CHAPTER 5

ELASTIC-PLASTIC AND CREEP STRESS ANALYSIS OF FUNCTIONALLY GRADED THICK-WALLED CIRCULAR CYLINDER UNDER EXTERNAL PRESSURE

5.1 INTRODUCTION

Thick-walled cylinders are widely used in chemical, petroleum, military industries as well as in nuclear power plants. Therefore, reliability of materials and structures in the form of thick-walled cylinders is of critical importance to many industries including power, nuclear and food processing industries etc. Cylindrical vessels are usually subjected to high pressure and temperature which may be constant or varying. The solution of a hollow circular cylinder subjected to uniform pressures on its inner and outer surfaces was found by the French scientist Lame’s around 1852 and is still widely used. The broad use of cylindrical structural components in mechanical engineering and constructions have stimulated great interest in the calculation of their characteristics. Elastic analysis of cylinders under external or internal pressure can be found in some text book of elasticity [8, 9]. Thick walled circular cylinders made up of non-homogenous material under internal pressure have been analyzed by many authors [11, 34, 41] for elastic-plastic states. Some degree of non-homogeneity is present in wide class of materials such as hot rolled copper, aluminum and magnesium alloys etc. Also, this non-homogeneity can be generated by certain external field i.e. by thermal field, as the elastic module of the material vary with temperature or co-ordinates etc. [6, 69, 101, 102]. Olszak et al. [103] presented the problem of a thick-walled non-homogeneous cylinder subjected to internal pressure and showed that plastic flow may start from either surface depending upon the character and intensity of the non-homogeneity. Hasegawa [104] found the exact solutions for the axisymmetric stress and displacements field caused by a solid or hollow circular cylindrical inclusion in an infinite elastic solid. Rimrott [105] has used the assumptions of zero axial strain, constant density and distortion energy law to derive equations for creep stresses and strain rates in a thick-walled closed ended circular hollow cylinder made up of isotropic homogeneous material under internal pressure. Rimrott and Luke [106] presented creep analysis of rotating isotropic cylinder considering finite strain theory. The appropriate solution has been obtained
for large strain. Creep analysis of thick-walled orthotropic cylinder made of isotropic monolithic material subjected to internal pressure has been presented by [107, 108]. Assuming the plain strain condition [27] and obtained stresses for a homogenous orthotropic rotating internally pressurized cylinder subjected to steady state creep condition. All these authors uses the assumptions of incompressibility, yield condition and jump continuity. Seth [59-62], introduced a transition theory that does not require these assumptions. Gupta et. al. [31] derived elastic-plastic and creep stresses in transversely isotropic rotating hollow and solid circular cylinders under internal pressure. Gupta and Sharma [35] obtained thermal creep stresses and strains in a non-homogenous thick hollow internally pressurized cylinder. In addition to this, Sharma [42] obtained the thermal creep stresses and strain rates in a non-homogenous thick walled rotating cylinder.

In this chapter, elastic-plastic and creep stresses for functionally graded thick-walled cylinder under external pressure have been obtained by using Seth’s transition theory [60]. In this present study, our main aim is to eliminate the need for yield condition, elastic-plastic strain hardening laws, semi-empirical laws, creep stress-strain laws and jump conditions etc. The objective of this chapter is to show that not only the stresses and strains may be obtained in transition, plastic and creep states but also the constitutive equation could be obtained in transition state. Results obtained have been discussed numerically and depicted graphically.

5.2 MATHEMATICAL FORMULATION

We consider an axi-symmetric functionally graded thick-walled circular cylinder of internal and external radii $a$ and $b$ respectively, subjected to external pressure $p_0$ as shown in Figure 5.1.

Non-homogeneity is defined in terms of compressibility of material in the cylinder as

$$C = C_0 \ r^k,$$

where $a \leq r \leq b$, $C_0$ and $k \ (\geq 0)$ are constants.

The generalized principal strain measure [59] is defined as

$$e_i = \int_0^{e_i} \left[ 1 - 2 \varepsilon_i \right]^{\frac{n-1}{2}} \ v_d e_i = \frac{1}{n} \ \left[ 1 - \left( 1 - 2 \varepsilon_i \right)^{\frac{n}{2}} \right].$$

In cylindrical polar co-ordinates $(r, \theta, z)$, the displacement components $(u, v, w)$ are given by

$$u = r(1 - \beta), \ v = 0, \ w = dz,$$

where $\beta$ is a function of $r = \sqrt{x^2 + y^2}$ and $d$ is a constant.
The generalized components of strain are
\[ e_{rr} = \frac{1}{n} \left[ 1 - (r \beta' + \beta)^n \right], \quad e_{\theta\theta} = \frac{1}{n} \left[ 1 - \beta^n \right], \quad e_{zz} = \frac{1}{n} \left[ 1 - (1 - d)^n \right], \quad e_{r\theta} = e_{\theta z} = e_{zr} = 0, \quad (5.4) \]

where \( n \) is the measure and \( \beta' = \frac{d \beta}{dr} \).

The stress strain relation for isotropic material is
\[ T_{ij} = \lambda \delta_{ij} + 2\mu e_{ij} \quad (i, j = 1, 2, 3) \quad (5.5) \]

where \( T_{ij}, e_{ij} \) are stress and strain tensors respectively, \( \lambda, \mu \) are Lame’s constants, \( \delta_{ij} \) is Kronecker’s delta.

The equations of equilibrium in the absence of body force is
\[ \frac{d}{dr} \left( r T_{rr} \right) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0. \quad (5.6) \]

Using Equation (5.5) in Equation (5.6), we get a nonlinear differential equation in \( \beta \) as
\[ n P \beta (P+1)^{n-1} \frac{dP}{d\beta} = \left[ r \left( \frac{\mu'}{\mu} - \frac{C'}{C} \right) \left[ (3 - 2C) - (1 - C)(1 - d)^n \right] \frac{1}{\beta^n} - (1 - C) - (P + 1)^n \right] + \left[ C[1 - (P + 1)^n] + r C'[1 - (2 - (1 - d)^n)] \frac{1}{\beta^n} - n P[(1 - C) + (P + 1)^n] \right] \quad (5.7) \]

where \( r \beta' = \beta P \), \( C = \frac{2\mu}{(\lambda + 2\mu)} \).

The transition point of \( \beta \) in Equation (5.7) are \( P \to -1 \) and \( P \to \pm \infty \).
The boundary conditions are
\[ T_r = 0 \text{ at } r = a, \quad T_r = -p_0 \text{ at } r = b. \]  
(5.8)

The resultant axial force in the cylinder is given by
\[ L_a = 2\pi \int_a^b r T_z \, dr. \]  
(5.9)

5.3 SOLUTION THROUGH PRINCIPAL STRESSES

It has been shown [29, 32, 60-62, 67] that the asymptotic solution through the principal stress
leads from elastic to plastic state at transition point \( P \to \pm \infty \). For finding the transitional and
plastic stresses at the transition point \( P \to \pm \infty \), we define the transition function \( R_f \) as
\[ R_f = T_r - \frac{\lambda}{n} k = \frac{2\mu}{Cn} \left[ C - \beta^a \left\{ (1-C) + (P+1)^a \right\} \right]. \]  
(5.10)

Taking the logarithmic differentiation of Equation (5.10) with respect to \( r \)
\[ \frac{d}{dr} \log R_f = \left[ r C' (1+\beta^a) - nP \beta^{a+1} (P+1)^{a+1} \frac{dP}{d\beta} - nP \beta^a \left\{ (1-C) + (P+1)^a \right\} \right] + 
\]  
\[ + r \left( \frac{\mu}{\mu - C} \right) \left[ C - \beta^a \left\{ (1-C) + (P+1)^a \right\} \right] + \]  
\[ \right] \]  
\[ = \]  
(5.11)

Substituting the value of \( \frac{dP}{d\beta} \) from Equation (5.7) in Equation (5.11) and taking the asymptotic
value \( P \to \pm \infty \), we have
\[ \frac{d}{dr} \log R_f = -\frac{C}{r}. \]  
(5.12)

Integrating Equation (5.12), we get
\[ R_f = A \exp f(r), \]  
(5.13)

where \( A \) is a constant of integration and
\[ f(r) = -\int \frac{C}{r} \, dr. \]  
(5.14)

From Equations (5.10) and (5.13), we have
\[ T_r = A \exp f(r) + B, \]  
(5.15)

where \( B = \frac{\lambda}{n} k \) and \( k = \left[ 3 - (1-d)^a \right] \).

Using boundary condition (5.8) in Equation (5.15), we have
\[ T_{rr} = A \left[ \exp f(r) - \exp f(b) \right] - p_0. \]  
\text{(5.16)}

Using Equation (5.16) in Equation (5.6), we get
\[ T_{\theta\theta} = A \left[ (1 - C) \exp f(r) - \exp f(b) \right] - p_0. \]  
\text{(5.17)}

Equations (5.5) give
\[ T_{zz} = \left( \frac{1 - C}{2 - C} \right) (T_{rr} + T_{\theta\theta}) + \frac{C\lambda}{(1 - C)} \left( \frac{3 - 2C}{2 - C} \right) e_z, \]  
\text{(5.18)}

where \[ e_z = \frac{\left( \frac{L_o}{2\pi} \right) - \int_a^b r \frac{C(1 - C)}{2 - C} (T_{rr} + T_{\theta\theta}) \, dr}{\lambda} \frac{r C (3 - 2C)}{(1 - C)(2 - C)} \, dr. \]

Taking the non-homogeneity Equation (5.1) in the cylinder due to varying compressibility, Equations (5.16)-(5.18) yields
\[ T_{rr} = A_1 \left[ \exp \left( - \frac{C_0 r^k}{k} \right) - \exp \left( - \frac{C_0 b^k}{k} \right) \right] - p_0, \]  
\text{(5.19)}

\[ T_{\theta\theta} = A_1 \left[ (1 - C_0 r^k) \exp \left( - \frac{C_0 r^k}{k} \right) - \exp \left( - \frac{C_0 b^k}{k} \right) \right] - p_0, \]  
\text{(5.20)}

\[ T_{zz} = \left( \frac{1 - C_0 r^k}{2 - C_0 r^k} \right) (T_{rr} + T_{\theta\theta}) + \frac{C_0 r^k}{(1 - C_0 r^k)} \left( \frac{3 - 2C_0 r^k}{2 - C_0 r^k} \right) e_z, \]  
\text{(5.21)}

where \[ A_1 = \frac{p_0}{B_2}, \quad B_2 = \left\{ \exp \left( - \frac{C_0 a^k}{k} \right) - \exp \left( - \frac{C_0 b^k}{k} \right) \right\}. \]

According to Tresca’s yield criterion, yielding in any material occurs when maximum shear stress is equals to yield stress of the material. This maximum shear stress is equals to half the maximum difference of principle stresses. In the classical theory, authors assumed this yield criterion to join the two spectrums i.e. elastic region and plastic region while in the case of transition theory this yield criterion has been calculated from the constitutive equations in transition state. Thus, from Equations (5.19) and (5.20), we have Tresca’s yield criterion [53, 109] as,
\[ T_{\theta\theta} - T_{rr} = -A_1 C_0 r^k \exp \left( - \frac{C_0 r^k}{k} \right). \]  
\text{(5.22)}

From Equation (5.22), we observed that \[ T_{\theta\theta} - T_{rr} \] is maximum at \[ r = \left( e^2 b^k \right)^{\frac{1}{2}} = r_1, \quad C_0 = -e^2 b^k k, \]
therefore initial yielding starts at \[ r = r_1. \]
\[ |T_{0\theta} - T_{rr}|_{r=r_1} = -A_i C_0 r_i^k \exp \left( -\frac{C_0 b}{k} \right) \equiv Y, \] (5.23)

which can be rewritten as
\[ \left| \frac{p_0 k}{e B_2} \right| = 1, \] (5.24)

where \( Y \) is the yield stress.

For fully plastic state \((C_0 \rightarrow 0)\), Equation (5.22) yields
\[ |T_{0\theta} - T_{rr}|_{r=b} = \frac{-p_0 k b^k}{(a^k - b^k)} \equiv Y_1, \]

where \( Y_1 \) is the yield stress.

Now we introduce the following non-dimensional component as:
\[ R = \frac{r}{b}, \quad R_0 = \frac{a}{b}, \quad \sigma_{rr} = \frac{T_{rr}}{Y}, \quad \sigma_{0\theta} = \frac{T_{0\theta}}{Y}, \quad \sigma_{zz} = \frac{T_{zz}}{Y}, \quad \rho = \frac{p_0}{Y_i}. \]

The necessary external pressure required for initial yielding is given by
\[ P_i = \left[ e \left\{ \exp \left( -e^2 R_0^k \right) - \exp \left( -e^2 \right) \right\} \right] \] (5.25)

Using Equation (5.25), transitional stresses from Equations (5.19)-(5.21) are given as
\[ \sigma_{rr} = \frac{P_i}{B_3} \left\{ \exp \left[ -\frac{C_0 b^k}{k} (R^k - 1) \right] - 1 \right\} - P_i, \]
\[ \sigma_{0\theta} = \frac{P_i}{B_3} \left\{ (1 - C_0 b^k R^k) \exp \left[ -\frac{C_0 b^k}{k} (R_0^k - 1) \right] - 1 \right\} - P_i, \] (5.26)
\[ \sigma_{zz} = \frac{1 - C_0 b^k R^k}{2 - C_0 b^k R^k} \left( \sigma_{rr} + \sigma_{0\theta} \right) + \frac{\lambda C_0 b^k R^k (3 - 2 C_0 (bR)^k)}{1 - C_0 (bR)^k (2 - C_0 (bR)^k)} e_{zz}, \]
\[ \lambda = \frac{L_i}{2\pi} \int_0^{\frac{1}{2} b R \left( \frac{1 - C_0 (bR)^k}{2 - C_0 (bR)^k} \right)} (e_{rr} + e_{0\theta}) dR, \]
where
\[ e_{zz} = \frac{1}{2\pi} \int_0^{\frac{1}{2} b R \left( \frac{1 - C_0 (bR)^k}{2 - C_0 (bR)^k} \right)} \left( \sigma_{rr} + \sigma_{0\theta} \right) dR, \]
\[ B_3 = \left\{ \exp \left[ -\frac{C_0 b^k}{k} (R_0^k - 1) \right] - 1 \right\}. \]
The external pressure required for fully plastic state is given by

\[ P_f = \left| -\frac{R_0^k - 1}{k} \right|. \] (5.27)

The stresses from Equation (5.26) for full plasticity are obtained by taking \( C_0 \rightarrow 0 \) as

\[ \sigma_{rr} = (-P_f) \left( \frac{R^k - 1}{R_0^k - 1} \right) - P_f, \quad \sigma_{\vartheta \vartheta} = \sigma_{rr} - \frac{k R^k (P_f)}{(R_0^k - 1)}, \] \] (5.28)

\[ \sigma_z = -\frac{k \lambda R^k}{2 \pi} - \frac{1}{2} \int_{R_0}^{R} R b^2 \left( \sigma_{rr} + \sigma_{\vartheta \vartheta} \right) dR \left( \frac{R_0^k - 1}{R_0^k - 1} \right). \]

5.4 SOLUTION THROUGH PRINCIPAL STRESSES DIFFERENCE

The asymptotic solution at each transition point gives solution to the transition state of a particular configuration of the problem. We observe that the material from elastic state can go over into creep state or plastic state or first to plastic and then to creep or vice versa under external loading system. All these final states are reached through a transition state. As only principal stresses are considered therefore, the transition can take place either through the principal stresses \( T_{rr} \) or \( T_{\vartheta \vartheta} \) becoming critical or through principal stress difference \( T_{rr} - T_{\vartheta \vartheta} \) becoming critical. It has been shown that [25, 29, 35, 42] transition through \( T_{rr} - T_{\vartheta \vartheta} \) leads to the creep state for the critical point \( P \rightarrow -1 \). Thus, transition function \( R_f \) can be defined as

\[ R_f = T_{rr} - T_{\vartheta \vartheta} = \frac{2 \mu \beta^n}{n} \left[ 1 - (P + 1)^n \right]. \] (5.29)

Taking logarithmic differentiation of the Equation (5.29) with respect to \( r \), we get

\[ \frac{d}{dr} \left( \log R_f \right) = \frac{n P + \mu'}{\mu} \left[ \frac{n P \beta (P + 1)^{n-1} \frac{dP}{d\beta}}{r \left[ 1 - (P + 1)^n \right]} \right]. \] (5.30)

Substituting the value of \( \frac{dP}{d\beta} \) from Equation (5.7) in Equation (5.30), we get
Taking asymptotic value of $\beta$ ($D = \frac{P}{r}$, $D$ being a constant) as $P \to -1$ in Equation (5.31), we have
\[
\frac{d}{dr} (\log R_f) = \frac{nP}{r} + \frac{\mu'}{\mu} - \frac{C' - 2n}{r[1 - (P + 1)^n]}.
\] (5.31)

Integrating Equation (5.32), we get
\[
R_f = A \frac{\mu^2}{C r^{2n}} \exp f, \tag{5.33}
\]
where $A$ is constant of integration and $f = \int X \, dr$.

From Equations (5.29) and (5.33), we found that
\[
T_{rr} - T_{\theta \theta} = A \frac{rF}{r}, \tag{5.34}
\]
where $F = \frac{\mu^2}{C r^{2n+1}} \exp f$.

Substituting the value of principal stress difference $T_{rr} - T_{\theta \theta}$ from Equation (5.34) in Equation (5.6) and integrating, we get
\[
T_{rr} = B - A \int F \, dr, \tag{5.35}
\]
where $B$ is a constant of integration.

The constants $A$ and $B$ are obtained by using the boundary conditions (Equation (5.8)) in Equation (5.35) as
\[
A = \frac{P_0}{b_a} \int_a^b F \, dr, \quad B = \frac{P_0}{b_a} \left[ \int_a^b F \, dr \right]_{r=b} - p_0.
\]

As the non-homogeneity in the cylinder is due to variable compressibility $C$ given by Equation (5.1), the creep stresses in a functionally graded cylinder under external pressure have been obtained as
Equation (5.36) gives creep stresses for a thick-walled circular cylinder having variable compressibility.

The following non-dimensional components are expressed as

\[
R_0 = \frac{a}{b}, \quad R = \frac{r}{b}, \quad \sigma_{rr} = \frac{T_{rr}}{E}, \quad \sigma_{\theta\theta} = \frac{T_{\theta\theta}}{E}, \quad \sigma_z = \frac{T_z}{E}, \quad p_0 = \frac{P_0}{E}.
\]

The Equation (5.36) in non-dimensional form can be written as

\[
\sigma_{rr} = -P_0 + P_0 \frac{\int_{r_0}^{1} F_2 \, dr}{\int_{r_0}^{1} F_2 \, dr}, \quad \sigma_{\theta\theta} = -P_0 + P_0 \frac{\int_{R_0}^{R} F_2 \, dR - RF_2}{\int_{R_0}^{R} F_2 \, dR}, \quad \sigma_z = \left(1 - \frac{C_0 b^k}{2 - C_0 b^k}\right) \left(\sigma_{\theta\theta} + \sigma_{rr}\right) + e_z,
\]

where \( e_z = \frac{L_0}{2\pi} - \int_{R_0}^{1} Rb \left(1 - \frac{C_0 b^k}{2 - C_0 b^k}\right) \left(\sigma_{\theta\theta} + \sigma_{rr}\right) dR \),

\[
f_z = \frac{(n-1)C_0 b^k}{k} + \frac{2kC_0 b^{n+k} b^{n+k}}{D^k (n+k)} - \frac{kC_0}{D^k} \int_{1}^{R_0} \left(1 - \frac{C_0 b^k}{2 - C_0 b^k}\right) R^{n+k-1} b^{n+k} dR + \log\left(1 - \frac{C_0 b^k}{2 - C_0 b^k}\right),
\]
\[ F_2 = \frac{E^2}{4C_0} \left( \frac{2-C_0 R^k b^k}{3-2C_0 R^k b^k} \right)^2 b^{-k-2 \eta-1} R^{-k-2 \eta-1} \exp f_2, \quad A_2 = \frac{P_0}{\int_{\kappa_0} bF_2 dR}. \]

Equation (5.37) gives creep stresses in thick-walled circular cylinder under external pressure in non-dimensional form.

### 5.4.1 STRAIN RATES

When the creep sets in, the strain should be replaced by strain rates. The stress-strain relation in Equation (5.5) can be written as

\[ (5.38) \]

\[ \dot{\epsilon}_{ij} = \frac{(1+\nu)}{E} \epsilon_{ij} - \frac{\nu}{E} \delta_{ij} \Theta, \]

where \( \dot{\epsilon}_{ij} \) is the strain rate tensor with respect to flow parameter \( t \) and \( \Theta = T_{11} + T_{22} + T_{33} \) and \( \nu = \frac{1-C}{2-C} \) is the Poisson’s ratio.

Differentiating Equation (5.4) with respect to \( t \), we get

\[ \dot{\epsilon}_{\theta\theta} = -\beta_{\eta-1} \dot{\beta}. \]  

(5.39)

For Swainger measure \( (n=1) \)

\[ \dot{\epsilon}_{\theta\theta} = -\dot{\beta}, \]

(5.40)

where \( \dot{\epsilon}_{\theta\theta} \) is Swainger strain measure.

The transition value of Equation (5.27) as \( P \rightarrow -1 \) gives

\[ \beta = \left( \frac{n}{2 \mu} \right)^{\frac{1}{n}} (T_{rr} - T_{\theta\theta})^{\frac{1}{n}}. \]  

(5.41)

Using Equations (5.39), (5.40) and Equation (5.41) in Equation (5.38), we have

\[ \dot{\epsilon}_{\theta\theta} = \left[ \frac{n}{2 \mu} (T_{rr} - T_{\theta\theta}) \right]^{\frac{1}{n-1}} \left[ \frac{(1+\nu)}{E} \epsilon_{rr} - \frac{\nu}{E} \delta_{rr} \Theta \right], \]

i.e.

\[ \dot{\epsilon}_{rr} = \left[ \frac{(3-2C)}{N(2-C)} \right]^{N-1} (\sigma_{rr} - \sigma_{\theta\theta})^{N-1} \left[ \sigma_{rr} - \left( \frac{1-C}{2-C} \right) (\sigma_{rr} + \sigma_{\theta\theta}) \right], \]  

(5.42)

\[ \dot{\epsilon}_{\theta\theta} = \left[ \frac{(3-2C)}{N(2-C)} \right]^{N-1} (\sigma_{rr} - \sigma_{\theta\theta})^{N-1} \left[ \sigma_{\theta\theta} - \left( \frac{1-C}{2-C} \right) (\sigma_{rr} + \sigma_{\theta\theta}) \right], \]  

(5.43)

and
\[
\dot{\varepsilon}_z = \left[ \frac{(3 - 2C)}{N(2-C)} \right]^{N-1} \left( \sigma_{rr} - \sigma_{\theta\theta} \right)^{N-1} \left[ \sigma_{zz} - \left( \frac{1-C}{2-C} \right) \left( \sigma_{rr} + \sigma_{\theta\theta} \right) \right].
\] (5.44)

These are the constitutive equations for finding the creep strains for \( N = \frac{1}{n} \).

### 5.5 RESULTS AND NUMERICAL DISCUSSION

1. Behavior of elastic-plastic stresses in thick-walled cylinder under external pressure:

To observe the effects of pressure on a cylinder made of up of functionally graded material, Figure 5.2 and Figure 5.3 have been drawn between pressure and radii ratio \( R \).

For a homogeneous circular cylinder, initial yielding starts at internal surface while for a circular cylinder made up of functionally graded material, initial yielding takes place at any point of radius \( r \) \((a \leq r \leq b)\) depending upon values of \( C_0 \) and \( k \). Pressure required for initial yielding is maximum at internal surface for cylinder made up of homogeneous as well as functionally graded materials. From Figure 5.2(a) it has been noticed that high pressure is required for initial yielding in case of homogeneous cylinder and less pressure is required for initial yielding in case of functionally graded cylinder with high compressibility as can be seen in Figure 5.2(b). Also, this pressure goes on decreasing with the increase in radii ratio \( R \).

It has also been observed from Figures 5.3(a)-(b) that for homogeneous as well as functionally graded circular cylinder, pressure required for fully plastic state is maximum at the internal surface. For functionally graded highly compressible cylinder, less pressure is required for fully plastic state as can be seen in Figure 5.3(b). It is also observed that pressure required for fully plastic state is more for cylinder made of homogeneous material than that of functionally graded material.

Figures 5.4-5.6 have been drawn between radii ratio and stresses to observe the influence of pressure and stresses with various non-homogeneity parameters.

From the Figure 5.4, it has been observed that for functionally graded cylinder these transitional circumferential stresses are maximum at external surface. Also, transitional circumferential stresses are maximum for highly compressible cylinder as compared to less compressible cylinder. It has been noticed that with the increase in pressure, transitional stresses increases significantly. From Figure 5.5 and Figure 5.6, it has been observed that in fully plastic state circumferential stress in homogeneous cylinder are less as compared to functionally graded cylinder. Further, it has been observed from the Figure 5.6 that in fully plastic state these circumferential stresses are high for less compressible cylinder as compared to highly
compressible cylinder. With the increase in pressure, these fully plastic compressible stresses increases significantly.

2. Behavior of creep stresses in thick-walled cylinder under external pressure:

In order to show the effect of pressure and non-homogeneity on creep stresses in cylinders made up of functionally graded material, Figures 5.7-5.9 have been drawn between radii ratio $R$ and creep stresses.

From Figure 5.7(a) it has been observed that circumferential stresses are compressible and maximum at internal surface for cylinder made up of homogenous and functionally graded material under external pressure with linear measure. However, as the measure changes from linear to nonlinear, these circumferential stresses with compressibility parameter $(k = 5, 3)$ are again maximum at internal surface while for $k = 1$, stresses are maximum near to $R = 0.6$ with external pressure $P_0 = 1$ as can be seen in Figure 5.7(b)-(c). From Figure 5.7(b), it has been examined that for homogeneous cylinder, circumferential stress is approximately constant. As measure changes from $N = 3$ to $N = 5$, circumferential stress is maximum at external surface.

With the increase in external pressure, we can observe from Figure 5.8 that circumferential stresses are compressible and are maximum at internal surface for cylinder made up of homogenous and functionally graded material with linear measure ($N = 1$). Also, these circumferential stresses are maximum for highly compressible cylinder as compared to less compressible cylinder. It has been observed from Figures 5.8 and 5.9 that with the increase in external pressure, circumferential stresses increases significantly.

For calculating creep deformation, Figures 5.10-5.12 have been drawn between radii ratio and creep strain rates with different parameters of compressibility and strain measure under external pressure.

From Figure 5.10, we can observe that creep strain rates are maximum at internal surface for homogenous cylinder and functionally graded cylinder with linear measure. From Figure 5.10, we can see that as measure changes from linear to nonlinear, creep strain rates decreases significantly for functionally graded cylinder as well as for homogenous cylinder. It can be observed from Figures 5.11 and 5.12 that when external pressure changes from $P_0 = 1$ to $P_0 = 2$, creep strain rates increases significantly but as measure changes from linear to nonlinear, creep strain rates keep going on decreasing. Similarly, from Figures 5.11 and 5.12, it has been noticed that with increase in pressure, creep strain rates also increases but these
strain rates decreases with the change in strain measure from linear to nonlinear. The solution of the problem has been obtained using the software Mathematica 5 and Mathematica 9.

5.6 CONCLUSION

From the analysis of elastic-plastic stresses, we can conclude that highly compressible cylinder is on the safer side of the design as compared to less compressible cylinder. This is because of the reason that functionally graded cylinder made up of highly compressible material requires high pressure for initial yielding as compared to less compressible functionally graded cylinder. From the analysis of creep stresses, we can conclude that functionally graded cylinder with linear measure is better choice from the designing point of view as compared to homogenous cylinder. This is because of the reason that circumferential stress is less for functionally graded cylinder with compressibility parameter \( k = 1 \) under external pressure for linear measure as compared to homogenous cylinder and functionally graded cylinder with other parameters of compressibility.
Figure 5.2: External pressure required for initial yielding for (a) Homogeneous cylinder and (b) Functionally graded cylinder

Figure 5.3: External pressure required for fully plastic for (a) Homogeneous cylinder and (b) Functionally graded cylinder
Figure 5.4: Transitional stresses for a thick-walled functionally graded circular cylinder under external pressure $P_0 = 5, 10$ and $15$
Figure 5.5: Fully plastic stresses for a thick-walled homogeneous circular cylinder under external pressure $P_e = 5, 10$ and $15$

Figure 5.6: Fully plastic stresses for a thick-walled functionally graded circular cylinder under external pressure $P_e = 5, 10$ and $15$
Figure 5.7: Creep stresses in a functionally graded thick-walled cylinder under external pressure \( P_0 = 1 \) with different strain measures: (a) \( N = 1 \) (b) \( N = 3 \) (c) \( N = 5 \)

Figure 5.8: Creep stresses in a functionally graded thick-walled cylinder under external pressure \( P_0 = 2 \) with different strain measures: (a) \( N = 1 \) (b) \( N = 3 \) (c) \( N = 5 \)
Figure 5.9: Creep stresses in a functionally graded thick-walled cylinder under external pressure ($P_0 = 3$) with different strain measures: (a) $N = 1$ (b) $N = 3$ (c) $N = 5$

Figure 5.10: Creep strain rates in a functionally graded thick-walled cylinder under external pressure ($P_0 = 1$) with different strain measures: (a) $N = 1$ (b) $N = 3$ (c) $N = 5$
Figure 5.11: Creep strain rates in a functionally graded thick-walled cylinder under external pressure ($P_0 = 2$) with different strain measure: (a) $N = 1$ (b) $N = 3$ (c) $N = 5$

Figure 5.12: Creep strain rates in a functionally graded thick-walled cylinder under external pressure ($P_0 = 3$) with different strain measure: (a) $N = 1$ (b) $N = 3$ (c) $N = 5$