CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Reinforced concrete structures were and are being designed on the basis of elastic theory. The need for inelastic analysis of structures with imperfect plasticity as in the case of concrete structures is explained in chapter 1.

The analysis of reinforced concrete structures calls for properties of materials, namely concrete and steel. Mechanical properties are expressed in terms of stress-strain relationship. These stress-strain relationships are required for developing moment-rotation relations and are explained in this chapter.

Attempts have been made in the past to design the structures based on the results of inelastic analysis. Also, optimization has been as a tool for determining minimum cost of frame applying different constraints. In this chapter a review of stress-strain relationship of concrete and steel, inelastic analysis, optimization and hexa-linear moment rotation laws is presented.

2.2 Review of Stress-Strain Relationship

The behaviour of plain and reinforced concrete is very much conflicting due to complexity of concrete as a material and to the numerous factors that affect its mechanical properties.

Stress-strain curves are based on uni-axial compression or tension tests. Stress-strain relationships for steel both in tension and compression are easily determined. They are not too sensitive to factors, such as rate of loading, strain gradient etc., and can be closely approximated by analytical functions. The stress-strain relationship of concrete in compression has a major effect on the behaviour of section and is extremely difficult to express by analytical functions because of its dependence on a large number of factors.

Up to 1950, the elastic theory was used as the basis of design and a particular form of stress-strain curve for concrete was defined through the assumption of particular shape of the compression stress block and adoption of Bernoulli’s hypothesis. (Plane sections before loading remain plane after loading). Whitney [11] reported complete stress-strain curves for concrete in compression in 1942, based on the results of tests on concrete cylinders of 8-inch
diameter and 16 inch height. It was felt that the brittle failure of concrete was mainly due to
the flexibility of testing machine.

Lee [12] obtained stress-strain curves from tests on reinforced concrete beams and concluded that the stress-strain relationship of concrete under flexural compression can be approximated by an equation
where,

\[ \sigma = \sigma_0 \left\{ \frac{(\varepsilon_u - 0.85 \varepsilon_0 - 0.15 \varepsilon)}{(\varepsilon_u - \varepsilon_0)} \right\} \quad \text{for} \quad \varepsilon > \varepsilon_0 \]  \hspace{1cm} -(2.3)

where, \( \sigma \) and \( \varepsilon \) have the same meaning as given earlier and other terms are shown in the figure (2.1).

\[ \varepsilon_u = 0.38 \% , \quad \varepsilon_0 = 2 \sigma_0 / E_c \]

\[ E_c = 18000 + 0.460 f'_c \] \text{(in fps units)} \quad \text{and} \quad \sigma_0 = 0.85 f'_c

Concrete strength \( f'_c \) is the only variable considered. The equation is simple but has limited application.
FIG. 2.1 STRESS - STRAIN DIAGRAM FOR CONCRETE IN FLEXURE
Smith and Young [15] proposed the exponential expression to represent the stress-strain relationship of concrete under flexural compression.

\[
s = s_o \left( \frac{e}{e_o} \right) e^{\left( t - \frac{e}{e_o} \right)}
\]  \hspace{1cm} (2.4)

Smith and Young assumed that \( s_o = f'_c \) and \( e_o = 0.17\% \) to 0.20\%. The value of \( e_o \), which is taken equal to 0.20\%, co-relates with the maximum value of parabolic curve given by Prentis, Hognestad etc. Actually concrete can be strained beyond this value for complete definition of stress-strain relationship. Moreover exponential relation is not very much suitable computationally.

\[
s = \left( E_c e_o - 2 f'_c \right) \left( \frac{e}{e_o} \right)^3 - \left( 2 E_c e_o - 3 f'_c \right) \left( \frac{e}{e_o} \right)^2 + \left( E_c e \right)
\]  \hspace{1cm} (2.5)

\[
s = f'_c \sin \left( \frac{\pi e}{2 e_o} \right)
\]  \hspace{1cm} (2.6)

In the development of ultimate flexural theory Young [16] used the following two relationships in addition to that given by equation (2.4).

Values of 1000 \( f'_c \) and 0.20\% for \( E_c \) and \( e_o \), respectively were adopted. Since the acceptable range of strain is beyond 0.20\%, the above expressions are not suitable for present work.

Desayi and Krishnan [17] represented the concrete stress-strain relationship by a simple equation of the form

\[
s = \frac{E_c e}{1 + \left( \frac{e}{e_o} \right)^2}
\]  \hspace{1cm} (2.7)

where, \( E_c = 2s_o / e_o \) and \( s_o = f'c \).

Equation 2.6 is complete only when \( e_o \) (or \( e_u \)) and corresponding stress are known. Desayi and Krishnan assumed that \( e_u = 0.30\% \) and corresponding stress equals \((7/8) f'_c \).

Ahmed and Shah [18] proposed following stress-strain relationship for concrete based on experimental stress-strain curve. The aim of their experiments was to predict the moment curvature relationship of reinforced concrete sections. The analytical model found to most accurately represent the experimental data was

\[
y = \frac{Ax + Bx^2}{1 + Cx + Dx^2}
\]  \hspace{1cm} (2.8)
where, \( x = \varepsilon / \varepsilon_0 \) and \( Y = f / f_0 \), \( \varepsilon_0 \) and \( f_0 \) are strain and stress at peak and \( A,B,C,D \), are constants which are different for ascending and descending parts. These four constants can be determined from the knowledge of the four key points on the stress-strain curve, secant modulus of elasticity at 45 percent of the peak stress, peak stress and the corresponding strain, the inflection point and an arbitrary point on descending portion. Curve given by them is shown in figure (2.2) By expressing these four constants in terms of compressive strength of concrete one can generate an entire curve from only the knowledge of the compressive strength of concrete. The expression and curve given by Ahmed and Shah appears to be rational to define the stress-strain relationship of concrete in compression. However, the determination of four constants \( A, B, C, D \), which are different for ascending and descending branches of the curve is difficult. Moreover their curve as shown in figure (2.2) and as expressed by them matches closely with that given by Hognestad. Also for the choice of arbitrary point on the curve no specific mention is made. However, it can be seen that the curve adopted in this work closely matches with the curve given by Ahmed and Shah [18] up to peak point.

2.3 Stress-Strain Curve of Concrete for Present Work

Use of stress-strain relationship is done for the development of moment-rotation laws for reinforced concrete sections, based on the properties of materials namely steel and concrete and properties of section i.e. breadth, depth and area of steel. The stress-strain relationship of concrete in compression plays an important role in finding the moment rotation laws of the sections. It is therefore thought that the stress-strain relationship for concrete in compression should be a curve, which is universally adopted. At the same time the stress-strain relationship be exact and not a complicated model which will pose computational difficulties.

As the stress-strain curve of concrete in compression is parabolic up to a strain of 0.002 and a straight line beyond, as is also recommended by IS: 456-2000 [1] the same is adopted in this work (figure 2.3). The values of \( \sigma_0 = 0.85 f'c \) and \( \varepsilon_0 = 0.002 \) which are adopted in this work are as per many codes of practice for design of reinforced concrete. The analytical expression for stress-strain relationship of concrete used in this work is as under:

\[
\sigma = \sigma_0 \left[ 2 \left( \frac{\varepsilon}{\varepsilon_0} \right) - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \text{for} \ \varepsilon \leq \varepsilon_0
\]  

(2.9)

The stress-strain curve is a straight line beyond strain value of 0.002.
FIG. 2.2 ANALYTICAL STRESS-STRAIN CURVE FOR CONCRETE

\[ Y = \frac{AX}{1 + CX + DX^2} \]

\[ X = \frac{\epsilon}{\epsilon_t}, \ Y = \frac{f}{f_t} \]
FIG. 2.3 RECTANGULAR PARABOLIC STRESS - STRAIN CURVE FOR CONCRETE
FIG. 2.4 STRESS-STRAIN DIAGRAM FOR MILD STEEL

$E_s = 200000$ N/mm$^2$

FIG. 2.5 STRESS-STRAIN CURVE FOR HIGH YIELD STRENGTH DEFORMED STEEL

$E_s = 200000$ N/mm$^2$
2.4 Stress-Strain Relationship for Steel

For Mild steel, stress is proportional to strain up to yield point and therefore the strain increases at constant stress. Young’s modulus of steel is given by the elastic slope. For mild steel the change from elastic to plastic condition is abrupt as shown in figure (2.4).

For high yield strength-deformed steel, there is no exact yield point. Therefore, the yield stress is taken at 0.2% proof strain. The change from elastic to plastic condition is gradual for this type of steel. The stress-strain relationship as adopted by Indian standard code of practice for plain and reinforced concrete, IS: 456-2000 [1] is shown in figure (2.5). Since HYSD steel is very much popular in the present day industry this type of steel is used in the present work.

2.5 Review of Inelastic Analysis

In the past, the theory of reinforced concrete was influenced by the philosophy under which steel structures were designed. The experimental evidence shows that steel is an elastoplastic material. Reinforced concrete, however, is a more complex material because of peculiar non-linear properties of concrete as regards stress-strain variation. The load deformation relationship of reinforced concrete structures is non-linear. Moment-rotation relation for a typical section of reinforced concrete frame member is shown in figure (1.1). For deformation calculations in the analysis process, this curve is approximated by linear segments.

Baker [19] has proposed a simplified limit method for the design of reinforced concrete frames using bi-linear, elastic, perfectly plastic approximation of moment rotation relation. He has also suggested a tri-linear approximation of moment-rotation curve. Baker’s method reduces the statically indeterminate structure to a statically determinate one by introducing number of hinges (number of hinges being equal to one plus degree of static indeterminacy). The positions of these hinges are chosen from the knowledge of approximate elastic moment distribution. The hinges are located at the points where the elastic bending moments are critical for the members. The moments at the hinges, i.e. plastic moment capacities are known from the sectional properties at the hinge sections. The techniques proposed by Baker for carrying out the analysis and design are fully described in references [2], [19] and [20].

A tri-linear approximation of moment rotation diagram as shown in figure (2.6) has been used by Macchi [21] in his method of ultimate load analysis called as the Method of
FIG. 2.6 TRI-LINEAR APPROXIMATION OF MOMENT-ROTATION LAW
Imposed Rotation. The method is based on the general interpretation of inelastic phenomena due to Colonetti and developed by Levi. It consists of considering inelastic rotations artificially imposed in critical sections of a still elastic structure. This method is explained fully in the chapter on formulation. Macchi's method got wide acceptance and was incorporated into the European Concrete Committee (CEB) recommendations. However, being trial and error solution scheme, it was practically applicable to simple structures (mainly continuous beams) with few critical sections.

In 1972, based on imposed rotation method, mathematical programming methods for the inelastic analysis of reinforced concrete frames were developed by Donato and Maier [22]. These methods consisted of Linear Complementarity Problem (LCP) and Quadratic Programming Problem. These methods were developed allowing limited rotation capacity and allowing a tri-linear moment rotation law in positive quadrant and/or a bi-linear moment-rotation law in negative quadrant.

Corradi, Donato and Maier [23] formulated the problem of inelastic analysis of reinforced concrete frames as a linear complementarity problem using tri-linear moment-rotation law. This formulation of imposed rotation method using L.C.P. requires a matrix of size $2n \times 2n$ for a frame, where $n$ is number of critical sections. An improvement upon the computational efficiency of this L.C.P. was made by I. Kaneko [24] as a condensed L.C.P. requiring less than half the computer storage in comparison with that due to Corradi, Donato and Maier. Kaneko's L.C.P. formulation requires a matrix of size $n \times 2n$ for a frame with 'n' critical sections.

Krishnamoorthy and Mosi [25] developed a computer program CONFAP for inelastic analysis of reinforced concrete frames.

Guruje and Agashe [26] developed a computer program INACOF for inelastic analysis of reinforced concrete frames. Their formulation is based on Kaneko's LCP formulation, and uses hexa-linear moment-rotation law.

Kaneko's LCP formulation of inelastic analysis is based on multi-stage loading process where each stage consists of a proportional loading and a tri-linear moment-rotation law. It assumes that the critical section does not undergo unloading at any of the loading stages. It implies therefore, that, there is no reversal of sign of bending moment at any critical section. Thus, the solution process of Kaneko has to be terminated if unloading is encountered, and the problem cannot be solved further. Since, unloading and reversal of sign of bending moment [23] is experienced in practice, Agashe [10] developed a program to account for such a solution. His formulation is based on Hexa-linear moment-rotation law.
and Kaneko’s LCP formulation. As this program is used in the present work, the Hexa-linear moment-rotation law is explained in details in section 1.5. Achawal [9] carried out inelastic analysis of reinforced concrete frames using moment rotation laws based on the properties of materials used and the properties of section.

2.6 Review of Optimization

Optimal design of reinforced concrete frames has been the subject of active research following the development of structural optimization techniques, mathematical programming techniques and availability of large-memory high-speed computational facilities. Cohn and Grierson [27] proposed two formulations for optimal design. In their first formulation, they obtained the optimal design of reinforced concrete beams and frames of given geometry such that under any possible load combination, certain specified minimum load factors could be guaranteed against both the collapse of the structure and yield of its first critical section. By linearising the merit function and developing a method to generate all limit equilibrium constraints the problem has been solved with the help of linear programming. The other formulation [28] is most general and, therefore, results into a non-linear programming problem of considerable complexity. This formulation has constraints associated with compatibility, limited ductility, limit equilibrium of the collapse mechanisms and serviceability criteria. However, only simple two-span beam example was solved with simplified assumption making the problem linear. The objective function was total volume of reinforcement.

Another linear programming formulation for optimal design of reinforced concrete frame which includes compatibility, limited ductility, equilibrium and serviceability criteria as constraints, has been presented by Krishnamoorthy [29], Munro-Krishnamoorthy-Yu [30] and Krishnamoorthy-Munro [31]. The simplex algorithm has been used to solve the linear programming problem to arrive at the minimum amount of reinforcement.

Krishnamoorthy and Mosi [32] have presented a formulation for the optimal design of reinforced concrete frames as a non-linear programming problem, which incorporates inelastic analysis based on Kaneko’s LCP formulation using tri-linear moment-rotation laws. The non-linear problem was solved by using penalty function approach developed by Fiacco and McCormick [33] with the use of Davidon-Fletcher-Powell’s algorithm. To reduce the computational efforts in the optimization process i.e. in minimization of cost of frame due to volume of concrete, volume of reinforcement and area of formwork, these authors have considered a reinforced concrete frame as an assembly of groups of identical beams and
groups of identical columns. Thus each group consists of either identical beams or columns. A typical beam element of a group is optimized subject to constraints belonging to that element to give optimum cost of the group. An element for a column group is similarly optimized separately subject to constraints related to that group to give optimum cost of the column group. The minimum cost of entire frame is obtained by simply summing the minimum cost of each group.

This approach was not rational though simple. Agashe [10] presented the formulation of optimization problem, considering cost of entire frame as objective function and constraints on all the design variables were treated together. He considered breadth, depth and reinforcement area at different critical sections for each member of the frame as the design variables. His formulation was based on hexa-linear moment-rotation relations. However, he used moment-rotation laws from the literature.

Sun Huanchun and Cao Zheng (34) presented optimum design of reinforced concrete frames by a two-level optimization technique, considering global constraints and local constraints, with iterations in each cycle. In this formulation global constraints are those relevant to all design variables, and the local constraints relate to the design variables of a single member only. In the first level the top horizontal displacement of the frame is taken as the objective function, and is maximized to satisfy all global constraints. The optimum solution from first level is applied to construct bounds upon design variables. In the second level, using these values from first level, the most economical structure satisfying all local constraints such as those of size, strength, and percentage of reinforcement is obtained.

A structural optimization algorithm, which includes global displacements as decision variables, was presented by Marc H., Alfredo S. and Fernando F. [35]. In this formulation objective function was the total cost of the materials. The constraints comprised of the equilibrium equalities, imposed maximum global displacements, and required element reliabilities. Section sizes and areas of longitudinal reinforcements were the design variables. It was shown that the simultaneous utilization of reliability and optimization techniques is a good combination for design of reinforced concrete frames.

Chang-Koon, C. and Hyo-Gyoung, K. [36] presented a simplified algorithm for optimum reinforced concrete member design, using direct search method to find the optimum member sections from the predetermined section data base. Entire structure is optimized through individual element optimization.

Dinno, K.S., and Mekha, B.B.[37], presented an algorithm for use in conjunction with the imposed rotation method. The inelastic tri-linear moment-rotation law, which is modelled
to reflect behaviour at critical sections, is adjusted during the incremental analysis in order to adhere to the non-holonomic behaviour of reinforced concrete when stress unloading occurs. This algorithm incorporates two relevant failure criteria, namely local failure and failure by formation of collapse mechanism.

A computer based optimality criteria (OC) method for the minimum cost design of reinforced concrete frameworks under gravity and lateral loading considering the constraints formulated from the member capacity sensitivities and structure response sensitivities was presented by Moharrami H. and Grierson D.E.[38]. This iterative optimization process converges into relatively few cycles, to a least cost design of frameworks.

Balling R. J. and Yao X.[39], presented a simplified method for optimization of 3-D reinforced concrete frames based on the philosophy that strength controls reinforcement design, and stiffness, weight, architectural limitations, or other factors may control the concrete dimension design.

A genetic algorithm procedure for discrete optimization of reinforced concrete frames, minimizing the material and construction costs of reinforced concrete structural elements subjected to serviceability and strength requirements as per American Concrete Institute Building Code was presented by Camp C.V., Pezeshk S. and Hanson H.[40]. Beam elements are evaluated based on their flexural response considering moment magnification factors due to frame stability. A rectilinear column strength diagram is used to evaluate the feasibility of columns with moment magnification due to slenderness effects. The limitations and specifications of the ACI Code are formulated as a series of constraints to the discrete cost optimization problem and applied as penalties on the fitness function of the genetic algorithm.

2.7 Hexa-Linear Moment – Rotation Law

Kaneko’s formulation uses tri-linear moment-rotation laws and multi-stage-loading process where each stage consists of fractional part of the total load, which the given frame is supposed to carry at the final stage. The loads in each of the final stage are expressed in terms of $\alpha \bar{F}$, where $\bar{F}$ is a load vector and $\alpha$ is a load factor, which increases stage wise. The LCP algorithm determines this $\alpha$ at each stage. Therefore $\alpha$ does not increase in equal increments. In the first stage as $\alpha$ increases, from zero to $\alpha_1$, all the n critical cross-sections of the frame are loaded with elastic moments, which are less than or equal to cracking moment of the corresponding sections. A typical cross-section starts cracking when the applied cracking moment due to the loads $\alpha_1 \bar{F}$ is reached. As $\alpha$ increases in the second stage from $\alpha_1$ to $\alpha_2$,
this cross-section develops inelastic rotations and may yield in subsequent higher load stages, when the yielding moment is attained. The moment effects at this section due to inelastic rotation are transferred to other critical sections, as $\alpha$ increases from $\alpha_1$ to $\alpha_2$. Kaneko’s formulation assumes that, the moment reversal due to local unloading does not take place at any of the sections during any two loading stages. The members of a frame are subjected to given loading and develop resistance in terms of bending moment and axial forces carried by them. As the load increases the moments carried by the members increase. The magnitude of moment does not remain the same for all the sections. With the increase in the load, some sections reach their resisting capacity while some are stressed much below their capacity. Under such situations, with the increase in the load, the sections, which have reached their resisting capacity, transfer the moment due to additional load to other sections, which are yet to reach their cracking limit. This transferred moment to the less stressed sections induces reversal of sign of bending moment in such sections. It is observed that this type of reversal of sign of bending moment occurs in elastic region only. Since $L_0$ is the point on moment-rotation curve coinciding with the cracking moment, in both positive and negative quadrants, the reversal takes place within the range of $+L_0$ to $-L_0$. Therefore, to accommodate such a reversal, program developed by Agashe [10] assumes that the material of cross section is reversible in the two elastic states and moment-rotation relationship for a cross-section is hexa-linear. In other words, it is assumed that every critical cross section has three linear segments in the positive moment-rotation plane and the three linear segments in the negative moment-rotation plane. The method using hexa-linear moment rotation law is explained in formulation.

In the limit analysis of reinforced concrete frames, the moment-rotation laws for sections of a frame member are extremely important. In the works mentioned earlier, on the inelastic analysis of reinforced concrete frames, these laws were assumed to be available, found experimentally and no attention had been paid to develop them from basic properties of materials and the properties of the sections being used for the frame. The absence of procedure to develop moment-rotation laws based on properties of materials and the properties of the section is an important lacuna in the literature. It is therefore sought to overcome the lacuna for moment-rotation laws in the present work. Baker, in his method of limit analysis of reinforced concrete frames, has used the idealized rectangular parabolic stress-strain variation for concrete in compression. Present work uses the rectangular parabolic stress-strain relationship for concrete in compression.