<td>60.328</td>
<td>17.277</td>
<td>2.097</td>
</tr>
</tbody>
</table>
Figure 6.1.2: Creep stresses in a thick-walled FGM cylinder for $P_i = 1.5$ and $P_o = 0.5$ with $N = 1, 3, 7$

Figure 6.1.3: Creep stresses in a thick-walled FGM cylinder for $P_i = 3$ and $P_o = 1$ with $N = 1, 3, 7$
Figure 6.1.4: Creep stresses in a thick-walled FGM cylinder for $P_f = 0.5$ and $P_o = 1.5$ with $N = 1, 3, 7$

Figure 6.1.5: Creep stresses in a thick-walled FGM cylinder for $P_f = 1$ and $P_o = 3$ with $N = 1, 3, 7$
Figure 6.1.6: Creep shear stresses in a thick-walled FGM cylinder for $P_i = 1.5$ and $P_0 = 0.5$ with $N = 1, 3, 7$

Figure 6.1.7: Creep shear stresses in a thick-walled FGM cylinder for $P_i = 3$ and $P_0 = 1$ with $N = 1, 3, 7$
**Figure 6.1.8:** Creep shear stresses in a thick-walled FGM cylinder for $P_i = 0.5$ and $P_o = 1.5$ with $N = 1, 3, 7$

**Figure 6.1.9:** Creep shear stresses in a thick-walled FGM cylinder for $P_i = 1$ and $P_o = 3$ with $N = 1, 3, 7$
SECTION 6.2

THERMAL CREEP TORSION IN FUNCTIONALLY GRADED THICK-WALLED CYLINDER UNDER INTERNAL AND EXTERNAL PRESSURE

6.2.1 INTRODUCTION

The design and manufacturing of cylindrical vessels is an important area of research to provide safe and better solutions for industries which require pressure vessels for storage of liquids and gases. The applications of pressure vessels vary from manufacturing of engineering equipment to medical components. Therefore, the highest quality is necessary to manufacture these systems for safe and reliable operation. Functionally graded materials (FGMs) are new generation non-homogeneous engineering materials whose composition changes over volume fraction so that a certain variety of the local material properties can be achieved [110, 118]. Functionally graded materials are used as thermal barriers for designing structural component in aerospace applications and nuclear reactors. The applications of functionally graded materials found in structural components to reduce the possibility of fracture for those structural components which operates under extremely high-temperature environment due to their better thermal resistance and mechanical performance. The investigation of torsion in a cylinder made up of homogeneous materials has been analyzed by many researchers using classical and transition theory [5, 28, 65]. The classical theory of deformation [6, 8] considered the jump conditions, yield criterion and linear strain measure to determine the stresses using the concept of infinitesimal strain theory. The transition theory [22] does not require any of the above assumptions and thus solves a more general problem using the concept of generalized strain measure [59]. This generalized strain measure not only gives the well-known strain measures but can also be used to find the stresses in plasticity and creep problems by determining the asymptotic solution at the transition points of the governing differential equations. Shear stresses for a circular cylinder made up of transversely isotropic material subjected to torsion have been determined by Gupta and Rana [30] and they found that the value of shear stresses for a cylinder made up of transversely isotropic material is more than that of cylinder made up of isotropic incompressible material. However, cylindrical
pressure vessels made up of functionally graded materials have received very little attention. Lekhnitskii [119] elaborated elastic problem for non-homogeneous materials. Rooney et al. [113] studied cylindrical bar made of inhomogeneous material with constant modulus and discussed the impact of material inhomogeneity on the torsion response. Further, Horgan et al. [114] investigated torsion in a solid cylindrical bar made up of isotropic material whose shear modulus is varying with position and concluded that maximum shear stress does not occur on the boundary of the rod. Batra [120] has analyzed analytically the torsion of circular cylindrical bar made of isotropic linear elastic material with varying material moduli in the longitudinal direction. Bayat et al. [121] gave a general solution for torsion of hollow cylinders made up of functionally graded materials and determine the angle of twist and shear stress for material whose Young’s modulus and Poisson’s ratio varying in radial direction. Uscilowska [122] studied the torsion problem of a hollow rod made up of functionally graded material.

In this chapter, thermal creep stresses have been investigated for a torsion of a thick-walled circular cylinder made up of functionally graded material with varying moduli in the radial direction under internal and external pressure using transition theory. The results have been discussed numerically and depicted graphically.

### 6.2.2 MATHEMATICAL FORMULATION

Consider a functionally graded thick-walled circular cylinder whose internal and external radii are $a$ and $b$ respectively, subjected to internal pressure $p_i$ and external pressure $p_o$. At internal surface, temperature is $T_0$ and at the outer surface, temperature is zero as can be seen in Figure 6.2.1.

The components of displacement in cylindrical polar coordinates are given by

$$u = r(1 - \beta), \ v = \eta rz \ \text{and} \ w = dz,$$

where $\beta$ is a function of $r = \sqrt{x^2 + y^2}$, $d$ is a constant and $\eta$ is the angle of twist per unit length.

The compressibility of functionally graded cylinder is taken as

$$C = C_0 r^k,$$

where $a \leq r \leq b$, $C_0$ and $k \ (\geq 0)$ are constants.
Figure 6.2.1: A FGM thick-walled pressurized circular cylinder under torsion and thermal loading

The generalized components of strain \([65]\) are given as

\[
e_{rr} = \frac{1}{n^m} \left[ 1 - (r \beta' + \beta)^n \right]^m, \quad e_{\theta\theta} = \frac{1}{n^m} \left[ 1 - \beta^n \right]^m, \quad e_{zz} = \left( \frac{D^n}{n} \right)^m \left[ 1 - \left( \frac{\eta r \beta}{D} \right)^n \right]^m,
\]

\[
e_{\theta z} = \frac{1}{n^m} \left[ \eta^{n/2} r^{n/2} \beta^n \right]^m, \quad e_{r\theta} = e_{xz} = 0,
\]

where \(D^n = \left[ 1 - (1 - d)^n \right]^m\), \(n\) is the measure of deformation and \(\beta' = \frac{d \beta}{dr}\).

In this study, we consider \(m = 1\), which holds for secondary state of creep.

For isotropic materials, the thermal stress-strain relation in elastic region is given by

\[
T_{ij} = \lambda \delta_{ij} I_1 + 2 \mu e_{ij} - \xi T \delta_{ij}, \quad (i, j = 1, 2, 3)
\]

where \(T_{ij}\), \(e_{ij}\) are stress and strain tensors respectively, \(I_1 = e_{kk}\) are strain invariants, \(\lambda\), \(\mu\) are Lame’s constant, \(\delta_{ij}\) is Kronecker’s delta, \(\xi = \alpha_0 (3 \lambda + 2 \mu)\), \(\alpha_0\) being coefficient of thermal expansion and \(T\) is the temperature.

The temperature \(T = \left( T_0 \log \frac{r}{b} \right) / \left( \log \frac{a}{b} \right)\) has to satisfy the Laplace equation

\[
T_{,tt} = 0.
\]
The temperature field satisfying Equation (6.2.5) is

\[ T = T_0 \quad \text{at} \quad r = a, \quad T = 0 \quad \text{at} \quad r = b, \quad \text{where} \quad T_0 \quad \text{is a constant.} \]

Equation (6.2.4) using Equation (6.2.3) with \( m = 1 \) can be rewritten as

\[
T_{rr} = \left( \frac{\lambda + 2\mu}{n} \right) \left( 1 - (r\beta' + \beta)^n \right) + \frac{\lambda}{n} \left( 1 - \beta^n \right) + \frac{\lambda}{n} \left[ D^n - (\eta r \beta)^n \right] - \xi T,
\]

\[
T_{\theta\theta} = \left( \frac{\lambda}{n} \right) \left( 1 - (r\beta' + \beta)^n \right) + \left( \frac{\lambda + 2\mu}{n} \right) \left( 1 - \beta^n \right) + \frac{\lambda}{n} \left[ D^n - (\eta r \beta)^n \right] - \xi T,
\]

\[
T_{zz} = \left( \frac{\lambda}{n} \right) \left( 1 - (r\beta' + \beta)^n \right) + \left( \frac{\lambda}{n} \right) \left( 1 - \beta^n \right) + \left( \frac{\lambda + 2\mu}{n} \right) \left[ D^n - (\eta r \beta)^n \right] - \xi T,
\]

\[
T_{\theta z} = \frac{2\mu}{n} \left[ \eta^{n/2} r^{n/2} \beta^n \right], \quad T_{r z} = T_{r \theta} = 0.
\]

(6.2.6)

Equations of equilibrium are all satisfied except

\[
\frac{d}{dr} \left( T_{rr} \right) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0.
\]

(6.2.7)

### 6.2.3 IDENTIFICATION OF TRANSITION POINT

As there exists a intermediate state i.e. transition state in between elastic and creep state and at transition, the differential system defining the elastic state should reach some kind of criticality. The nonlinear differential equation at transition state is obtained by substituting Equation (6.2.6) in Equation (6.2.7) as,

\[
n\beta P(P + 1)^{n-1} \frac{dP}{d\beta} = r \left( \frac{\mu'}{\mu} - \frac{C'}{C} \right) \left[ \left( 3 - 2C \right) - (1 - C) \left( 1 - d \right)^n \right] \frac{1}{\beta^n} - \left[ \left( 1 - C \right) \left( 1 + \eta^n r^n \right) + (P + 1)^n \right]
\]

\[
C \left[ 1 - (P + 1)^n \right] + rC' \left[ (1 + \eta^n r^n) - \left( 2 - (1 - d)^n \right) \frac{1}{\beta^n} \right] - nP \left[ (1 - C) + (P + 1)^n \right] - n\eta^n r^n (1 - C) (P + 1)
\]

\[-n r^{n+1} \eta^{n-1} \eta' (1 - C) - \frac{T_0 n C}{2 \mu^2 \beta^n} \left[ \xi' r \log \left( \frac{r}{b} \right) + \xi \right],
\]

(6.2.8)

where \( r\beta' = \beta P \), \( C = \frac{2\mu}{\lambda + 2\mu} \), \( \frac{T_0}{\log \frac{a}{b}} \), \( \eta = \frac{2\eta_1 (3 - 2C)}{(2 - C)} \) and \( \eta_1 \) is a constant.

Equation (6.2.8) shows that the transition points of \( \beta \) are \( P = -1 \) and \( P = \pm \infty \). The asymptotic solution through \( P \to -1 \) gives creep stresses [22, 28, 30, 59] depends upon the transition function used.
The boundary conditions are
\[ T_{rr} = -p_i \text{ at } r = a \text{ and } T_{rr} = -p_b \text{ at } r = b. \] (6.2.9)

The resultant axial force in the circular cylinder is given by
\[ \int_a^b r T_{rr} \, dr = L_a. \] (6.2.10)

6.2.3.1 SOLUTION THROUGH PRINCIPAL STRESS DIFFERENCE

For finding the creep stresses at the transition point \( P \to -1 \), we define the transition function \([22, 28, 30, 59, 62, 65]\) through the principal stress difference as follows:
\[ R_f = T_{rr} - T_{\theta\theta} = \frac{2\mu}{n} \beta^n \left[ 1 - (P + 1)^n \right]. \] (6.2.11)

Taking the logarithmic differentiation of the Equation (6.2.11) with respect to \( r \), we have
\[ \frac{d}{dr} (\log R_f) = \frac{nP}{r} + \frac{\mu'}{\mu} - \frac{n\beta P (P + 1)^{n-1} dP}{r \left[ 1 - (P + 1)^n \right]}. \] (6.2.12)

Substituting the value of \( \frac{dP}{d\beta} \) from Equation (6.2.8) in Equation (6.2.12) and taking the asymptotic value \( P \to -1 \), we get
\[ \frac{d}{dr} (\log R_f) = 2 \frac{\mu'}{\mu} - \frac{C'}{C} - \frac{2n}{r} + X, \] (6.2.13)
where
\[ X = \left( \frac{n-1}{r} \right) - C \frac{\mu'}{\mu} + \frac{C'}{\beta^n} \left[ 2 - (1-d)^n \right] - \left( \frac{\mu'}{\mu} - \frac{C'}{C} \right) \left[ (3-2C) - (1-C)(1-d)^n \right] \frac{1}{\beta^n} \]
\[ + (1-C) n r^n \eta^{n-1} \eta' + \frac{nCT_0}{2\mu r \beta^n} \left[ \xi + \xi' r \log \left( \frac{r}{b} \right) \right]. \]

Integration of Equation (6.2.13) yields
\[ R_f = A \frac{\mu^2}{C r^{2n}} \exp f(r), \] (6.2.14)
where \( f(r) = \int X \, dr \) and \( A \) is a constant of integration.

Using Equation (6.2.11) and Equation (6.2.14), we have
\[ R_f = T_{rr} - T_{\theta \theta} = A \frac{\mu^2}{C r^{2n}} \exp f(r) = A r F , \quad (6.2.15) \]

where \( F = \frac{\mu^2}{C r^{2n+1}} \exp f(r) . \)

The asymptotic value of \( \beta \) from Equations (6.2.11) and (6.2.15) is

\[ \beta^n = A \frac{n \mu}{2 Cr^{2n}} \exp f(r) . \quad (6.2.16) \]

Substituting Equation (6.2.15) in Equation (6.2.7) and integrating, we get

\[ T_{rr} = B - A \int F \, dr , \quad (6.2.17) \]

where \( B \) is a constant of integration and asymptotic value of \( \beta \) is \( \frac{D}{r} \) as \( P \to -1 \), \( D \) is a constant.

The constants \( A \) and \( B \) are obtained by using the boundary conditions (6.2.9) in (6.2.17) as

\[ A = \frac{P_0 - p_i}{b}, \quad B = -p_0 + A \int_{a}^{b} F \, dr . \quad (6.2.18) \]

Substituting the value of \( B \) in Equation (6.2.17), we get

\[ T_{rr} = -p_0 + A \int_{a}^{b} F \, dr . \quad (6.2.19) \]

The circumferential, axial and shearing stresses are obtained from Equations (6.2.15), (6.2.18) and Equation (6.2.6) are

\[ T_{\theta \theta} = -p_0 + A \int_{a}^{b} F \, dr - r F , \quad T_{zz} = (1 - C) (T_{rr} + T_{\theta \theta}) + 2 \mu \left( \frac{3 - 2C}{2 - C} \right) e_z - 2 \mu \left( \frac{3 - 2C}{2 - C} \right) \alpha_a T , \]

\[ T_{\theta z} = A \left( \frac{\mu^2 \eta^{n/2} r^{-3n/2}}{C} \exp f(r) \right) , \quad (6.2.20) \]

The twisting couple \( M \) is given by

\[ M = 2\pi \int_{a}^{b} r^2 T_{\theta z} \, dr = 2\pi \int_{a}^{b} A \left( \frac{\mu^2 \eta^{n/2} r^{(-3n+4)/2}}{C} \exp f(r) \right) \, dr , \quad (6.2.21) \]

where
\[
D^r = \left[ \frac{L_a + \lambda}{2\pi} \int_a^b \frac{rC(3-2C)}{(1-C)(2-C)} \alpha_0 Tdr - \int_a^b \frac{r(1-C)}{(2-C)} \left[ T_{rr} + T_{\theta\theta} \right] dr \right] + \frac{\lambda}{n} \int_a^b \frac{rC(3-2C)}{(1-C)(2-C)} dr
\]
and \(e_{zz} = \frac{\lambda}{n} \int_a^b \frac{rC(3-2C)}{(1-C)(2-C)} dr
\)

Stresses for functionally graded cylinder with compressibility variation \((C = C_0 r^k)\) in the radial direction are given as

\[
T_{rr} = -p_0 + A_1 \int_r^b F_i dr, \quad T_{\theta\theta} = -p_0 + A_1 \int_r^b F_i dr - rF_1,
\]

\[
T_{zz} = \left( \frac{1-C_0 r^k}{2-C_0 r^k} \right) \left[ -2p_0 + A_1 \left( \int_r^b F_i dr - rF_1 \right) \right] + \frac{\lambda C_0 r^k (3-2C_0 r^k)}{(1-C_0 r^k)(2-C_0 r^k)}(e_{zz} - \alpha_0 T),
\]

\[
T_{\theta z} = A_4 r^{-k-2} \frac{E^2 (2-C_0 r^k)^2}{4C_0 (3-2C_0 r^k)^2} \left[ \frac{2\eta(3-2C_0 r^k)r}{(2-C_0 r^k)} \right]^{n/2} \exp f_1,
\]

\[
M = 2\pi \int_a^b A_4 r^{k-2n+2} \frac{E^2 (2-C_0 r^k)^2}{4C_0 (3-2C_0 r^k)^2} \left[ \frac{2\eta(3-2C_0 r^k)r}{(2-C_0 r^k)} \right]^{n/2} \exp f_1 dr,
\]

where \(A_4 = \frac{p_0 - p_i}{\int_a^b F_i dr}, \quad F_i = \frac{E^2 (2-C_0 r^k)^2}{4C_0 (3-2C_0 r^k)^2} r^{-k-2n-1} \exp f_1,[/itex]
Now we introduce the components in non-dimensional form as

\[ R = \frac{r}{b}, \quad R_0 = \frac{a}{b}, \quad P_0 = P_i = \frac{P_0}{E}, \quad \sigma_{rr} = \frac{T_{rr}}{E}, \quad \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{E}, \quad \sigma_{zz} = \frac{T_{zz}}{E}, \quad \sigma_{\varphi z} = \frac{T_{\varphi z}}{E}. \]

Thermal creep principal and shear stresses under internal and external pressure in non-dimensional form from Equation (6.2.22) are expressed as

\[
\sigma_{rr} = -P_0 + A_2 \int_{\varphi_0}^{\frac{\pi}{2}} bF_2 \, dR, \quad \sigma_{\theta\theta} = -P_0 + bA_2 \left[ \int_{\varphi_0}^{\frac{\pi}{2}} F_2 \, dR - RF_2 \right],
\]

\[
\sigma_{zz} = \frac{\left(1 - C_0 b^k R^k\right)}{\left(2 - C_0 b^k R^k\right)} \left[ -2P_0 + bA_2 \left( \int_{\varphi_0}^{\frac{\pi}{2}} 2F_2 \, dR - RF_2 \right) + \left( e_{zz} - T_1 \frac{\log R}{\log R_0} \right) \right],
\]

\[
\sigma_{\varphi z} = bRA_2F_2 \left( 2\eta \frac{bR}{b} \right)^{\frac{n-1}{2}} \left[ \frac{(3 - 2C_0 b^k R^k)}{(2 - C_0 b^k R^k)} \right]^{\frac{n}{2}},
\]

\[
M = 2\pi \int_{\varphi_0}^{\frac{\pi}{2}} A_2 b^{-k-2n+3} R^{-k-2n+2} \left[ \frac{2\eta(3 - 2C_0 b^k R^k)bR}{(2 - C_0 b^k R^k)} \right]^{\frac{n}{2}} \exp F_2 \, dR,
\]

(6.2.23)

where \( \alpha_0 T_0 = T_1, \quad A_2 = \frac{P_0 - P_i}{\int_a^b F_2 \, dR}, \quad F_2 = \frac{E^2(2 - C_0 b^k R^k)^2}{4C_0(3 - 2C_0 b^k R^k)^2} b^{-k-2n-1} R^{-k-2n-1} \exp F_2, \)

\[
f_2 = \frac{(n-1)C_0 b^k R^k}{k} + \frac{2kC_0 b^{n+k} R^{n+k}}{D^n(n+k)} - \frac{C_0 b^k}{D^n} \left[ \int b^{n+k} R^{n+k-1} \frac{(3 - 2C_0 b^k R^k)}{(1 - C_0 b^k R^k)} \, dR \right]
\]

\[+ \log(1 - C_0 b^k R^k) + \frac{nT_1 b^n}{\log R_0 D^n} \left[ \left( 3 - 2C_0 b^k R^k \right) - \frac{2k(3 - 2C_0 b^k R^k) \log R}{(2 - C_0 b^k R^k)} \right] R^{n-1} \, dR \]
\[-2\eta IC_0nk \left[ \frac{b^{n+k} R^{n+k-1} (1 - C_0 b^k R^k)}{(2 - C_0 b^k R^k)} \right] \left[ \frac{2\eta (3 - 2C_0 b^k R^k)}{(2 - C_0 b^k R^k)} \right]^{n-1} dR \text{ and} \]

\[e_{zz} = - \frac{L_a}{2\pi} \int_{R_0}^{1} b^2 R \log \frac{R}{R_0} dR - \int_{R_0}^{1} b^2 R (1 - C_0 b^k R^k) \left[ \sigma_{rr} + \sigma_{\theta \theta} \right] dR \]

Equation (6.2.23) represents thermal radial, circumferential, axial, shear stresses and twisting couple in a non-dimensional form for secondary state of creep.

### 6.2.4 RESULTS AND NUMERICAL DISCUSSION

The material properties of the cylinder made up of functionally graded material are defined as:

- thermal expansion coefficient \( \alpha_0 = 17.3 \times 10^{-6} [\text{C}^{-1}] \) (stainless steel),
- compressibility coefficient \( C_0 = 0.5 \),
- Poisson’s ratio \( \nu = 0.3 \).
- The inner and outer radii of the cylinder are taken as \( a = 1 [\text{m}] \) and \( b = 2 [\text{m}] \) respectively.
- The parameters of the compressibility are \( k = 0, 0.5, 1, 1.5 \).

To observe the effect of temperature and pressure with different parameters of strain measure and compressibility, Figures 6.2.2 to 6.2.7 have been drawn between radii ratio and stresses with angle of twist \( \eta_i = 50 \) when internal pressure is greater than that of external pressure.

It is observed from Figure 6.2.2 and Table 6.2.1 that without thermal effects, circumferential stresses are tensile in nature while radial stresses are compressive in nature and are maximum at internal surface with linear strain measure. Also, it has been noticed that circumferential stresses are maximum for cylinder made up of less functionally graded material as compared to highly functionally graded material or homogeneous material. It is noticed that with the change in measure from linear to nonlinear, circumferential stresses are maximum at external surface. It is observed that circumferential stress decreases significantly with the change in measure from linear to nonlinear which further decreases with the increase in nonlinearity of the measure. Also, it has been noticed that circumferential stresses are high for highly functionally graded cylinder as compared to homogeneous and less functionally graded cylinder.

From Figure 6.2.3 and Table 6.2.2, it has been noticed that with the introduction of thermal effects, circumferential stresses with linear measure are tensile in nature. Also, circumferential stress increases for homogeneous cylinder while decreases for functionally graded cylinder. With the
increase in temperature, circumferential stresses increased for homogeneous cylinder and decreased for functionally graded cylinder significantly as can be seen from Figure 6.2.4 and Table 6.2.3. From Figure 6.2.5, it is noticed that with the increase in internal and external pressure, circumferential stresses increase for linear and nonlinear measure. It is also observed that these stresses are tensile in nature throughout the radii excluding highly functionally graded cylinder for nonlinear measure. These stresses are maximum at internal surface for less functionally graded cylinder with linear measure while maximum at external surface for highly functionally graded cylinder with nonlinear measure. With the introduction of temperature, circumferential stress increases for homogeneous and less functionally graded cylinder while decreases for highly functionally graded cylinder as can be seen in Figure 6.2.6. Also, it is observed from Figure 6.2.7 that with the increase in temperature, these stresses again increased for homogeneous and less functionally graded cylinder while decreased for highly functionally graded cylinder.

Figures 6.2.8-6.2.13 and Tables 6.2.4-6.2.6 describes the behavior of creep stresses against radii ratio when internal pressure is less than that of external pressure.

From Figure 6.2.8 and Table 6.2.4, it is observed that circumferential stresses are compressive and are maximum at internal surface for linear measure and at external surface for nonlinear measure. However, these stresses are maximum for less functionally graded cylinder with linear measure and for highly functionally graded cylinder with nonlinear measure. With the introduction of temperature, creep stresses decrease for homogeneous and less functionally graded cylinder with linear measure but increase for cylinder made up of highly functionally graded material. From Figure 6.2.9 and Table 6.2.5, it is observed that with the introduction of temperature, creep stress decreases for homogeneous and functionally graded cylinder with nonlinear measure. For linear measure circumferential stresses are maximum at internal surface. However, for highly functionally graded cylinder these circumferential stresses are maximum at external surface. With the increase in temperature, circumferential stress decreases for homogeneous cylinder while increases for functionally graded cylinder for linear measure. However, creep circumferential stress decreases for nonlinear measure. It is clear from Figure 6.2.11-6.2.13 that with the increase in internal and external pressure, these stresses increased significantly but compressive in nature throughout the radii. For homogeneous and highly functionally graded cylinder, circumferential stresses are maximum at internal surface for linear measure, while these stresses are maximum at external surface for nonlinear measure. Also, circumferential stresses for less functionally graded
cylinder are maximum at internal surface as can be seen in Figure 6.2.11. With the introduction of temperature, these stresses decrease for linear and nonlinear measure except for highly functionally graded cylinder with linear measure. With the increase in temperature, these stresses again decreased for linear as well as nonlinear measure excluding highly functionally graded cylinder.

A shear stress generated when a structural component is twisted. Due to torsion, shear stress generated in addition to principal stresses. The localized parallel shear forces will be highest when the normal forces are high. Due to combined loading i.e. axial and torsional loading, stresses generated in functionally graded material intersecting stresses generating in homogeneous material. As cylinder is subjected to different normal forces on each side and thus when internal pressure is greater than that of external pressure, forces on outer side are small and inner side are high. Non-uniform distribution of pressure and twist creates shear stress that tries to distort the cube. Shear stress act as support force and support force are not evenly distributed because of non-uniform distribution of pressure. Figures 6.2.14-6.2.19 and Tables 6.2.7-6.2.9 have been drawn for shear stresses against radii ratio when internal pressure is greater than that of external pressure to show the effect of temperature and strain measure on stresses.

Creep shear stresses with angle of twist (= 50, say) at room temperature are compressive and are maximum at internal surface for functionally graded and homogeneous cylinder with linear and nonlinear strain measure which can be noticed from Figure 6.2.14 and Table 6.2.7. It is also found that shear stresses are maximum for less functionally graded cylinder as compared to homogeneous and highly functionally graded cylinder. From Figure 6.2.15 and Table 6.2.8, it is noticed that with the introduction of thermal effects, shear stress increases at internal surface while decreases at external surface. From Figure 6.2.16 and Table 6.2.9, it has been noticed that with the increase in temperature, shear stress increases significantly. Also, it is noticed that shear stresses are more for linear measure as compared to non-linear measure. From Figures 6.2.17-6.2.19, it is noticed that, with the increase in internal and external pressure, shear stresses increases significantly. Also, it is found that these shear stresses are maximum for less functionally graded cylinder as compared to cylinder made up of highly functionally graded material.

Figures 6.2.20 to 6.2.25 and Tables 6.2.10 to 6.2.12 have been drawn for shear stresses against radii ratio when internal pressure is less than that of external pressure.

It is observed that creep shear stresses are tensile in nature and maximum at internal surface with linear and nonlinear measure as can be seen from Figure 6.2.20 and Table 6.2.10. Also, it is noticed
that with the increase in nonlinearity shear stress decreases. It is observed that shear stresses are maximum for cylinder made up of less functionally graded material as compared to cylinder made up of highly functionally graded material and homogenous material for linear and nonlinear measure. From Figure 6.2.21 and Table 6.2.11, it is observed that with linear measure and temperature, shear stress decreases at external surface and increases at internal surface for homogeneous and less functionally graded cylinder while decreases at internal surface and increases at external surface for highly functionally graded cylinder. With the increase in thermal effects and pressure, these shear stresses increase significantly as can be seen from Figure 6.2.22 and Table 6.2.12 with linear and nonlinear measure. It has also observed from Table 6.2.11 and Table 6.2.12 that under thermal loading, shear stresses are maximum for cylinder made up of less functionally graded material as compared to cylinder made up of highly functionally graded material and homogeneous material for all the values of strain measure. With the increase in internal and external pressure these shear stress increases significantly as can be seen from Figures 6.2.23 to 6.2.25. The solution of the problem has been obtained using the software Mathematica 9 running on a PC.

6.2.5 CONCLUSION

In this section, the influence of strain measure and temperature for creep behavior on functionally graded cylinder in torsion have been examined under internal and external pressure. It has been observed that with or without thermal effects, cylinder made up of less functionally graded material is on the safer side of design in torsion as compared cylinder made up of highly functionally graded material and homogeneous material for linear and nonlinear strain measures. This is because of the reason that shear stresses are maximum for less functionally graded cylinder as compared to cylinder made up of highly functionally graded material and homogeneous material.
Table 6.2.1: Creep stresses for FGM cylinder with $P_i = 1$, $P_o = 0.2$

<table>
<thead>
<tr>
<th>$\sigma_\theta$</th>
<th>$N = 1$</th>
<th>$N = 3$</th>
<th>$N = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$R$</td>
<td>$0.5$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>0</td>
<td>1.133</td>
<td>0.541</td>
<td>0.333</td>
</tr>
<tr>
<td>0.5</td>
<td>1.466</td>
<td>0.500</td>
<td>0.189</td>
</tr>
<tr>
<td>-1</td>
<td>1.782</td>
<td>0.460</td>
<td>0.076</td>
</tr>
<tr>
<td>-1.5</td>
<td>1.868</td>
<td>0.475</td>
<td>-0.022</td>
</tr>
</tbody>
</table>

Table 6.2.2: Thermal creep stresses ($T_i = 0.05$) for FGM cylinder with $P_i = 1$, $P_o = 0.2$

<table>
<thead>
<tr>
<th>$\sigma_\theta$</th>
<th>$N = 1$</th>
<th>$N = 3$</th>
<th>$N = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$R$</td>
<td>$0.5$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>0</td>
<td>1.235</td>
<td>0.529</td>
<td>0.286</td>
</tr>
<tr>
<td>0.5</td>
<td>1.387</td>
<td>0.511</td>
<td>0.218</td>
</tr>
<tr>
<td>1</td>
<td>1.706</td>
<td>0.470</td>
<td>0.103</td>
</tr>
<tr>
<td>1.5</td>
<td>1.951</td>
<td>0.460</td>
<td>-0.029</td>
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Table 6.2.3: Thermal creep stresses ($T_i = 0.1$) for FGM cylinder with $P_i = 1$, $P_o = 0.2$

<table>
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<th>$\sigma_\theta$</th>
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<td>$0.75$</td>
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<tr>
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<td>0.517</td>
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</tr>
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<td>0.5</td>
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<td>1.637</td>
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<td>1.978</td>
<td>0.454</td>
<td>-0.020</td>
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Table 6.2.4: Creep stresses for FGM cylinder with $P_i = 0.2$, $P_o = 1$

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</thead>
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<td>$0.75$</td>
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Table 6.2.5: Thermal creep stresses ($T_i = 0.05$) for FGM cylinder with $P_i = 0.2$, $P_o = 1$

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<th>$N = 5$</th>
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<tbody>
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<td>$k/R$</td>
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<td>0.75</td>
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</tr>
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<td>-1.808</td>
<td>-1.851</td>
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Table 6.2.6: Thermal creep stresses ($T_i = 0.1$) for FGM cylinder with $P_i = 0.2$, $P_o = 1$

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<th>$N = 5$</th>
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</tr>
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<tbody>
<tr>
<td>$k/R$</td>
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<td>0.75</td>
<td>1</td>
<td>0.5</td>
<td>0.75</td>
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Table 6.2.7: Creep shear stresses for FGM cylinder with $P_i = 1$, $P_o = 0.2$

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<th>$N = 3$</th>
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<th>$N = 5$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$k/R$</td>
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<td>1</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
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Table 6.2.8: Thermal creep shear stresses ($T_i = 0.05$) for FGM cylinder with $P_i = 1$, $P_o = 0.2$

<table>
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<tbody>
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Table 6.2.9: Thermal creep shear stresses \((T_i = 0.1)\) for FGM cylinder with \(P_i = 1, P_0 = 0.2\)

<table>
<thead>
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<th>(R)</th>
<th>(N = 1)</th>
<th>(N = 3)</th>
<th>(N = 5)</th>
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</thead>
<tbody>
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<td>-12.801</td>
<td>-7.416</td>
</tr>
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</tr>
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<td>-11.178</td>
<td>-4.525</td>
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<tr>
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<td>0.5</td>
<td>-26.753</td>
<td>-12.536</td>
<td>-6.905</td>
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<tr>
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<td>-26.800</td>
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<td>-11.178</td>
<td>-4.525</td>
</tr>
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<td>0.5</td>
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<td>-9.801</td>
<td>-1.230</td>
</tr>
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<td></td>
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<td>-30.408</td>
<td>-11.178</td>
<td>-4.525</td>
</tr>
<tr>
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<td>-1.230</td>
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<td>0.5</td>
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<td>-9.801</td>
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<td>-9.801</td>
<td>-1.230</td>
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</table>

Table 6.2.10: Creep shear stresses for FGM cylinder with \(P_i = 0.2, P_0 = 1\)

<table>
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<td>0.5</td>
<td>24.634</td>
<td>13.409</td>
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<td>12.071</td>
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<td>1.216</td>
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<td>0.75</td>
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Table 6.2.11: Thermal creep shear stresses \((T_i = 0.05)\) for FGM cylinder with \(P_i = 0.2, P_0 = 1\)

<table>
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<th>(N = 1)</th>
<th>(N = 3)</th>
<th>(N = 5)</th>
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<tbody>
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<td>6.512</td>
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Table 6.2.12: Thermal creep shear stresses \((T_i = 0.1)\) for FGM cylinder with \(P_i = 0.2, P_0 = 1\)

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<td>11.178</td>
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<td>4.525</td>
</tr>
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<td>0.5</td>
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<td>9.801</td>
<td>1.230</td>
</tr>
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Figure 6.2.2: Creep stresses in a functionally graded thick-walled cylinder for $P_t = 1$ and $P_o = 0.2$ with $N = 1, 3, 5$

Figure 6.2.3: Thermal creep stresses in a functionally graded thick-walled cylinder for $P_t = 1$ and $P_o = 0.2$ at temperature $T_1 = 0.05$ with $N = 1, 3, 5$
Figure 6.2.4: Thermal creep stresses in a functionally graded thick-walled cylinder for $P_i = 1$ and $P_o = 0.2$ at temperature $T_i = 0.1$ with $N = 1, 3, 5$

Figure 6.2.5: Creep stresses in a functionally graded thick-walled cylinder for $P_i = 2$ and $P_o = 0.4$ with $N = 1, 3, 5$
Figure 6.2.6: Thermal creep stresses in a functionally graded thick-walled cylinder for \( P_i = 2 \) and \( P_o = 0.4 \) at temperature \( T_i = 0.05 \) with \( N = 1, 3, 5 \)

Figure 6.2.7: Thermal creep stresses in a functionally graded thick-walled cylinder for \( P_i = 2 \) and \( P_o = 0.4 \) at temperature \( T_i = 0.1 \) with \( N = 1, 3, 5 \)
Figure 6.2.8: Creep stresses in a functionally graded thick-walled cylinder for $P_i = 0.2$ and $P_o = 1$ with $N = 1, 3, 5$

Figure 6.2.9: Thermal creep stresses in a functionally graded thick-walled cylinder for $P_i = 0.2$ and $P_o = 1$ at temperature $T_i = 0.05$ with $N = 1, 3, 5$
Figure 6.2.10: Thermal creep stresses in a functionally graded thick-walled cylinder for \( P_i = 0.2 \) and \( P_o = 1 \) at temperature \( T_i = 0.1 \) with \( N = 1, 3, 5 \)

Figure 6.2.11: Creep stresses in a thick-walled cylinder for \( P_i = 0.4 \) and \( P_o = 2 \) with \( N = 1, 3, 5 \)
Figure 6.2.12: Thermal creep stresses in a functionally graded thick-walled cylinder for $P_i = 0.4$ and $P_o = 2$ at temperature $T_i = 0.05$ with $N = 1, 3, 5$

Figure 6.2.13: Thermal creep stresses in a functionally graded thick-walled cylinder for $P_i = 0.4$ and $P_o = 2$ at temperature $T_i = 0.1$ with $N = 1, 3, 5$
Figure 6.2.14: Creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 1$ and $P_o = 0.2$ with $N = 1, 3, 5$

Figure 6.2.15: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 1$ and $P_o = 0.2$ at temperature $T_i = 0.05$ with $N = 1, 3, 5$
Figure 6.2.16: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 1$ and $P_o = 0.2$ at temperature $T_i = 0.1$ with $N = 1, 3, 5$

Figure 6.2.17: Creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 2$ and $P_o = 0.4$ with $N = 1, 3, 5$
Figure 6.2.18: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 2$ and $P_0 = 0.4$ at temperature $T_t = 0.05$ with $N = 1, 3, 5$

Figure 6.2.19: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 2$ and $P_0 = 0.4$ at temperature $T_t = 0.1$ with $N = 1, 3, 5$
Figure 6.2.20: Creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 0.2$ and $P_o = 1$ with $N = 1, 3, 5$

Figure 6.2.21: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 0.2$ and $P_o = 1$ at temperature $T_i = 0.05$ with $N = 1, 3, 5$
Figure 6.2.22: Thermal creep shear stresses in a thick-walled cylinder for $P_i = 0.2$ and $P_0 = 1$ at temperature $T_i = 0.1$ with $N = 1, 3, 5$

Figure 6.2.23: Creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 0.4$ and $P_0 = 2$ with $N = 1, 3, 5$
Figure 6.2.24: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 0.4$ and $P_o = 2$ at temperature $T_i = 0.05$ with $N = 1, 3, 5$

Figure 6.2.25: Thermal creep shear stresses in a functionally graded thick-walled cylinder for $P_i = 0.4$ and $P_o = 2$ at temperature $T_i = 0.1$ with $N = 1, 3, 5$