CHAPTER V: PULSE FORMING LINE (PFL):

WORKING PRINCIPLE
For generation of short duration high voltage pulses with fast current rise time required in applications to pulsed lasers and electron beam generators etc., it is common to use Pulse Forming Lines (PFL) of the coaxial or flat plate type/1/ in transmission line (simple PFL)/2-4/ or Blumlein line/5-8/ configuration. The pulse forming lines have been employed in single/7,8/ and multiple arrays/9/ and also as an element of the marx generator/10/. The advantage of using PFL is that the electrical performance is quite predictable if the load is impedance matched to the line/11,19/. This makes it possible to design a suitable PFL to meet the requirements of a given application by appropriate choice of parameters such as dimensions of conductors and the type of dielectric separating them/12/. Lines with various solid dielectrics such as Mylar/13/ fiberglass reinforced epoxy/14,15/ polyesteer/16/ etc. and liquid dielectrics such as castor oil/1/ glycerine/7/ and water/17/ etc. have successfully been operated.

A transmission line may be made of parallel wires, or parallel plates, coaxial conductors or in general of any two conductors separated by a dielectric medium. The distributed inductance and capacitance along the length of the line play the main role/18/.
The current flow is affected by a distributed series inductance representing the back-induced voltage effects of magnetic flux surrounding the conductor; the voltage between conductors act across a distributed shunt capacitance. The other losses being of less or no significance are neglected in the present design considerations.

Consider a differential length of line $dz$ with distributed inductance $L$ per unit length and distributed capacitance $C$ per unit length. The length $dz$ then has inductance $Ldz$ and a capacitance $Cdz$ (fig.1). The voltage drop across the length is then equal to the product of the inductance and the rate of change of current. For such a differential length voltage change along it at any instant may be written as the length multiplied by the rate of change of voltage with respect to the length.

$$\text{Voltage change} = \frac{\partial V}{\partial z} \, dz = -(Ldz) \frac{\partial I}{\partial t} \quad \ldots(1)$$

Similarly the decrease in current across the element at any instant is merely the current that is shunted across the distributed capacity. The rate of decrease of current with distance is given by the capacity multiplied by time rate of change of voltage.
FIG. 1

\[ I \rightarrow L \, dz \rightarrow I + \frac{\partial I}{\partial z} \, dz \]

\[ V + \frac{\partial V}{\partial z} \, dz \]

FIG. 2

\[ I_+ \rightarrow I_- \]

\[ I_L \rightarrow R_L \rightarrow V_L \]
Current change = $\frac{\partial I}{\partial z} \, dz = - (Cdz) \frac{\partial V}{\partial t}$  \hspace{1cm} \ldots(2)

Cancelling length $dz$ in (1) and (2)

$$\frac{\partial V}{\partial z} = - L \frac{\partial I}{\partial t} \hspace{1cm} \ldots(3)$$

$$\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t} \hspace{1cm} \ldots(4)$$

Equations (3) and (4) are the fundamental differential equations for the analysis of an ideal transmission line.

Differentiating (3) partially with respect to $z$ and (4) with respect to $t$, one has

$$\frac{\partial^2 V}{\partial z^2} = - L \frac{\partial^2 I}{\partial z \partial t} \hspace{1cm} \ldots(5)$$

$$\frac{\partial^2 I}{\partial t \partial z} = - C \frac{\partial^2 V}{\partial t^2} \hspace{1cm} \ldots(6)$$

using (6) in (5)

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2} \hspace{1cm} \ldots(7)$$

where $v = 1/\sqrt{LC}$ \hspace{1cm} \ldots(7a)

is the velocity of propagation of the electromagnetic wave along the line.
A similar equation may be obtained for current.

\[
\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2}
\]  \ ...(8)

A solution of (7) is of the form

\[ F(t - \frac{z}{v}) \text{ or } F(t + \frac{z}{v}) \]

A complete solution of equation (7) can be written as

\[ V = F_1(t - \frac{z}{v}) + F_2(t + \frac{z}{v}) \]  \ ...(9)

Substituting (9) in transmission line equation (3), we have

\[- L \frac{\partial I}{\partial t} = - \frac{1}{v} F_1 \left(t - \frac{z}{v}\right) + \frac{1}{v} F_2 \left(t + \frac{z}{v}\right) \]

Integrating partially with respect to \( t \), one has

\[ I = \frac{1}{LV} \left( F_1(t - \frac{z}{v}) - F_2(t + \frac{z}{v}) \right) \]  \ ...(10)

\[ I = \frac{1}{Z_0} \left( F_1(t - \frac{z}{v}) - F_2(t + \frac{z}{v}) \right) \]  \ ...(11)

\[ Z_0 = LV = \sqrt{L/C} \]  \ ...(12)

The constant \( Z_0 \) as defined in (12) is the characteristic impedance of the line and is seen from (11) to be the ratio of voltage to current for a single travelling wave at any given point and given instant.
For a length 'l' of transmission line, with
total series inductance \( L' = lL \) and shunt capacity
\( C' = lC \)

\[
Z_0 = \frac{L}{\nu} = \sqrt{\frac{L}{C}}
\]

The time required by the wave to travel full length of
the line (from one end to the other) or the transit time
can be written using equation (7a) as

\[
T = \frac{1}{\nu} = \sqrt{\frac{L'}{C'}}
\]

Using equation for \( Z_0 \), the transit time or the electrical
length of the line can also be written as

\[
T = Z_0 C'
\]

with \( Z_0 \) and \( C' \) as the characteristic impedance and the
capacity of the line respectively.

**Reflection at discontinuity**:

In coupling transmission lines to other lines
or loads, the characteristic impedance of the line is not
always matched to the load, and this mismatch introduces
a discontinuity/19/. This gives rise to reflection of
waves travelling along the line. By Kirchoff's law,
total voltage and current must be continuous across the
discontinuity. For voltage at the discontinuity, the
sum of voltage in positively travelling wave $V^+$ and the reflected negatively travelling wave $V^-$ is equal to the voltage across the load $R_L$ fed by the transmission line (fig.2).

$$V^+ + V^- = V_L \quad \ldots(13)$$

Similarly

$$I^+ + I^- = I_L \quad \ldots(14)$$

which can also be written as

$$\frac{V^+}{Z_0} - \frac{V^-}{Z_0} = \frac{V_L}{R_L} \quad \ldots(15)$$

From these equations the ratio of voltage in the reflected wave to that in the incident wave is the reflection coefficient $K_r$

$$K_r = \frac{V^-}{V^+} = \frac{R_L - Z_0}{R_L + Z_0} \quad \ldots(16)$$

The ratio of voltage in the load to that in the incident wave is the transmission coefficient $K_t$

$$K_t = \frac{V_L}{V^+} = \frac{2Z_0}{R_L + Z_0} \quad \ldots(17)$$
From (16) and (17) it is obvious that when a transmission line is driving a load (or terminated into a load), $R_L$ equal to the characteristic impedance of the line, reflection coefficient becomes zero. There is no reflected wave if the terminating resistance $R_L$ is exactly equal to the characteristic impedance $Z_0$ of the line. All energy of the incident wave is then transferred to the load.

**dc voltage applied to an infinite line:**

When a voltage $V_0$ is suddenly applied to an infinite line (fig.3) the line starts to charge to voltage $V_0$, and the wave front travels with a velocity $v = l/\sqrt{LC}$. Since there is no discontinuity there is no reflected wave and the only current present is that flowing in the wave $I = V_0/Z_0$. At any time $t$ after the voltage $V_0$ is impressed, there is voltage $V_0$ and current $V_0/Z_0$ in the line up to the point $Z = vt$ and no voltage or current beyond.

**dc voltage applied to shorted line:**

The mechanism of current build-up in a shorted line is interesting (fig.4a). If the line is shorted at some point $z = l$, when the incident wave with voltage $V_0$ reaches the short circuit, a reflected or negatively
travelling wave of voltage $-V_0$ is sent back so that the sum of the voltages is zero, and the current in the negatively travelling wave is $= -(-V_0/Z_0) = V_0/Z_0$ and hence adds directly to the current in the positively travelling wave when zero voltage reaches the source. A fresh voltage wave $V_0$ is sent making a total current in the line $3V_0/Z_0$. The current gradually builds up to infinity step by step as shown in fig.(4b). $T$ is the time $1/v$ required for a wave to travel one way down the line, the electrical transit time.

**Charged line connected to a resistor:**

When a resistance $R$ is connected at time $t=0$ to a line length $l$ charged to $V_0$, at any time $t$ the voltage across the resistance is the sum of the dc line voltage and the positive wave

$$V_R = V_0 + V_+ \quad \quad \quad \quad \quad (18)$$

and the current flowing into the resistor is the negative of the current for the positively travelling wave (fig.5a)

$$I_R = -I_+ \quad \quad \quad \quad \quad (19)$$

or

$$\frac{V_R}{R} = -\frac{V_+}{Z_0} \quad \quad \quad \quad \quad (20)$$
[a]  

[b]  

[c]  

**FIG. 5**

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**FIG. 6**

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**FIG. 7**
Using (18)

\[ V_R = V_o \left( \frac{R}{R+Z_o} \right) = -V + \frac{R}{Z_o} \]  \hspace{1cm} ...(21)

or

\[ V_+ = -V_o \left( \frac{Z_o}{R+Z_o} \right) \]  \hspace{1cm} ...(22)

**Case I - Matched load: R = Z_o:**

When such a matched resistance is connected to a line charged to \( V_o \), at the first moment, a voltage equal to half the line voltage (\( V_o/2 \)) appears across \( R \) and a travelling wave of \( V_o/2 \) is sent away from the load \( R \); when this wave reaches the open end a reflected wave starts such that the total current is zero. Therefore, the current in the reflected wave must be

\[ -I_+ = \frac{V_o}{2R} \]

and a voltage \( -V_o/2 \) as seen from (22).

Thus in case of \( R = Z_o \) original wave wipes off half the voltage, i.e. \( V_o/2 \), and corresponding current \( -V_o/2 Z_o \) is that which flows through \( R \) during a time \( 0 - T \).

\[ I_R = -I_+ = \frac{V_o}{2Z_o} \]  \hspace{1cm} ...(23)

The reflected wave then wipes out the remaining half of the voltage. The voltage \( V_R = V_o/2 \) remains across \( R \) for a time \( t = 0-2T \) after which there is no voltage on the line and no current through \( R \). The voltage across \( R \)
and the current through R as a function of time is shown in fig.5, b and c respectively.

Case II: \( R \geq Zo \):

If the load resistance \( R \) is greater than \( Zo \), as is seen from equation (21), voltage appearing across \( R \) is greater than \( Vo/2 \) and the reflected travelling wave sent along the line will have smaller magnitude accordingly, and will sweep a smaller portion of the voltage on the line. After reflecting from the open end it will further sweep some part, leaving a fraction behind. The same process will till the voltage across the load reaches zero. Current through \( R \) is shown in fig.(6).

Case III: \( R < Zo \):

When \( R \) is less than \( Zo \), the travelling wave sent away from \( R \) has a magnitude greater than \( Vo/2 \), sweeping greater part of the voltage.

After reflection from the open end the sum of voltages on the line becomes negative, after \( t = 2T \). Fig.(7) shows current through \( R \) when \( R \) is less than \( Zo \), the characteristic impedance of line.

Blumlein line operation:

A typical Blumlein line schematic is shown in fig.(8a) which consists of 3 elements, a common electrode
FIG. 8

[Diagram a] Conductor, Sections I and II, Load R_L, Insulator, Switch S

[Diagram b] Tubular structure with R_L

[Diagram c] Similar to diagram b

FIG. 9

[Graph a] Voltage V vs. Time t, R_L = \infty, V = 2 V_0 for 2T, 0 before and after

[Graph b] Voltage V vs. Time t, Matched Load R_L = Z_0, V = V_0 for 2T, 0 before and after
and a two equal live electrodes, separated from the common electrode by a dielectric with a switch shown in position. This arrangement is found to be very useful for high voltage pulse generation. The two upper electrodes are charged to a certain voltage say $V_o$. Closure of the switch develops a voltage pulse across the gap between the two electrodes (at the output), equal to $2V_o$ with no load connected for a duration of time equal to the two way transit time of one section of the Blumlein line. For a matched load the output is equal to the charging voltage $V$.

The working of a Blumlein line can be explained by considering the transmission line behaviour of the Blumlein line. Considering a Blumlein line fig.(8a) with both the sections charged to $V_o$ and closure of switch $S$ occurring at a time $t = 0$, say to short circuit the section with zero resistance in the switch. This sends a voltage wave travelling towards the other end of the section (output side), of magnitude

$$V_+ = -V_o\left(\frac{Z_o}{R+Z_o}\right) = -V_o$$ \hspace{1cm} ...(24)

in course of time the wave proceeds and finally reaches the other end in a time $T$ equal to the one way transit time of the line.
The case when no load is connected:

When Blumlein line switch \( S \) (fig. 8a) is closed at time \( t_0 = 0 \) with both the sections charged to voltage \( V_0 \) and no load connected at the output, a voltage wave \(-V_0\) is sent down the line towards the output side (open end). While doing so it sweeps the entire voltage on the line making the voltage equal to zero at all the points it has traversed. Detailed time history of development of the pulse is given in fig. (10). After it reaches the open end, again it sees a discontinuity (open circuit) with a reflection coefficient \( = +1 \) (equation 16), and gets reflected back and starts travelling towards the switch, in turn charging the line to \(-V_0\). As the wave proceeds towards \( S \), all the points on the line traversed by the wave are charged to \(-V_0\), till it reaches \( S \) at a time \( t = 2T \) as shown in fig. (10), when it reaches \( S \) it sees a discontinuity again but with a short circuit and the reflection coefficient being \( = -1 \) (from equation 16); therefore gets reflected with reverse polarity \((-V_0) = +V_0\) and travels towards the output end (open end) making the voltage \( = 0 \) at all the points traversed discharging the line as shown in fig. (10), till it reaches the open end. The same cycle repeats again and again resulting in an output of the form shown in fig. (9a) under ideal no load condition.
FIG. 10

| $t_0 = 0$ | \( \bar{v}_0 \) | 0 |
| $t_1 = T/2$ | \( \bar{v}_0 \) | 0 |
| $t_2 = T$ | \( \bar{v}_0 \) | 0 |
| $t_3 = T + \delta t$ | \( \bar{v}_0 \) | 0 |
| $t_4 = 3T/2$ | \( \bar{v}_0 \) | 0 |
| $t_5 = 2T$ | \( \bar{v}_0 \) | 0 |
| $t_6 = 5T/2$ | \( \bar{v}_0 \) | 0 |
| $t_7 = 3T + \delta t$ | \( \bar{v}_0 \) | 0 |
| $t_8 = 4T$ | \( \bar{v}_0 \) | 0 |

| \( s \) | \( \bar{v}_0 \) | \( \bar{v}_0 \) | \( \bar{v}_0 \) | \( \bar{v}_0 \) | 0 |
Matched load case:

When the Blumlein line output (impedance $Z_0$) is terminated into a resistive load $R = Z_0$, the time history of development of pulse across the load is shown in fig.(11) at different instants of time.

When the line is charged to $V_0$ as above the voltage across $R = Z_0$ is zero since both the electrodes are at the same potential. At the time of closure of switch $S$ (fig.11) a voltage wave is sent towards the load of magnitude $-V_0$ discharging the line to zero voltage. When the wave reaches the load the load sees a difference in potential equal to $V_0$ ignoring the effect of load the reflected wave would be $-V_0$ as in the previous case but with $Z_0$ as the load. When the wave front reaches the load it sees an impedance $3Z_0/2 = (Z_0+Z_0/2)$, the load impedance $Z_0$ and the characteristic impedance of the other section $Z_0/2$ in series with it. From (16) and (17) reflection coefficient

$$K_r = \frac{3Z_0/2 - Z_0}{3Z_0/2 + Z_0} \quad \ldots (25)$$

$$= 0.5$$

Transmission coefficient

$$K_t = \frac{2Z_0/2}{3Z_0/2 + Z_0/2} = \frac{Z_0}{2Z_0} = 0.5 \quad \ldots (26)$$
\[ t_0 = 0 \]\[ t_1 = T/2 \]\[ t_2 = T \]\[ t_3 = t_2 + 6t \]\[ t_4 = 3T/2 \]\[ t_5 = 2T \]\[ t_6 = 5T/2 \]\[ t_7 = t_6 + 8t \]\[ t_8 = 3T \]

S \rightarrow R-Z_0

**FIG. 11**
Reflected current \( I_r = K_r I_i \) 
\[ = 0.5 I_i \] 
where \( I_i \) is the incident current given by

\[ I_i = \frac{E}{Z} = -\frac{V_0}{Zo/2} \] 

Transmitted current \( I_t = I_i - I_r = 0.5 I_i \) 

Reflected voltage \( E_R = I_R \frac{Zo}{2} \) 
\[ = 0.5 I_i \frac{Zo}{2} \]

But \( E = I_i(Zo/2) \) 
\[ E_r = -\frac{E}{2} \]

Reflected wave \( E_R \) is half in magnitude and has same polarity as that of the incident wave (-E).

Voltage transmitted is

\[ E_t = I_t \times Zo \] 
\[ = K_t I_i \times Zo \]
\[ = 0.5 I_i \frac{3}{2} Zo \]
\[ = 0.5 \left( -\frac{V_0}{Zo/2} \right) \frac{3}{2} Zo \]
\[ = -\frac{3}{2} V_0 \] 

Out of the transmitted voltage \(-3V_0/2\), load and other section of line share in proportion their impedance i.e. \(-V_0\) appears across the load and \(-V_0/2\) is delivered to the other section of the line. This transmitted portion
\(-Vo/2\) travels toward the open end of the other section, partially discharging the line to Vo/2 (the line was initially at Vo) (fig.11). As this wave front moves toward the open end of the second section the reflected wave front from the load travels toward the switch charging the line to \(-Vo/2\). The wave front travelling toward the switch after reflection at a short circuit with \(Kr = -1\) gets inverted and a wave front \(+Vo/2\) travels toward the load discharging the line to zero potential. The wave front (\(-Vo/2\)) travelling toward the open end of second section sees open circuit reflects with same polarity (\(Kr = +1\) at open circuit), sending a front \(-Vo/2\) toward the load which discharges this section too to zero potential. When both the wave fronts reach the load, there is no charge and no currents afterwards (fig.11). The voltage \(-Vo\) appears across the load for a period of time \(t = T\) to \(t = 3T\) i.e. for a period \(2T\), the two way transit time of the line and has a magnitude equal to the charging voltage \(Vo\). Output voltage pulse is seen in fig.(9b).

When the switch \(S\) is not ideal:

A more realistic approach would be to consider the associated values of RLC with the spark gap switch (which is the case) rather than considering an ideal switch with \(R_s\), \(L_s\) and \(C_s = 0\). In actual practice \(C_s\) can be
low enough, but the role of $R_s$ and $L_s$ in general cannot be ignored in low $Z$ lines. This is because of the condition that the current rise time in the spark gap $L_s/Z_0$ should be less than the two way transmit time on the line. Where $L_s$ is the switch inductance and $Z_0$ is the characteristic impedance of line being switched. In actual working the output of the Blumlein line (across an open load) can be written as/22/

$$V = V_0 \left(1 - e^{-t/T} \cos \omega t \right)$$

where $T = 2L_s/R_s$ and $\omega = 1/\sqrt{L_s C_2}$

$V_0 =$ charge (voltage) on the line.

with $C_2$ as the capacity of the section of Blumlein line being switched, $L_s$ the spark gap inductance and $R_s$ as the damping resistance.

The voltage across the laser head with the breakdown in the head tends to decrease because of the current $I$ flowing through it and can be written as/22/

$$V_{\text{effective}} = V_0 \left(1 - e^{-t/T} \cos \omega t \right) - \frac{2}{C_1} \int Idt$$

which is the open circuit voltage of the voltage minus a term that represents the discharge of Blumlein line capacitor.
Other configurations:

The same principle of working applies to both parallel plate Blumlein or a coaxial one (fig.8c) and can be extended to multiple Blumlein lines. A Blumlein line can be folded one/1/ as shown in fig.(8b) i.e. the common electrode need not extend separately for both electrodes but a single electrode between two live electrodes serves the purpose equally well and results in a relative compact design. In such a design, instead of charging the outer electrodes, middle electrode alone can be charged without affecting the performance, on the other hand outer conductors being at ground potential it seems as a sort of shielding.

Comparison (Simple PFL and Blumlein line):

For comparison of the two types of system driving the same load we assume the case of a matched load, when the load resistance $R$ is equal to the output impedance of the line. In practice, however, the laser plasma offers a time varying impedance collapsing with time, starting with a very high value, near infinity to a very low value near zero as the discharge in the laser head proceeds/20/. For a short time in the uniform discharge a reasonable impedance, however, can be assumed.
In the case of a simple PFL type system among plate and coaxial cable type, coaxial type design is expected to superior under ideal conditions. For practical application of this type however a parallel plate arrangement was found to be suitable.

When a simple PFL with capacity C, inductance L and characteristic output impedance $Z_0 = \sqrt{\frac{L}{C}}$ charged to a voltage $V_0$ is connected to a matched resistive load $R = Z_0$, the voltage $V_R$ across load can be calculated from equation (21)

$$V_R = V_0 \frac{R}{R + Z_0} \quad \ldots (35)$$

$$= \frac{V_0}{2}$$

and the current through the load (from 23) is

$$I_R = \frac{V_0}{2} \frac{Z_0}{Z_0} \quad \ldots (36)$$

which remains for a time $t = 2T = 2ZC$, the two way transit time of the line.

A Blumlein line charged to $V_0$ with output impedance $Z_0$ (characteristic impedance of each section being $Z_0/2$) while driving a matched resistive load $R = Z_0$ gives a voltage pulse of duration $t = 2T = 2ZC = Z_0C$ (where C is capacity of one section) with a load voltage of
\[ V_R = V_o \quad \ldots(37) \]

and current \[ I_R = V_o/Z_o \quad \ldots(38) \]

From (35) and (37) it is seen the system gain (ratio of output to input voltages) is 1 for Blumlein line as compared to 0.5 in simple PFL. Equation (36) and (38) become identical if the value of Zo used is accounted for the characteristic impedance of line. The total current through the Blumlein line is equal to the current through the switch. In practical applications of Blumlein line the peak current is dominated by the switch performance and in low impedance systems the role of the switch becomes very crucial/21/. In addition when the switch rise times tends to be comparable with the transit time of the line, the transmission line behaviour deviates from what is expected.
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