Chapter 5
Stochastic Analysis of the System incorporating Manual Network Restoration through Hardware/Software Expansion

In the preceding chapters, the system was analysed considering situation of traffic congestion in network besides other faults/failures in the system. It was considered there that the network is restored from traffic congestion automatically at BSC level. In today’s competitive environment, performance of a mobile communication system is judged/evaluated by the subscribers on the basis that how fast and without any loss of voice or data, its network provides services to the subscribers. This all is possible when besides failure-free operations of the system, no or least traffic congestion is in its network. This has forced mobile companies and its service providers to focus a lot on the problems of traffic congestion in networks for highest performance of the system. Consequently, some researchers in recent years extensively investigated and tried to resolve the problem of traffic congestion in networks. For instance, Hammer and Michael (1999) analysed traffic congestion patterns while Medhi (2005) presented perspective on network restoration. Sugawara (2013) provided solution to congestion problems of mobile communication services during major natural disasters.

While collecting data on the system, it has been observed that whenever traffic congestion occurs, network is generally restored automatically from traffic congestion at BSC level. But in case, congestion in the network occurs frequently and persists for longer period or not restored automatically due to one reason or the other, the network is manually restored from the traffic congestion by the service provider. This is done by expanding hardware/software components of the system, e.g. by adding transceiver cards or installing additional BTS.

Keeping above practical situation in view, the present chapter is devoted to analyses of the system incorporating the possibility of manual restoration of the network from persistent traffic congestion through software/hardware expansion of the system. It is considered that when the system is unable to restore network traffic congestion automatically or there is persistent traffic congestion in the network then manual restoration of the network is done through hardware/software expansion of the system and further hardware/software
expansion times have arbitrary time distributions. Two stochastic models are discussed with this provision. In the Model-I, situation of the system when there is no hardware-software interactions faults due to failures or delayed/improper repairs in hardware/software components is considered whereas in Model-II, the situation of presence of both hardware-software interactions faults in the system due to failures or delayed/improper repairs in hardware/software components is considered. Other assumptions are same that are taken up in the previous chapter.

The models are analysed by making use of Markov process and regenerative point technique. The following measures of system performance are obtained:

- Mean time to system failure
- Expected uptime of the system
- Expected degradation time of the system
- Expected congestion time of the system
- Busy period of repair team (Inspection time only)
- Busy period of repair team (Repair time only)
- Busy period of repair team (Replacement time only)
- Expected number of hardware expansions
- Expected number of software expansions

**Notations**

- \( \delta_2 \): Manual network restoration rate
- \( i_s(t)/I_s(t) \): P.d.f./C.d.f of inspection time on network traffic congestion
- \( p_1/q_1 \): Probability that automatic/manual network restoration from traffic congestion is done, \( q_1 = 1-p_1 \)
- \( p_2/q_2 \): Probability that hardware/software expansion is carried out, \( q_2 = 1-p_2 \)
- \( u_{h_i}(t)/U_{h_i}(t) \): P.d.f./C.d.f of hardware expansion time of the system
- \( u_{s_i}(t)/U_{s_i}(t) \): P.d.f./C.d.f of software expansion time of the system

**States of the System**

- \( O_c \): Congestion state under inspection
- \( O_{w_s}/O_{w_s} \): Congestion state under hardware/software expansion

Other notations are same as taken in the previous chapters.
Model-I

The various states of transition of the system are shown in transition diagram given in fig. 5.1. The epochs of entry in to state 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 are regenerative point, i.e. all the states are regenerative states.

Fig. 5.1 State Transition Diagram
Transition Probabilities and Mean Sojourn Times

The transition probabilities are

\[
\begin{align*}
\mathrm{d}Q_{01}(t) &= \lambda_1 e^{-(\lambda_1 + \lambda_2 + \eta)t} \mathrm{d}t \\
\mathrm{d}Q_{02}(t) &= \lambda_2 e^{-(\lambda_1 + \lambda_2 + \eta)t} \mathrm{d}t \\
\mathrm{d}Q_{12}(t) &= c_1 i_1(t) \mathrm{d}t \\
\mathrm{d}Q_{17}(t) &= d_i i_1(t) \mathrm{d}t \\
\mathrm{d}Q_{29}(t) &= c_2 i_2(t) \mathrm{d}t \\
\mathrm{d}Q_{30}(t) &= p_1 \delta e^{-\delta t} \mathrm{d}t \\
\mathrm{d}Q_{40}(t) &= g_{b_1}(t) \mathrm{d}t \\
\mathrm{d}Q_{50}(t) &= g_{b_2}(t) \mathrm{d}t \\
\mathrm{d}Q_{60}(t) &= g_{c_1}(t) \mathrm{d}t \\
\mathrm{d}Q_{80}(t) &= g_{c_2}(t) \mathrm{d}t \\
\mathrm{d}Q_{90}(t) &= g_{c_3}(t) \mathrm{d}t \\
\mathrm{d}Q_{10,0}(t) &= g_{s_2}(t) \mathrm{d}t \\
\mathrm{d}Q_{11,13}(t) &= q_2 i_3(t) \mathrm{d}t \\
\mathrm{d}Q_{12,0}(t) &= u_{b_1}(t) \mathrm{d}t \\
\mathrm{d}Q_{13,0}(t) &= u_{c_1}(t) \mathrm{d}t
\end{align*}
\]

The non-zero elements \( p_{ij} = \lim_{s \to 0} q_{ij}^*(s) \) are obtained as under:

\[
\begin{align*}
p_{01} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \eta} \\
p_{02} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \eta} \\
p_{03} &= \frac{\eta}{\lambda_1 + \lambda_2 + \eta} \\
p_{14} &= a_1 i_1^*(0) \\
p_{15} &= c_1 i_1^*(0) \\
p_{16} &= b_j i_1^*(0) \\
p_{17} &= d_i i_1^*(0) \\
p_{28} &= a_2 i_2^*(0) \\
p_{29} &= c_2 i_2^*(0) \\
p_{2,10} &= b_2 i_2^*(0) \\
p_{30} &= p_1 \\
p_{3,11} &= q_i \\
p_{40} &= g_{b_1}(0) \\
p_{50} &= g_{b_2}(0) \\
p_{60} &= g_{b_3}(0) \\
p_{70} &= g_{c_1}(0) \\
p_{80} &= g_{c_2}(0) \\
p_{90} &= g_{c_3}(0) \\
p_{10,0} &= g_{s_2}(0) \\
p_{11,12} &= p_2 i_3^*(0) \\
p_{11,13} &= q_2 i_3^*(0) \\
p_{12,0} &= u_{b_1}(0) \\
p_{13,0} &= u_{c_1}(0)
\end{align*}
\]

By these transition probabilities, it can be verified that

\[
p_{01} + p_{02} + p_{03} = p_{14} + p_{15} + p_{16} + p_{17} = p_{28} + p_{29} + p_{2,10} = 1
\]
p_{30} + p_{3,11} = p_{11,12} + p_{11,13} = 1
p_{40} = p_{56} = p_{60} = p_{70} = p_{80} = p_{9,10} = p_{10,0} = p_{12,0} = p_{13,0} = 1

The mean sojourn time ($\mu_i$) in the regenerative state $i$ is defined as the time of stay in that state before transition to any other state. If $T$ denotes the sojourn time in regenerative state $i$, then

$$\mu_i = \int_0^\infty P(T_i > t) \, dt$$

which gives

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2 + \eta} \quad \mu_1 = -i_1^*(0) \quad \mu_2 = -i_2^*(0)$$

$$\mu_3 = \frac{p_4}{\delta_1} + \frac{q_1}{\delta_2} \quad \mu_4 = -g_{h_1}^*(0) \quad \mu_5 = -g_{h_2}^*(0)$$

$$\mu_6 = -g_{h_3}^*(0) \quad \mu_7 = -g_{h_4}^*(0) \quad \mu_8 = -g_{h_5}^*(0)$$

$$\mu_9 = -g_{h_6}^*(0) \quad \mu_{10} = -g_{h_7}^*(0) \quad \mu_{11} = -i_3^*(0)$$

$$\mu_{12} = -u_{h_1}^*(0) \quad \mu_{13} = -u_{h_2}^*(0)$$

The unconditional mean time taken by the system to transit for any regenerative state $j$, when it is counted from epoch of entrance into that state $i$, is mathematically stated as

$$m_{ij} = \int_0^\infty q_j^*(t) \, dt = q_j^*(s)$$

Thus,

$$m_{01} + m_{02} + m_{03} = m_0 \quad m_{14} + m_{15} + m_{16} + m_{17} = m_1$$

$$m_{28} + m_{29} + m_{2,10} = m_2 \quad m_{30} + m_{3,11} = m_3$$

$$m_{40} = m_4 \quad m_{56} = m_5$$

$$m_{60} = m_6 \quad m_{70} = m_7$$

$$m_{80} = m_8 \quad m_{9,10} = m_9$$

$$m_{10,0} = m_{10} \quad m_{11,12} + m_{11,13} = m_{11}$$

$$m_{12,0} = m_{12} \quad m_{13,0} = m_{13}$$
Mean Time to System Failure

To determine the MTSF of the system, the failed states of the system are taken as absorbing states. By probabilistic arguments, the following recursive relations for \( \phi_i(t) \), c.d.f of the first passage time from regenerative state \( i \) to failed state are obtained:

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) + Q_{02}(t) \phi_1(t) + Q_{03}(t) \phi_3(t) \\
\phi_1(t) &= Q_{28}(t) \phi_8(t) + Q_{29}(t) \phi_9(t) + Q_{2,10}(t) \phi_{10}(t) \\
\phi_2(t) &= Q_{30}(t) \phi_9(t) + Q_{3,11}(t) \phi_{11}(t) \\
\phi_3(t) &= Q_{80}(t) \phi_9(t) \\
\phi_4(t) &= Q_{9,10}(t) \phi_{10}(t) \\
\phi_5(t) &= Q_{100}(t) \phi_9(t) \\
\phi_{10}(t) &= Q_{11,12}(t) \phi_{12}(t) + Q_{11,13}(t) \phi_{13}(t) \\
\phi_{11}(t) &= Q_{12,0}(t) \phi_9(t) \\
\phi_{12}(t) &= Q_{13,0}(t) \phi_9(t)
\end{align*}
\]

Using L.S.T., the above recursive relations are solved in terms of \( \phi_0^*(s) \), we get

\[
\phi_0^*(s) = \frac{N(s)}{D(s)},
\]

where

\[
N(s) = Q_0^{**}(s),
\]

and

\[
D(s) = 1 - Q_0^{**}(s)Q_{28}^{**}(s)Q_{90}^{**}(s) - Q_{02}^{**}(s)Q_{29}^{**}(s)Q_{9,10}^{**}(s)Q_{10,0}^{**}(s) - Q_{03}^{**}(s)Q_{2,10}^{**}(s)Q_{0,0}^{**}(s) - Q_{03}^{**}(s)Q_{30}^{**}(s) - Q_{03}^{**}(s)Q_{3,11}^{**}(s)[Q_{11,12}^{**}(s)Q_{12,0}^{**}(s) + Q_{11,13}^{**}(s)Q_{13,0}^{**}(s)]
\]

The mean time to system failure (\( T_{51} \)) when the system starts from the state 0, is

\[
T_{51} = \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \frac{1 - \phi_0^*(s)}{s} = \frac{N}{D},
\]

where \( R^*(s) \) is the L.T. of the reliability \( R(t) \). The reliability \( R(t) \) of the system at time ‘t’ can be obtained by taking inverse L.T. of \( R^*(s) \).
Using L’ Hospital rule and substituting the value of \( f_0''(s) \), we get

\[
T_{s1} = \frac{N}{D},
\]

where,

\[
N = \mu_0 + p_{02} \mu_2 + p_{03} \mu_3 + p_{02} p_{28} \mu_8 + p_{02} p_{29} \mu_9 + \left( p_{02} p_{29} + p_{02} p_{210}\right) \mu_{10}
\]

\[
+ p_{03} p_{3,11} \mu_{11} + p_{03} p_{3,11} p_{11,12} \mu_{12} + p_{03} p_{3,11} p_{11,13} \mu_{13}
\]

and \( D = p_{01} \).

**Expected Uptime of the System**

Using the arguments of the theory of regenerative processes, the expected uptime of the system \( UT_i(t) \), the probability that the system is up at instant \( t \) given that it entered regenerative state \( i \) at \( t = 0 \), satisfies the following recursive relations:

\[
UT_0(t) = M_0(t) + q_{i0}(t)\circ UT_1(t) + q_{i0}(t)\circ UT_2(t) + q_{i0}(t)\circ UT_3(t)
\]

\[
UT_1(t) = q_{i1}(t)\circ UT_4(t) + q_{i1}(t)\circ UT_5(t) + q_{i1}(t)\circ UT_6(t) + q_{i1}(t)\circ UT_7(t)
\]

\[
UT_2(t) = q_{i2}(t)\circ UT_8(t) + q_{i2}(t)\circ UT_9(t) + q_{i2}(t)\circ UT_{10}(t)
\]

\[
UT_3(t) = q_{i3}(t)\circ UT_0(t) + q_{i3}(t)\circ UT_{11}(t)
\]

\[
UT_4(t) = q_{i4}(t)\circ UT_0(t)
\]

\[
UT_5(t) = q_{i5}(t)\circ UT_0(t)
\]

\[
UT_6(t) = q_{i6}(t)\circ UT_0(t)
\]

\[
UT_7(t) = q_{i7}(t)\circ UT_0(t)
\]

\[
UT_8(t) = q_{i8}(t)\circ UT_0(t)
\]

\[
UT_9(t) = q_{i9}(t)\circ UT_0(t)
\]

\[
UT_{10}(t) = q_{i10}(t)\circ UT_0(t)
\]

\[
UT_{11}(t) = q_{i11}(t)\circ UT_0(t)
\]

\[
UT_{12}(t) = q_{i12}(t)\circ UT_0(t)
\]

\[
UT_{13}(t) = q_{i13}(t)\circ UT_0(t)
\]

where

\[
M_0(t) = e^{-(\lambda_1 + \lambda_2 + \eta)t}
\]

Using L.T., the above recursive relations are solved in terms of \( UT_0^*(s) \), we get

\[
UT_0^*(s) = \frac{N_i(s)}{D_i(s)}
\]

where

\[
N_i(s) = M_i^*(s)
\]
and

\[ D_1(s) = 1 - q_{01}^*(s)q_{14}^*(s)q_{16}^*(s) - q_{01}^*(s)q_{15}^*(s)q_{16}^*(s)q_{46}^*(s) - q_{01}^*(s)q_{16}^*(s)q_{46}^*(s) \\
- q_{01}^*(s)q_{17}^*(s)q_{70}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{80}^*(s) - q_{02}^*(s)q_{29}^*(s)q_{9,10}^*(s)q_{10,0}^*(s) \\
- q_{03}^*(s)q_{3,10}^*(s)q_{10,0}^*(s) - q_{03}^*(s)q_{30}^*(s) - q_{03}^*(s)q_{3,11}^*(s)q_{11,12}^*(s)q_{12,0}^*(s) \\
- q_{03}^*(s)q_{3,11}^*(s)q_{11,13}^*(s)q_{13,0}^*(s) \]

In steady state, the expected uptime of the system (UT$_{51}$) at is given by

\[ UT_{51} = \lim_{s \to 0} (sUT_0^*(s)) = \frac{N_1}{D_1}, \]

where

\[ N_1 = \mu_0 \]

and

\[ D_1 = \mu_0 + p_{01} \mu_1 + p_{02} \mu_2 + p_{03} \mu_3 + p_{01} \mu_4 + p_{01} \mu_5 + p_{01} \mu_6 + p_{01} \mu_{14} + p_{01} \mu_{15} + p_{01} \mu_{16} \]

\[ + p_{01} \mu_7 + p_{02} \mu_8 + p_{02} \mu_9 + p_{02} \mu_{29} + p_{02} \mu_{10} + p_{02} \mu_{11} + p_{03} \mu_{3,11} \mu_{12} + p_{03} \mu_{3,11} \mu_{13}. \]

**Expected Degradation Time of the System**

Using the arguments of the theory of regenerative processes, the expected degradation time of the system DT$_i$(t), the probability that the system is degraded at instant t given that it entered regenerative state i at t = 0, satisfies the following recursive relations:

\[
\begin{align*}
DT_0(t) &= q_{01}(t)\cdot DT_1(t) + q_{02}(t)\cdot DT_2(t) + q_{03}(t)\cdot DT_3(t) \\
DT_1(t) &= q_{14}(t)\cdot DT_2(t) + q_{15}(t)\cdot DT_3(t) + q_{16}(t)\cdot DT_4(t) + q_{17}(t)\cdot DT_7(t) \\
DT_2(t) &= M_2(t) + q_{28}(t)\cdot DT_4(t) + q_{29}(t)\cdot DT_9(t) + q_{10,0}(t)\cdot DT_{10}(t) \\
DT_3(t) &= q_{30}(t)\cdot DT_5(t) + q_{31}(t)\cdot DT_{11}(t) \\
DT_4(t) &= q_{40}(t)\cdot DT_6(t) \\
DT_5(t) &= q_{50}(t)\cdot DT_6(t) \\
DT_6(t) &= q_{60}(t)\cdot DT_0(t) \\
DT_7(t) &= q_{70}(t)\cdot DT_0(t) \\
DT_8(t) &= M_6(t) + q_{80}(t)\cdot DT_0(t) \\
DT_9(t) &= M_9(t) + q_{9,10}(t)\cdot DT_{10}(t) \\
DT_{10}(t) &= M_{10}(t) + q_{10,0}(t)\cdot DT_0(t) \\
DT_{11}(t) &= q_{11,12}(t)\cdot DT_{12}(t) + q_{11,13}(t)\cdot DT_{13}(t)
\end{align*}
\]
\[ DT_{12}(t) = q_{12,0}(t)DT_0(t) \]
\[ DT_{13}(t) = q_{13,0}(t)DT_0(t) \]

where

\[ M_2(t) = \overline{I}_2(t); \quad M_8(t) = \overline{G}_{h_2}(t); \quad M_9(t) = \overline{G}_{h_1}(t); \]
\[ M_{10}(t) = \overline{G}_{s_2}(t) \]

Using L.T., the above recursive relations are solved in terms of \( DT_0^*(s) \), we get

\[ DT_0^*(s) = \frac{N_2(s)}{D_1(s)}, \]

where

\[ N_2(s) = q_{02}^*(s)M_2(s) + q_{02}^*(s)q_{28}(s)M_8(s) + q_{02}^*(s)q_{29}(s)M_9(s) + q_{02}^*(s)q_{02}(s)M_{10}(s) \]
\[ + q_{02}^*(s)[q_{29}(s)q_{10,10}(s) + q_{12,10}(s)]M_{10}(s) \]

and \( D_1(s) \) is as already given.

In steady state, the expected degradation time of the system \( DT_{51} \) is given by

\[ DT_{51} = \lim_{s \to 0} \left( sDT_0^*(s) \right) = \frac{N_2}{D_1}, \]

where

\[ N_2 = p_{02} \mu_2 + p_{02} p_{28} \mu_8 + p_{02} p_{29} \mu_9 + p_{02} (p_{29} + p_{2,10}) \mu_{10} \]

and \( D_1 \) is as already defined.

**Expected Congestion Time of the System**

Using the arguments of the theory of regenerative processes, the expected congestion time of the system \( CT_i(t) \), the probability that the system is in traffic congestion at instant \( t \) given that it entered regenerative state \( i \) at \( t = 0 \), satisfies the following recursive relations:

\[ CT_0(t) = q_{01}(t)CT_1(t) + q_{02}(t)CT_2(t) + q_{03}(t)CT_3(t) \]
\[ CT_1(t) = q_{14}(t)CT_4(t) + q_{15}(t)CT_5(t) + q_{16}(t)CT_6(t) + q_{17}(t)CT_7(t) \]
\[ CT_2(t) = q_{28}(t)CT_8(t) + q_{29}(t)CT_9(t) + q_{2,10}(t)CT_{10}(t) \]
\[ CT_3(t) = M_3(t) + q_{30}(t)CT_0(t) + q_{3,11}(t)CT_{11}(t) \]
\[ CT_4(t) = q_{40}(t)CT_0(t) \]
\[ CT_5(t) = q_{50}(t)CT_6(t) \]

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where

\[ M_5(t) = p_5 e^{-\delta t} + q_5 e^{-\delta t} \]; \quad M_{11}(t) = \overline{I}_1(t); \quad M_{12}(t) = \overline{U}_{h_1}(t); \]

\[ M_{13}(t) = \overline{U}_{h_2}(t) \]

Using L.T., the above recursive relations are solved in terms of \( CT_0^*(s) \), we get

\[ CT_0^*(s) = \frac{N_3(s)}{D_1(s)}, \]

where

\[ N_3(s) = q_{03}^* (s) M_3^*(s) + q_{03}^* (s) q_{3,11}^* (s) M_{11}^*(s) + q_{03}^* (s) q_{3,12}^* (s) M_{12}^*(s) + q_{03}^* (s) q_{3,13}^* (s) q_{11,13}^* (s) M_{13}^*(s) \]

and \( D_1(s) \) is as already given.

In steady state, the expected congestion time of the system (\( CT_{51} \)) is given by

\[ CT_{51} = \lim_{s \to 0} (sCT_0^*(s)) = \frac{N_3}{D_1}, \]

where

\[ N_3 = p_{03} \mu_3 + p_{03} p_{3,11} \mu_{11} + p_{03} p_{3,12} \mu_{12} + p_{03} p_{3,11} p_{11,12} \mu_{13} \]

and \( D_1 \) is as already defined.

**Busy Period of Repair Team (Inspection Time Only)**

Using arguments for regenerative process, the following recursive relations are obtained:

\[ BI_0(t) = q_{01}(t) \oplus BI_1(t) + q_{02}(t) \oplus BI_2(t) + q_{03}(t) \oplus BI_3(t) \]

\[ BI_1(t) = W_1(t) + q_{14}(t) \oplus BI_4(t) + q_{15}(t) \oplus BI_5(t) + q_{16}(t) \oplus BI_6(t) + q_{17}(t) \oplus BI_7(t) \]

\[ BI_2(t) = W_2(t) + q_{28}(t) \oplus BI_8(t) + q_{29}(t) \oplus BI_9(t) + q_{2,10}(t) \oplus BI_{10}(t) \]
\[ BI_1(t) = q_{30}(t) BI_0(t) + q_{3,1}(t) BI_1(t) \]
\[ BI_4(t) = q_{40}(t) BI_0(t) \]
\[ BI_5(t) = q_{50}(t) BI_6(t) \]
\[ BI_6(t) = q_{60}(t) BI_0(t) \]
\[ BI_7(t) = q_{70}(t) BI_6(t) \]
\[ BI_8(t) = q_{80}(t) BI_0(t) \]
\[ BI_9(t) = q_{9,10}(t) BI_{i0}(t) \]
\[ BI_{10}(t) = q_{10,0}(t) BI_0(t) \]
\[ BI_{11}(t) = W_{11}(t) + q_{1,1,2}(t) BI_{12}(t) + q_{1,1,1}(t) BI_{13}(t) \]
\[ BI_{12}(t) = q_{1,2,0}(t) BI_0(t) \]
\[ BI_{13}(t) = q_{1,3,0}(t) BI_0(t) \]

where
\[ W_i(t) = \Gamma_i(t); \quad W_2(t) = \Gamma_2(t); \quad W_{11}(t) = \Gamma_{11}(t) \]

Using L.T., the above recursive relations are solved in terms of \( BI_0'(s) \), we get
\[ BI_0'(s) = \frac{N_4(s)}{D_1(s)} \]

where
\[ N_4(s) = q^*_{01}(s) W_i^*(s) + q^*_{02}(s) W_2^*(s) + q^*_{03}(s) q_{3,1,1}(s) W_{11}^*(s) \]

and \( D_1(s) \) is as already given.

In steady-state, the total fraction of time for which the system is under inspection (\( BI_{51} \)) is given by
\[ BI_{51} = \lim_{s \to 0} s BI_0'(s) = \frac{N_4}{D_1} \]

where
\[ N_4 = p_{01} \mu_1 + p_{02} \mu_2 + p_{03} p_{3,1,1} \mu_{11} \quad \text{and} \quad D_1 \text{ is as already specified.} \]

**Busy Period of Repair Team (Repair Time Only)**

Using arguments for regenerative process, the following recursive relations are obtained:
\[ BR_0(t) = q_{01}(t) BR_1(t) + q_{0,2}(t) BR_2(t) + q_{03}(t) BR_3(t) \]
\[ BR_4(t) = q_{4,1}(t) BR_4(t) + q_{1,5}(t) BR_5(t) + q_{1,6}(t) BR_6(t) + q_{1,7}(t) BR_7(t) \]
\[ BR_2(t) = q_{20}(t)BR_4(t) + q_{21}(t)BR_5(t) + q_{2,10}(t)BR_{10}(t) \]
\[ BR_3(t) = q_{30}(t)BR_4(t) + q_{311}(t)BR_{11}(t) \]
\[ BR_4(t) = W_4(t) + q_{40}(t)BR_9(t) \]
\[ BR_5(t) = W_5(t) + q_{50}(t)BR_9(t) \]
\[ BR_6(t) = W_6(t) + q_{60}(t)BR_9(t) \]
\[ BR_7(t) = W_7(t) + q_{70}(t)BR_9(t) \]
\[ BR_8(t) = W_8(t) + q_{80}(t)BR_9(t) \]
\[ BR_9(t) = W_9(t) + q_{9,10}(t)BR_{10}(t) \]
\[ BR_{10}(t) = W_{10}(t) + q_{10,0}(t)BR_9(t) \]
\[ BR_{11}(t) = q_{11,12}(t)BR_{12}(t) + q_{11,13}(t)BR_{13}(t) \]
\[ BR_{12}(t) = q_{12,0}(t)BR_9(t) \]
\[ BR_{13}(t) = q_{13,0}(t)BR_9(t) \]

where
\[ W_4(t) = \bar{G}_{b_1}(t); \quad W_5(t) = \bar{G}_{b_1}(t); \quad W_6(t) = \bar{G}_{b_1}(t); \]
\[ W_7(t) = G_{c_1}(t); \quad W_8(t) = \bar{G}_{b_1}(t); \quad W_9(t) = \bar{G}_{b_1}(t); \]
\[ W_{10}(t) = \bar{G}_{s_2}(t) \]

Using L.T., the above recursive relations are solved in terms of \( BR_9^*(s) \), we get
\[ BR_9^*(s) = \frac{N_5(s)}{D_1(s)}, \]

where
\[ N_5(s) = q_{01}(s)q_{14}(s)W_4^*(s) + q_{01}(s)q_{15}(s)W_5^*(s) + q_{01}(s)[q_{15}(s)q_{50}(s) + q_{10}(s)q_{15}(s)W_6^*(s) + q_{10}(s)q_{15}(s)W_7^*(s) + q_{10}(s)q_{15}(s)W_8^*(s) + q_{10}(s)q_{15}(s)W_9^*(s) + q_{28}(s)W_{10}^*(s) + q_{28}(s)W_{10}^*(s) + q_{28}(s)W_{10}^*(s) + q_{28}(s)W_{10}^*(s) + q_{28}(s)W_{10}^*(s)]W_{10}^*(s) \]

and
\[ D_1(s) \text{ is as already given.} \]

In steady-state, the total fraction of time for which the system is under repair (\( BR_{51} \)) is given by
\[ BR_{51} = \lim_{s \to 0} (sBR_9^*(s)) = \frac{N_5}{D_1}, \]
where
\[
N_5 = p_{01} p_{14} \mu_4 + p_{01} p_{15} \mu_5 + (p_{01} p_{15} + p_{01} p_{16}) \mu_6 + p_{01} p_{17} \mu_7 + p_{02} p_{28} \mu_8 + p_{02} p_{29} \mu_9 + (p_{02} p_{29} + p_{02} p_{210}) \mu_{10}
\]
and \( D_1 \) is as already specified.

**Expected Number of Hardware Expansions**

Using arguments for regenerative process, the following recursive relations are obtained:

\[
\begin{align*}
HE_0(t) &= Q_{01}(t) \times HE_1(t) + Q_{02}(t) \times HE_2(t) + Q_{03}(t) \times HE_3(t) \\
HE_1(t) &= Q_{14}(t) \times HE_4(t) + Q_{15}(t) \times HE_5(t) + Q_{16}(t) \times HE_6(t) \\
& \quad + Q_{17}(t) \times HE_7(t) \\
HE_2(t) &= Q_{28}(t) \times HE_8(t) + Q_{29}(t) \times HE_9(t) + Q_{2,10}(t) \times HE_{10}(t) \\
HE_3(t) &= Q_{30}(t) \times HE_9(t) + Q_{3,11}(t) \times HE_{11}(t) \\
HE_4(t) &= Q_{40}(t) \times HE_9(t) \\
HE_5(t) &= Q_{56}(t) \times HE_9(t) \\
HE_6(t) &= Q_{60}(t) \times HE_9(t) \\
HE_7(t) &= Q_{70}(t) \times HE_9(t) \\
HE_8(t) &= Q_{80}(t) \times HE_9(t) \\
HE_9(t) &= Q_{9,10}(t) \times HE_{10}(t) \\
HE_{10}(t) &= Q_{10,0}(t) \times HE_9(t) \\
HE_{11}(t) &= Q_{11,12}(t) \times HE_{12}(t) + Q_{11,13}(t) \times HE_{13}(t) \\
HE_{12}(t) &= Q_{12,0}(t) \times [1 + HE_9(t)] \\
HE_{13}(t) &= Q_{13,0}(t) \times HE_9(t)
\end{align*}
\]

Using L.S.T., the above recursive relations are solved in terms of \( HE_0^*(s) \), we get

\[
HE_0^*(s) = \frac{N_7(s)}{D_1(s)},
\]

where
\[
N_7(s) = Q_{01}(s)Q_{3,11}(s)Q_{11,12}(s)Q_{12,0}(s)Q_{13,0}(s) \quad \text{and} \quad D_1(s) \text{ is already given.}
\]
In steady-state, the expected number of hardware expansions of the system ($HE_{51}$) is given by:

$$HE_{51} = \lim_{s \to 0} (sHE_{0}^{**}(s)) = \frac{N_7}{D_1},$$

where

$$N_7 = p_{03} p_{11,11} p_{11,12}, \quad \text{and} \quad D_1 \text{ is as already specified.}$$

**Expected Number of Software Expansions**

Using arguments for regenerative process, the following recursive relations are obtained:

$$SE_0(t) = Q_{00}(t) \circ \ SE_1(t) + Q_{01}(t) \circ \ SE_2(t) + Q_{02}(t) \circ \ SE_3(t)$$
$$SE_1(t) = Q_{10}(t) \circ \ SE_4(t) + Q_{11}(t) \circ \ SE_5(t) + Q_{12}(t) \circ \ SE_6(t)$$
$$+ Q_{13}(t) \circ \ SE_7(t)$$
$$SE_2(t) = Q_{20}(t) \circ \ SE_8(t) + Q_{21}(t) \circ \ SE_9(t) + Q_{22}(t) \circ \ SE_{10}(t)$$
$$SE_3(t) = Q_{30}(t) \circ \ SE_0(t) + Q_{31}(t) \circ \ SE_{11}(t)$$
$$SE_4(t) = Q_{40}(t) \circ \ SE_0(t)$$
$$SE_5(t) = Q_{50}(t) \circ \ SE_6(t)$$
$$SE_6(t) = Q_{60}(t) \circ \ SE_0(t)$$
$$SE_7(t) = Q_{70}(t) \circ \ SE_0(t)$$
$$SE_8(t) = Q_{80}(t) \circ \ SE_0(t)$$
$$SE_9(t) = Q_{90}(t) \circ \ SE_{10}(t)$$
$$SE_{10}(t) = Q_{10,0}(t) \circ \ SE_0(t)$$
$$SE_{11}(t) = Q_{11,0}(t) \circ \ SE_0(t) + Q_{11,1}(t) \circ \ SE_{12}(t)$$
$$SE_{12}(t) = Q_{12,0}(t) \circ \ SE_0(t)$$
$$SE_{13}(t) = Q_{13,0}(t) \circ [1 + SE_0(t)]$$

Using L.S.T., the above recursive relations are solved in terms of $SE_{0}^{**}(s)$, we get

$$SE_{0}^{**}(s) = \frac{N_0(s)}{D_1(s)},$$
where

\[ N_7(s) = Q_{03}^{**}(s)Q_{3,11}^{**}(s)Q_{11,13}^{**}(s)Q_{13,0}^{**}(s) \quad \text{and} \quad D_1(s) \text{ is already given.} \]

In steady-state, the expected number of software expansions of the system (SE51) is given by:

\[ \text{SE}_{51} = \lim_{s \to 0} \left( s \text{SE}_0^*(s) \right) = \frac{N_8}{D_1}, \]

where

\[ N_8 = p_{03} p_{3,13} p_{13,15} \quad \text{and} \quad D_1 \text{ is as already specified.} \]

**Profit Analysis**

The expected profit incurred from the system in steady state is given by

\[ P = C_0 \text{UT}_{51} + C_1 \text{DT}_{51} + C_2 \text{CT}_{51} - C_3 \text{BI}_{51} - C_4 \text{BR}_{51} - C_6 \text{HE}_{51} - C_7 \text{SE}_{51} - C \]

where

\[ C_0 = \text{revenue per unit uptime of the system} \]
\[ C_1 = \text{revenue per unit degradation time of the system} \]
\[ C_2 = \text{revenue per unit congestion time of the system} \]
\[ C_3 = \text{cost per unit time of inspection} \]
\[ C_4 = \text{cost per unit time of repair} \]
\[ C_6 = \text{cost per unit hardware expansion} \]
\[ C_7 = \text{cost per unit software expansion} \]
\[ C = \text{cost of installation of the system.} \]

**Particular Case**

For graphical analysis purpose following particular case is considered:

\[ i_1(t) = \alpha_1 e^{-\alpha_1 t}, \quad i_2(t) = \alpha_2 e^{-\alpha_2 t}, \quad i_3(t) = \alpha_3 e^{-\alpha_3 t}, \]
\[ g_{h_1}(t) = \beta_{h_1} e^{-\beta_{h_1} t}, \quad g_{h_2}(t) = \beta_{h_2} e^{-\beta_{h_2} t}, \quad g_{h_3}(t) = \beta_{h_3} e^{-\beta_{h_3} t}, \]
\[ g_{c_1}(t) = \beta_{c_1} e^{-\beta_{c_1} t}, \quad g_{c_2}(t) = \beta_{c_2} e^{-\beta_{c_2} t}, \quad g_{c_3}(t) = \beta_{c_3} e^{-\beta_{c_3} t}, \]
\[ g_{a_1}(t) = \beta_{a_1} e^{-\beta_{a_1} t}, \quad u_{h_1}(t) = \xi_{h_1} e^{-\xi_{h_1} t}, \quad u_{h_2}(t) = \xi_{h_2} e^{-\xi_{h_2} t}. \]
Then, transition probabilities and mean sojourn times are given by:

\[
\begin{align*}
p_{01} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \eta} & p_{02} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \eta} & p_{03} &= \frac{\eta}{\lambda_1 + \lambda_2 + \eta} \\
p_{14} &= a_1 & p_{15} &= c_1 & p_{16} &= b_1 \\
p_{17} &= d_1 & p_{28} &= a_2 & p_{29} &= c_2 \\
p_{2,10} &= b_2 & p_{30} &= p_1 & p_{3,11} &= q_1 \\
p_{1,1,12} &= p_2 & p_{11,13} &= q_2 & p_{30} &= p_{40} = 1 \\
p_{56} &= p_{60} = 1 & p_{70} &= p_{80} = 1 & p_{9,10} &= p_{10,0} = 1 \\
p_{12,0} &= p_{13,0} = 1 \\
\mu_0 &= \frac{1}{\lambda_1 + \lambda_2 + \eta} & \mu_i &= \frac{1}{\alpha_i} & \mu_2 &= \frac{1}{\alpha_5} & \mu_3 &= \frac{p_l + q_l}{\delta_l + \delta_2} \\
\mu_4 &= \frac{1}{\beta_{b_1}} & \mu_5 &= \frac{1}{\beta_{b_1}} & \mu_6 &= \frac{1}{\beta_{c_1}} & \mu_7 &= \frac{1}{\beta_{c_1}} \\
\mu_8 &= \frac{1}{\beta_{b_2}} & \mu_9 &= \frac{1}{\beta_{b_2}} & \mu_{10} &= \frac{1}{\beta_{s_2}} & \mu_{11} &= \frac{1}{\alpha_3} \\
\mu_{12} &= \frac{1}{\xi_{b_1}} & \mu_{13} &= \frac{1}{\xi_{c_l}}
\end{align*}
\]

Using the above particular case and the estimated values of the various parameters from the collected data, as given in Chapter 2, the values of the measures of the system performance are obtained as under:

- Mean time to system failure ($T_{51}$) = 689.80
- Expected up time of the system ($UT_{51}$) = 0.7271
- Expected degradation time of the system ($DT_{51}$) = 0.006
- Expected congestion time of the system ($CT_{51}$) = 0.6264
- Busy period of a repair team (Inspection time only) ($BI_{51}$) = 0.1244
- Busy period of a repair team (Repair time only) ($BR_{51}$) = 0.0046
- Expected number of hardware expansions ($HE_{51}$) = 0.0182
- Expected number of software expansions ($SE_{51}$) = 0.0727
**Graphical Interpretation**

Various graphs for measures of system performance viz. MTSF, expected uptime, expected degradation time, expected congestion time and profit are plotted for different values of rates of faults \((\lambda_1, \lambda_2)\), probabilities of hardware/ software/hardware based software faults and common cause failure \((a_1,a_2,b_1,b_2,c_1,c_2,d_1)\), probabilities of automatic/ manual network restoration \((p_1, q_1)\), probabilities of hardware/ software expansion of the system \((p_2,q_2)\), inspection rates \((a_1,a_2,a_3)\), hardware/ software/ hardware based software repair rates \((\beta_{b_1}, \beta_{b_2}, \beta_{s_1}, \beta_{s_2}, \beta_{h_1}, \beta_{h_2}, \beta_{a_1}, \beta_{a_2}, \beta_{a_3})\), common cause repair rates \((\beta_{c_{f}})\), traffic congestion, automatic and manual network restoration rates \((\eta, \delta_1, \delta_2)\), rates of hardware/ software expansions \((\xi_{h_1}, \xi_{s_1})\).

**Fig. 5.2** gives the graph between MTSF \((T_{51})\) and rate of major fault \((\lambda_1)\) for different values of probability of minor hardware fault \((a_2)\). The graph reveals that MTSF decreases with increase in the values of the rate of major faults. Further it can be observed that MTSF has lower values for higher values of the probability of minor hardware faults.

**Fig. 5.3** shows the graph between expected uptime of the system \((UT_{51})\) and rate of minor faults \((\lambda_2)\) for different values of rate of major faults \((\lambda_1)\). The graph reveals that expected uptime of the system decreases with increase in values of the
rate of minor faults and has smaller values for higher values of the rate of major faults.

**Fig. 5.3**

**Fig. 5.4** presents the graph between expected degradation time (DT$_{51}$) and rate of minor faults ($\lambda_2$) for different values of probability of minor software faults ($b_2$). The graph reveals that expected degradation time increases with increase in values of the rate of minor faults. Further it can be observed that the expected degradation time has higher values for higher values of the probability of minor software faults.

![Expected Uptime versus Rate of Minor Faults for Different Values of Rate of Major Faults](image)

**EXPECTED UPTIME VERSUS RATE OF MINOR FAULTS FOR DIFFERENT VALUES OF RATE OF MAJOR FAULTS**

$\delta_1=1, \delta_2=1, a_1=0.78, a_2=0.79, a_3=0.76, b_1=-0.54, b_2=0.98, c_1=0.57, c_2=0.83, b_3=0.52, d_1=0.39, d_2=0.3, d_3=0.88, n=0.5, a_4=0.71, c_4=0.06, b_4=0.11, d_4=0.12, c_5=0.09, b_5=0.16, a_6=0.71, p_1=0.75, p_2=0.2$

**Fig. 5.3**

**EXPECTED DEGRADATION TIME VERSUS RATE OF MINOR FAULTS FOR DIFFERENT VALUES OF PROBABILITY OF MINOR S/W FAULTS**

$\delta_1=1, \delta_2=1, a_1=0.78, a_2=0.79, a_3=0.76, b_1=-0.54, b_2=0.98, c_1=0.57, c_2=0.83, b_3=0.52, b_4=0.39, b_5=0.3, b_6=0.88, n=0.5, a_7=0.71, c_7=0.06, b_8=0.11, d_9=0.12, \lambda_1=0.0017, c_2=0.09, b_2=0.16, a_2=0.71, p_1=0.75, p_2=0.2$

**Fig. 5.4**
Fig. 5.5 gives the graph between expected congestion time (CT_{51}) of the system and network traffic congestion rate (\eta) for different values of probability of automatic network restoration (p_1). The graph indicates that expected congestion time increases with increase in values of the network traffic congestion rate and has lower values for higher values of the probability of automatic network restoration.

![EXPECTED CONGESTION TIME VERSUS TRAFFIC CONGESTION RATE FOR DIFFERENT VALUES OF PROBABILITY OF AUTOMATIC RESTORATION](image)

\[ \delta_1=1, \delta_2=1, a_1=0.78, a_2=0.72, a_3=0.76, b_1=0.54, b_2=0.98, b_3=0.57, \beta_1=0.83, b_{\theta_1}=0.52, b_{\theta_2}=0.39, b_{\theta_3}=0.3, \xi_1=0.88, \xi_2=0.5, a_1=0.06, b_1=0.11, d_1=0.12, \lambda_1=0.0017, \lambda_2=0.0032, c_1=0.09, b_2=0.16, a_2=0.71, p_1=0.2 \]

Fig. 5.5

Fig. 5.6 gives the graph between expected congestion time (CT_{51}) and manual network restoration rate (\delta_2) of the system for different values of probability of manual restoration.

![EXPECTED CONGESTION TIME VERSUS MANUAL RESTORATION RATE FOR DIFFERENT VALUES OF PROBABILITY OF MANUAL RESTORATION](image)

\[ \delta_1=1, a_1=0.78, a_2=0.72, a_3=0.76, b_1=0.54, b_2=0.98, b_3=0.57, \beta_1=0.83, b_{\theta_1}=0.52, b_{\theta_2}=0.39, b_{\theta_3}=0.3, \xi_1=0.88, \xi_2=0.5, a_1=0.06, b_1=0.11, d_1=0.12, \lambda_1=0.0017, \lambda_2=0.0032, c_1=0.09, b_2=0.16, a_2=0.71, p_1=0.2 \]

Fig. 5.6

Fig. 5.6 gives the graph between expected congestion time (CT_{51}) and manual network restoration rate (\delta_2) of the system for different values of probability of manual restoration.
manual network restoration \((q_1)\). The graph indicates that expected congestion time decreases with increase in values of the manual network restoration rate and has higher values for higher values of the probability of manual network restoration.

**Fig. 5.7** shows the graph between expected congestion time \((CT_{51})\) of the system and rate of hardware expansion \(\xi_{h1}\) for different values of probability of hardware expansion \((p_2)\) of the system. It can be concluded from the graph that the expected congestion time decreases with increase in values of the rate of hardware expansion and has higher values for higher values of the probability of hardware expansion.

![Expected Congestion Time Versus Rate of H/W Expansion for Different Values of Probability of H/W Expansion](image)

**Fig. 5.7**

The graph in the **Fig. 5.8** shows the pattern of profit \((P_{51})\) with respect to the rate of major faults \((\lambda_1)\) for different values of cost per unit hardware expansion of the system \((C_6)\). From the graph following is concluded:

(i) The profit of the system decreases with the increase in the values of rate of major faults and has smaller values for higher values of the cost per unit hardware expansion of the system.

(ii) For \(C_6 = \text{Rs.} \ 500\), the profit is positive or zero or negative according as \(\lambda_1 < \text{or} = \text{or} > 0.424\). Thus, in this case, the system is profitable whenever \(\lambda_1\) is less than 0.424.
(iii) For \( C_6 = \) Rs. 2500, the profit is positive or zero or negative according as \( \lambda_1 \) is \(<\) or \(=\) or \(>\) 0.394. Thus, in this case, the system is profitable whenever \( \lambda_1 \) is less than 0.394.

(iv) For \( C_6 = \) Rs. 4500, the profit is positive or zero or negative according as \( \lambda_1 \) is \(<\) or \(=\) or \(>\) 0.424. Thus, in this case, the system is profitable whenever \( \lambda_1 \) is less than 0.364.

**Fig. 5.8**

The graph in the fig. 5.9 reveals the behaviour of the profit (\( P_{S1} \)) with respect to the revenue per unit uptime of the system (\( C_0 \)) for different values of rate of hardware expansion of the system (\( \xi_{h(i)} \)). From the graph, the following conclusions are made:

(i) The profit of the system increases with the increase in values of the revenue per unit uptime of the system and has higher values for higher values of rate of hardware expansion of the system.

(ii) For \( \xi_{h(i)} = 0.3 \), the profit is positive or zero or negative according as \( C_0 \) is \(>\) or \(=\) or \(>\) Rs. 1150.07 and hence, in this case, for the system to be profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 1150.07.

(iii) For \( \xi_{h(i)} = 0.5 \), the profit is positive or zero or negative according as \( C_0 \) is \(>\) or \(=\) or \(>\) Rs. 1118.40 and hence, in this case, for the system to be...
profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 1118.40.

(iv) For $\xi_h = 0.7$, the profit is positive or zero or negative according as $C_0$ is $> =$ or $> Rs. \ 1104.83$ and hence, in this case, for the system to be profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 1104.83.

Fig. 5.9

Fig. 5.10 depicts the behavior of the profit ($P_{51}$) with respect to the network traffic congestion rate ($\eta$) for different values of probability of manual network restoration ($q_1$). Following is concluded from the graph:

(i) The profit of the system decreases with the increase in the values of network traffic congestion rate and has higher values for higher values of the probability of manual network restoration.

(ii) For $q_1 = 0.5$, the profit is positive or zero or negative according as $\eta$ is $< = > 4.13$. Hence, in this case, the system is profitable whenever $\eta$ is less than 4.13.

(iii) For $q_1 = 0.7$, the profit is positive or zero or negative according as $\eta$ is $< = > 1.98$. Hence, in this case, the system is profitable whenever $\eta$ is less than 1.98.
(iv) For $q_1 = 0.9$, the profit is positive or zero or negative according as
$\eta$ is $<$ or $=$ or $>$ 1.32. Hence, in this case, the system is profitable whenever $\eta$ is less than 1.32.

**Fig. 5.10**

The graph in the **Fig. 5.11** shows the pattern of profit ($P_{s1}$) with respect to the
cost per unit software expansion ($C_7$) for different values of rate of software expansion ($\xi_{s1}$) of the system. From the graph following conclusions are drawn:

(i) The profit of the system decreases with the increase in the values of the cost per unit software expansion and has higher values for higher values of the rate of software expansion of the system.

(ii) For $\xi_{s1} = 0.4$, the profit is positive or zero or negative according as $C_7$ is $<$ or $=$ or $>$ Rs. 2634.11. Thus, in this case, the system is profitable whenever $C_7$ is less than Rs. 2634.11.

(iii) For $\xi_{s1} = 0.6$, the profit is positive or zero or negative according as $C_7$ is $<$ or $=$ or $>$ Rs. 2800.79. Thus, in this case, the system is profitable whenever $C_7$ is less than Rs. 2800.79.

(iv) For $\xi_{s1} = 0.8$, the profit is positive or zero or negative according as $C_7$ is $<$ or $=$ or $>$ Rs. 2884.12. Thus, in this case, the system is profitable whenever $C_7$ is less than Rs. 2884.12.
The transition probabilities are regenerative point, i.e. all the states are regenerative states.

The epochs of entry into state 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 are regenerative point, i.e. all the states are regenerative states.

**Transition Probabilities and Mean Sojourn Times**

The transition probabilities are

\[dQ_{01}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2+\tau) t} dt\]
\[dQ_{12}(t) = a_1 i_1(t) dt\]
\[dQ_{13}(t) = d_1 i_1(t) dt\]
\[dQ_{2,10}(t) = b_2 i_2(t) dt\]
\[dQ_{40}(t) = e^{-\lambda_1} g_{b_1}(t) dt\]
\[dQ_{50}(t) = e^{-\lambda_1} g_{b_0}(t) dt\]
\[dQ_{80}(t) = e^{-\lambda_1} g_{b_2}(t) dt\]
\[dQ_{10,0}(t) = e^{-\lambda_1} g_{s_2}(t) dt\]
\[dQ_{12,0}(t) = h_{b_1}(t) dt\]
\[dQ_{14,0}(t) = u_{b_1}(t) dt\]

**Model-II**

A transition diagram showing the various states of transition is shown as Fig. 5.12. The epochs of entry into state 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 are regenerative point, i.e. all the states are regenerative states.
Fig. 5.12 State Transition Diagram
The non-zero elements $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$ are obtained as under:

$$
p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \eta} \quad p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \eta} \quad p_{03} = \frac{\eta}{\lambda_1 + \lambda_2 + \eta}
$$

$$
p_{14} = a_{i_1}^*(0) \quad p_{15} = c_{i_1}^*(0) \quad p_{16} = b_{i_1}^*(0)
$$

$$
p_{17} = d_{i_1}^*(0) \quad p_{28} = a_{i_2}^*(0) \quad p_{29} = c_{i_2}^*(0)
$$

$$
p_{2,10} = b_{i_2}^*(0) \quad p_{30} = p_1 \quad p_{3,13} = q_1
$$

$$
p_{40} = g_{h_1}^*(\lambda_3) \quad p_{45} = 1 - g_{h_1}^*(\lambda_3) \quad p_{56} = g_{h_2}^*(0)
$$

$$
p_{60} = g_{h_2}^*(\lambda_3) \quad p_{6,11} = 1 - g_{h_2}^*(\lambda_3) \quad p_{70} = g_{c_1}^*(0)
$$

$$
p_{80} = g_{h_2}^*(\lambda_4) \quad p_{89} = 1 - g_{h_2}^*(\lambda_4) \quad p_{9,10} = g_{h_1}^*(0)
$$

$$
p_{10,0} = g_{h_3}^*(\lambda_4) \quad p_{10,12} = 1 - g_{h_3}^*(\lambda_4) \quad p_{11,0} = h_{i_1}^*(0)
$$

$$
p_{12,0} = h_{h_4}^*(0) \quad p_{13,14} = p_{12,15}^* (0) \quad p_{13,15} = q_{2,15}^*(0)
$$

$$
p_{14,0} = u_{h_5}^*(0) \quad p_{15,0} = u_{h_5}^*(0)
$$

By these transition probabilities, it can be verified that

$$
p_{01} + p_{02} + p_{03} + p_{14} + p_{15} + p_{16} + p_{17} + p_{28} + p_{29} + p_{2,10} = p_{30} + p_{3,13} = 1
$$

$$
p_{40} + p_{45} = p_{60} + p_{6,11} = p_{80} + p_{89} = p_{10,0} + p_{10,12} = p_{13,14} + p_{13,15} = 1
$$

$$
p_{56} = p_{70} = p_{9,10} = p_{11,0} = p_{12,0} = p_{14,0} = p_{15,0} = 1
$$

The mean sojourn time ($\mu_i$) in the regenerative state $i$ is defined as the time of stay in that state before transition to any other state. If $T$ denotes the sojourn time in regenerative state $i$, then

$$
\mu_i = \int_0^\infty P(T > t)dt
$$

gives

$$
\mu_0 = \frac{1}{\lambda_1 + \lambda_2 + \eta} \quad \mu_1 = -i_1^*(0) \quad \mu_2 = -i_2^*(0)
$$

$$
\mu_3 = \frac{p_h}{\delta_1} + \frac{q_1}{\delta_2} \quad \mu_4 = \frac{1}{\lambda_3} (1 - g_{h_1}^*(\lambda_3)) \quad \mu_5 = -g_{h_1}^*(0)
$$

$$
\mu_6 = \frac{1}{\lambda_5} (1 - g_{h_1}^*(\lambda_5)) \quad \mu_7 = -g_{c_1}^*(0) \quad \mu_8 = \frac{1}{\lambda_4} (1 - g_{h_2}^*(\lambda_4))
$$

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\[ \mu_0 = -\mathbb{E}_{h_0}(0) \quad \mu_{10} = \frac{1}{h_0}(1 - \mathbb{E}_{h_2}(\lambda_0)) \quad \mu_{11} = -h_{h_1}(0) \]

\[ \mu_{12} = -h_{h_2}(0) \quad \mu_{13} = -i_{h_1}(0) \quad \mu_{14} = -u_{h_2}(0) \]

\[ \mu_{15} = -u_{h_3}(0) \]

The unconditional mean time taken by the system to transit for any regenerative state \( j \), when it is counted from epoch of entrance into that state \( i \), is mathematically stated as

\[ m_{ij} = \int_0^\infty t q_{ij}(t) dt = q_{ij}^s(s) \]

Thus,

\[ m_{01} + m_{02} + m_{03} = \mu_0 \quad m_{14} + m_{15} + m_{16} + m_{17} = \mu_1 \]

\[ m_{28} + m_{29} + m_{2,10} = \mu_2 \quad m_{30} + m_{3,13} = \mu_3 \]

\[ m_{40} + m_{45} = \mu_4 \quad m_{56} = \mu_5 \]

\[ m_{60} + m_{6,11} = \mu_6 \quad m_{70} = \mu_7 \]

\[ m_{80} + m_{89} = \mu_8 \quad m_{9,10} = \mu_9 \]

\[ m_{10,0} + m_{10,12} = \mu_{10} \quad m_{11,0} = \mu_{11} \]

\[ m_{12,0} = \mu_{12} \quad m_{13,14} + m_{13,15} = \mu_{13} \]

\[ m_{14,0} = \mu_{14} \quad m_{15,0} = \mu_{15} \]

**Mean Time to System Failure**

To determine the MTSF of the system, the failed states of the system are taken as absorbing states. By probabilistic arguments, the following recursive relations for \( \phi_i(t) \), c.d.f of the first passage time from regenerative state \( i \) to failed state are obtained:

\[ \phi_{i0}(t) = Q_{0i}(t) + Q_{02}(t) \cdot \phi_2(t) + Q_{03}(t) \cdot \phi_3(t) \]

\[ \phi_{i1}(t) = Q_{28}(t) \cdot \phi_8(t) + Q_{29}(t) \cdot \phi_9(t) + Q_{2,10}(t) \cdot \phi_{10}(t) \]

\[ \phi_{i2}(t) = Q_{30}(t) \cdot \phi_6(t) + Q_{3,13}(t) \cdot \phi_{13}(t) \]

\[ \phi_{i3}(t) = Q_{80}(t) \cdot \phi_6(t) + Q_{89}(t) \cdot \phi_9(t) \]
\[ \phi_1(t) = Q_{9,10}(t) \cdot \phi_{10}(t) \]
\[ \phi_{10}(t) = Q_{10,0}(t) \cdot \phi_0(t) + Q_{10,12}(t) \cdot \phi_{12}(t) \]
\[ \phi_{12}(t) = Q_{12,0}(t) \cdot \phi_0(t) \]
\[ \phi_{13}(t) = Q_{13,14}(t) \cdot \phi_{14}(t) + Q_{13,15}(t) \cdot \phi_{15}(t) \]
\[ \phi_{14}(t) = Q_{14,0}(t) \cdot \phi_0(t) \]
\[ \phi_{15}(t) = Q_{15,0}(t) \cdot \phi_0(t) \]

Using L.S.T., the above recursive relations are solved in terms of \( \phi_0^*(s) \), we get

\[ \phi_0^*(s) = \frac{N(s)}{D(s)}, \]

where
\[ N(s) = Q_{01}(s) \]
and
\[ D(s) = 1 - Q_{02}^*(s)Q_{28}^*(s)Q_{89}^*(s) - Q_{02}^*(s)[Q_{28}^*(s)Q_{89}^*(s) + Q_{29}^*(s)]Q_{10,10}^*(s)Q_{10,0}^*(s) - Q_{02}^*(s)Q_{2,10}^*(s)Q_{10,0}^*(s) - Q_{02}^*(s)[Q_{28}^*(s)Q_{89}^*(s) + Q_{29}^*(s)]Q_{10,10}^*(s)Q_{10,0}^*(s) + Q_{29}^*(s)Q_{10,12}^*(s)Q_{12,0}^*(s) - Q_{02}^*(s)Q_{2,10}^*(s)Q_{10,12}^*(s)Q_{12,0}^*(s) - Q_{03}^*(s)Q_{30}^*(s) - Q_{03}^*(s)Q_{3,13}^*(s)Q_{13,14}^*(s)Q_{14,0}^*(s) + Q_{13,14}^*(s)Q_{15,0}^*(s)] \]

The mean time to system failure \( (T_{52}) \) when the system starts from the state 0, is

\[ T_{52} = \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \frac{1 - \phi_0^*(s)}{s} = \frac{N}{D}, \]

where \( R^*(s) \) is the L.T. of the reliability \( R(t) \). The reliability \( R(t) \) of the system at time \( 't' \) can be obtained by taking inverse L.T. of \( R^*(s) \).

Using L’Hospital rule and substituting the value of \( \phi_0^*(s) \), we get

\[ T_{52} = \frac{N}{D}, \]

where
\[ N = \mu_0 + p_{02} \mu_2 + p_{03} \mu_3 + p_{02} p_{28} \mu_8 + (p_{02} p_{28} p_{89} + p_{02} p_{29}) \mu_9 + (p_{02} p_{28} p_{89} + p_{02} p_{29} p_{10,12} + p_{02} p_{29} p_{10,12} + p_{02} p_{2,10} p_{10,12}) \mu_{12} + p_{03} p_{3,13} \mu_{13} + p_{03} p_{3,13} p_{13,14} \mu_{14} + p_{03} p_{3,13} p_{13,14} \mu_{15} \]
and
\[ D = p_{01}. \]
Expected Uptime of the System

Using the arguments of the theory of regenerative processes, the expected uptime of the system $U_T(t)$, the probability that the system is up at instant $t$ given that it entered regenerative state $i$ at $t = 0$, satisfies the following recursive relations:

- $U_T(0) = M_0(t) + q_{01}(t)U_T(t) + q_{02}(t)U_T(t) + q_{03}(t)U_T(t)$
- $U_T(t) = q_{14}(t)U_T(t) + q_{15}(t)U_T(t) + q_{16}(t)U_T(t) + q_{17}(t)U_T(t)$
- $U_T(t) = q_{28}(t)U_T(t) + q_{29}(t)U_T(t) + q_{2,10}(t)U_T(t)$
- $U_T(t) = q_{30}(t)U_T(t) + q_{3,13}(t)U_T(t)$
- $U_T(t) = q_{40}(t)U_T(t) + q_{45}(t)U_T(t)$
- $U_T(t) = q_{50}(t)U_T(t)$
- $U_T(t) = q_{60}(t)U_T(t) + q_{6,11}(t)U_T(t)$
- $U_T(t) = q_{70}(t)U_T(t)$
- $U_T(t) = q_{80}(t)U_T(t) + q_{89}(t)U_T(t)$
- $U_T(t) = q_{90}(t)U_T(t)$
- $U_T(t) = q_{10,0}(t)U_T(t) + q_{10,12}(t)U_T(t)$
- $U_T(t) = q_{11,0}(t)U_T(t)$
- $U_T(t) = q_{12,0}(t)U_T(t)$
- $U_T(t) = q_{13,14}(t)U_T(t) + q_{13,15}(t)U_T(t)$
- $U_T(t) = q_{14,0}(t)U_T(t)$
- $U_T(t) = q_{15,0}(t)U_T(t)$

where

- $M_0(t) = e^{-(\lambda_1 + \lambda_2 + \eta)t}$

Using L.T., the above recursive relations are solved in terms of $UT_0^*(s)$, we get

- $UT_0^*(s) = \frac{N_1(s)}{D_1(s)}$

where

- $N_1(s) = M_0^*(s)$

and

- $D_1(s) = 1 - q_{01}^*(s)q_{14}^*(s)q_{40}^*(s) - q_{01}^*(s)q_{14}^*(s)q_{45}^*(s)q_{56}^*(s)q_{60}^*(s)$
- $- q_{01}^*(s)q_{14}^*(s)q_{45}^*(s)q_{56}^*(s)q_{6,11}^*(s)q_{11,0}^*(s) - q_{01}^*(s)q_{15}^*(s)q_{56}^*(s)q_{60}^*(s)$
- $- q_{01}^*(s)q_{15}^*(s)q_{56}^*(s)q_{6,11}^*(s)q_{11,0}^*(s) - q_{01}^*(s)q_{16}^*(s)q_{60}^*(s)$
- $- q_{01}^*(s)q_{16}^*(s)q_{6,11}^*(s)q_{11,0}^*(s) - q_{01}^*(s)q_{17}^*(s)q_{70}^*(s) - q_{01}^*(s)q_{28}^*(s)q_{40}^*(s)$
In steady state, the expected uptime of the system (UT52) is given by

\[
\text{UT}_{52} = \lim_{s \to 0^+} s \text{UT}_5^*(s) = \frac{N_1}{D_1},
\]

where

\[N_1 = \mu_0\]

and

\[D_1 = \mu_0 + p_{01} \mu_1 + p_{02} \mu_2 + p_{03} \mu_3 + p_{01} p_{14} \mu_4 + p_{01} (p_{14} p_{45} + p_{15}) \mu_5 + p_{01} (p_{14} p_{45} + p_{16}) \mu_6 + p_{01} p_{17} \mu_7 + p_{02} p_{28} \mu_8 + p_{02} (p_{28} p_{89} + p_{29}) \mu_9 + p_{02} (p_{28} p_{89} + p_{29} + p_{20,10}) \mu_{10} + p_{03} p_{3,13} \mu_{11} + p_{03} p_{3,13} p_{13,14} \mu_{12} + p_{03} p_{3,13} p_{13,15} \mu_{13} + p_{03} p_{3,13} p_{13,14} p_{14,1} + p_{03} p_{3,13} p_{13,15} p_{15,1}.\]

**Expected Degradation Time of the System**

Using the arguments of the theory of regenerative processes, the expected degradation time of the system DT_i(t), the probability that the system is degraded at instant t given that it entered regenerative state i at t = 0, satisfies the following recursive relations:

\[
\begin{align*}
- q_{01}^* (s) q_{28}^* (s) & q_{89}^* (s) q_{4,0}^* (s) q_{10,0}^* (s) - q_{02}^* (s) q_{28}^* (s) q_{9,0}^* (s) q_{10,12}^* (s) q_{12,0}^* (s) \\
- q_{01}^* (s) q_{29}^* (s) & q_{9,10}^* (s) q_{10,0}^* (s) - q_{02}^* (s) q_{28}^* (s) q_{9,10}^* (s) q_{10,12}^* (s) q_{12,0}^* (s) \\
- q_{01}^* (s) q_{2,10}^* (s) & q_{10,0}^* (s) - q_{02}^* (s) q_{2,10}^* (s) q_{10,12}^* (s) q_{12,0}^* (s) - q_{03}^* (s) q_{3,10}^* (s) \\
- q_{01}^* (s) q_{3,13}^* (s) & q_{13,14}^* (s) q_{14,0}^* (s) - q_{03}^* (s) q_{3,13}^* (s) q_{13,15}^* (s) q_{15,0}^* (s)
\end{align*}
\]
DT_8(t) = M_8(t) + q_{80}(t)DT_8(t) + q_{89}(t)DT_9(t) \\
DT_9(t) = M_9(t) + q_{9,10}(t)DT_{10}(t) \\
DT_{10}(t) = M_{10}(t) + q_{10,0}(t)DT_8(t) + q_{10,12}(t)DT_{12}(t) \\
DT_{11}(t) = q_{11,0}(t)DT_0(t) \\
DT_{12}(t) = M_{12}(t) + q_{12,0}(t)DT_8(t) \\
DT_{13}(t) = q_{13,14}(t)DT_{14}(t) + q_{13,15}(t)DT_{15}(t) \\
DT_{14}(t) = q_{14,0}(t)DT_8(t) \\
DT_{15}(t) = q_{15,0}(t)DT_0(t)

where

\[ M_2(t) = \bar{I}_2(t); \quad M_8(t) = e^{-\lambda_1 t}G_{h_2}(t); \quad M_9(t) = \bar{G}_{h_4}(t); \]

\[ M_{10}(t) = e^{-\lambda_1 t}G_{h_5}(t); \quad M_{12}(t) = \bar{H}_{h_1}(t) \]

Using L.T., the above recursive relations are solved in terms of DT^\ast_0(s), we get

\[ DT^\ast_0(s) = \frac{N_2(s)}{D_1(s)}, \]

where

\[ N_2(s) = q^{\ast}_{02}(s)M^{\ast}_2(s) + q^{\ast}_{02}(s)q_{28}(s)M^{\ast}_8(s) + q^{\ast}_{02}(s)[q_{28}(s)q_{89}(s) + q_{29}(s)]M^\ast_9(s) \]

\[ + q^{\ast}_{02}(s)[q_{28}(s)q_{89}(s)q_{9,10}(s) + q_{28}(s)q_{9,10}(s) + q^{\ast}_{02}(s)]M^{\ast}_{10}(s) \]

\[ + q^{\ast}_{02}(s)[q_{28}(s)q_{89}(s)q_{9,10}(s) + q_{28}(s)q_{9,10}(s) + q^{\ast}_{2,10}(s)]q^{\ast}_{10,12}(s)M^{\ast}_{12}(s) \]

and \( D_1(s) \) is as already given.

In steady state, the expected degradation time of the system (DT52) is given by

\[ DT_{52} = \lim_{s \to 0} (sDT^\ast_0(s)) = \frac{N_2}{D_1}, \]

where

\[ N_2 = p_{02} \mu_2 + p_{02} p_{28} \mu_8 + p_{02} (p_{28} p_{89} + p_{29}) \mu_9 + p_{02} (p_{28} p_{89} + p_{29} + p_{2,10}) \mu_{10} \]

\[ + p_{02} (p_{28} p_{89} p_{10,12} + p_{29} p_{10,12} + p_{2,10} p_{10,12}) \mu_{12} \]

and \( D_1 \) is as already defined.

**Expected Congestion Time of the System**

Using the arguments of the theory of regenerative processes, the expected congestion time of the system CT_i(t), the probability that the system is in traffic
congestion at instant $t$ given that it entered regenerative state $i$ at $t = 0$, satisfies the following recursive relations:

$$
\begin{align*}
CT_0(t) &= q_{01}(t)\circ CT_1(t) + q_{02}(t)\circ CT_2(t) + q_{03}(t)\circ CT_3(t) \\
CT_1(t) &= q_{14}(t)\circ CT_4(t) + q_{15}(t)\circ CT_5(t) + q_{16}(t)\circ CT_6(t) + q_{17}(t)\circ CT_7(t) \\
CT_2(t) &= q_{28}(t)\circ CT_8(t) + q_{29}(t)\circ CT_9(t) + q_{2,10}(t)\circ CT_{10}(t) \\
CT_3(t) &= M_3(t) + q_{30}(t)\circ CT_9(t) + q_{3,13}(t)\circ CT_{13}(t) \\
CT_4(t) &= q_{40}(t)\circ CT_0(t) + q_{45}(t)\circ CT_5(t) \\
CT_5(t) &= q_{56}(t)\circ CT_6(t) \\
CT_6(t) &= q_{60}(t)\circ CT_0(t) + q_{6,11}(t)\circ CT_0(t) \\
CT_7(t) &= q_{70}(t)\circ CT_0(t) \\
CT_8(t) &= q_{80}(t)\circ CT_0(t) + q_{89}(t)\circ CT_9(t) \\
CT_9(t) &= q_{90,10}(t)\circ CT_{10}(t) \\
CT_{10}(t) &= q_{10,0}(t)\circ CT_0(t) + q_{10,12}(t)\circ CT_{12}(t) \\
CT_{11}(t) &= q_{11,0}(t)\circ CT_0(t) \\
CT_{12}(t) &= q_{12,0}(t)\circ CT_0(t) \\
CT_{13}(t) &= M_{13}(t) + q_{13,14}(t)\circ CT_{14}(t) + q_{13,15}(t)\circ CT_{15}(t) \\
CT_{14}(t) &= M_{14}(t) + q_{14,0}(t)\circ CT_0(t) \\
CT_{15}(t) &= M_{15}(t) + q_{15,0}(t)\circ CT_0(t)
\end{align*}
$$

where

$$
\begin{align*}
M_3(t) &= p_1e^{-\delta t} + q_1e^{-\beta t}; & M_{13}(t) &= \overline{I_1}(t); & M_{14}(t) &= \overline{U_{h_1}}(t); \\
M_{15}(t) &= \overline{U_{h_2}}(t)
\end{align*}
$$

Using L.T., the above recursive relations are solved in terms of $CT_0^*(s)$, we get

$$
CT_0^*(s) = \frac{N_3(s)}{D_1(s)}
$$

where

$$
N_3(s) = q_{03}^*(s)M_1^*(s) + q_{03}^*(s)q_{3,13}^*(s)M_{13}^*(s) + q_{03}^*(s)q_{3,14}^*(s)M_{14}^*(s) + q_{03}^*(s)q_{3,15}^*(s)M_{15}^*(s)
$$

and

$D_1(s)$ is as already given.
In steady state, the expected congestion time of the system \( CT_{52} \) is given by

\[
CT_{52} = \lim_{s \to 0} \left( sCT_0^* (s) \right) = \frac{N_3}{D_1},
\]

where

\[
N_3 = p_0 \mu_3 + p_0 p_{3,13} \mu_{13} + p_0 p_{3,13} p_{13,14} \mu_{14} + p_0 p_{3,13} p_{13,15} \mu_{15}
\]

and \( D_1 \) is as already defined.

**Busy Period of Repair Team (Inspection Time Only)**

Using arguments for regenerative process, the following recursive relations are obtained:

- \( B_{I_0}(t) = q_{01}(t) \cdot B_{I_1}(t) + q_{02}(t) \cdot B_{I_2}(t) + q_{03}(t) \cdot B_{I_3}(t) \)
- \( B_{I_1}(t) = W_1(t) + q_{14}(t) \cdot B_{I_4}(t) + q_{15}(t) \cdot B_{I_5}(t) + q_{16}(t) \cdot B_{I_6}(t) + q_{17}(t) \cdot B_{I_7}(t) \)
- \( B_{I_2}(t) = W_2(t) + q_{28}(t) \cdot B_{I_8}(t) + q_{29}(t) \cdot B_{I_9}(t) + q_{2,10}(t) \cdot B_{I_{10}}(t) \)
- \( B_{I_3}(t) = q_{30}(t) \cdot B_{I_0}(t) + q_{3,13}(t) \cdot B_{I_{13}}(t) \)
- \( B_{I_4}(t) = q_{40}(t) \cdot B_{I_0}(t) + q_{45}(t) \cdot B_{I_5}(t) \)
- \( B_{I_5}(t) = q_{50}(t) \cdot B_{I_6}(t) \)
- \( B_{I_6}(t) = q_{60}(t) \cdot B_{I_0}(t) + q_{6,11}(t) \cdot B_{I_{11}}(t) \)
- \( B_{I_7}(t) = q_{70}(t) \cdot B_{I_6}(t) \)
- \( B_{I_8}(t) = q_{80}(t) \cdot B_{I_0}(t) + q_{89}(t) \cdot B_{I_9}(t) \)
- \( B_{I_9}(t) = q_{9,10}(t) \cdot B_{I_{10}}(t) \)
- \( B_{I_{10}}(t) = q_{10,0}(t) \cdot B_{I_6}(t) + q_{10,12}(t) \cdot B_{I_{12}}(t) \)
- \( B_{I_{11}}(t) = q_{11,0}(t) \cdot B_{I_0}(t) \)
- \( B_{I_{12}}(t) = q_{12,0}(t) \cdot B_{I_6}(t) \)
- \( B_{I_{13}}(t) = W_{13}(t) + q_{13,14}(t) \cdot B_{I_{14}}(t) + q_{13,15}(t) \cdot B_{I_5}(t) \)
- \( B_{I_{14}}(t) = q_{14,0}(t) \cdot B_{I_0}(t) \)
- \( B_{I_{15}}(t) = q_{15,0}(t) \cdot B_{I_6}(t) \)

where

\[
W_i(t) = \overline{I}_i(t); \quad W_2(t) = \overline{I}_2(t); \quad W_{13}(t) = \overline{I}_7(t)
\]

Using L.T., the above recursive relations are solved in terms of \( B_{I_0}(s) \), we get

\[
B_{I_0}(s) = \frac{N_4(s)}{D_1(s)}.
\]
where

\[ N_4(s) = q_{01}^*(s)W_1^*(s) + q_{02}^*(s)W_2^*(s) + q_{03}(s)q_{3,13}(s)W_{13}^*(s) \]

and \( D_1(s) \) is as already given.

In steady-state, the total fraction of time for which the system is under inspection \((BI_{52})\) is given by

\[ BI_{52} = \lim_{s \to 0} \left( sB_{I0}^*(s) \right) = \frac{N_4^*}{D_1}, \]

where

\[ N_4 = p_{01} \mu_1 + p_{02} \mu_2 + p_{03} p_{3,13} \mu_{13} \]

and \( D_1 \) is as already specified.

**Busy Period of Repair Team (Repair Time Only)**

Using arguments for regenerative process, the following recursive relations are obtained:

\[
\begin{align*}
BR_0(t) &= q_{01}(t) \odot BR_1(t) + q_{02}(t) \odot BR_2(t) + q_{03}(t) \odot BR_3(t) \\
BR_1(t) &= q_{14}(t) \odot BR_4(t) + q_{15}(t) \odot BR_5(t) + q_{16}(t) \odot BR_6(t) + q_{17}(t) \odot BR_7(t) \\
BR_2(t) &= q_{28}(t) \odot BR_8(t) + q_{29}(t) \odot BR_9(t) + q_{2,10}(t) \odot BR_{10}(t) \\
BR_3(t) &= q_{30}(t) \odot BR_{10}(t) + q_{3,13}(t) \odot BR_{13}(t) \\
BR_4(t) &= W_4(t) + q_{40}(t) \odot BR_5(t) + q_{45}(t) \odot BR_5(t) \\
BR_5(t) &= W_5(t) + q_{56}(t) \odot BR_6(t) \\
BR_6(t) &= W_6(t) + q_{60}(t) \odot BR_0(t) + q_{6,11}(t) \odot BR_{11}(t) \\
BR_7(t) &= W_7(t) + q_{70}(t) \odot BR_0(t) \\
BR_8(t) &= W_8(t) + q_{80}(t) \odot BR_0(t) + q_{89}(t) \odot BR_9(t) \\
BR_9(t) &= W_9(t) + q_{9,10}(t) \odot BR_{10}(t) \\
BR_{10}(t) &= W_{10}(t) + q_{10,0}(t) \odot BR_0(t) + q_{10,12}(t) \odot BR_{12}(t) \\
BR_{11}(t) &= q_{11,0}(t) \odot BR_0(t) \\
BR_{12}(t) &= q_{12,0}(t) \odot BR_0(t) \\
BR_{13}(t) &= q_{13,14}(t) \odot BR_{14}(t) + q_{13,15}(t) \odot BR_{15}(t) \\
BR_{14}(t) &= q_{14,0}(t) \odot BR_0(t) \\
BR_{15}(t) &= q_{15,0}(t) \odot BR_0(t)
\end{align*}
\]
where
\[ W_4(t) = e^{-\lambda t} \bar{G}_{b_1}(t); \quad W_5(t) = \bar{G}_{b_1}(t); \quad W_6(t) = e^{-\lambda t} \bar{G}_{b_1}(t); \]
\[ W_7(t) = \bar{G}_{b_2}(t); \quad W_8(t) = e^{-\lambda t} \bar{G}_{b_2}(t); \quad W_9(t) = \bar{G}_{b_2}(t); \]
\[ W_{10}(t) = e^{-\lambda t} \bar{G}_{b_2}(t) \]

Using L.T., the above recursive relations are solved in terms of \( BR_0(s) \), we get

\[ BR_0'(s) = \frac{N_5(s)}{D_1(s)}, \]

where
\[ N_5(s) = q_{01}^* (s)q_{14}^* (s)W_4^*(s) + q_{01}^* (s)[q_{14}^* (s)q_{45}^* (s) + q_{15}^* (s)]W_5^*(s) + q_{01}^* (s)[q_{14}^* (s)q_{45}^* (s) + q_{15}^* (s)q_{56}^* (s) + q_{16}^* (s)]W_6^*(s) + q_{01}^* (s)[q_{17}^* (s)W_7^*(s) + q_{02}^* (s)q_{28}^* (s)W_8^*(s) + q_{02}^* (s)[q_{28}^* (s)q_{89}^* (s) + q_{29}^* (s)]W_9^*(s) + q_{02}^* (s)[q_{28}^* (s)q_{89}^* (s)q_{9,10}^* (s) + q_{29}^* (s)q_{9,10}^* (s) + q_{2,10}^* (s)]W_{10}^*(s) \]

and \( D_1(s) \) is as already given.

In steady-state, the total fraction of time for which the system is under repair \((BR_{52})\) is given by

\[ BR_{52} = \lim_{s \to 0} (sBR_0^* (s)) = \frac{N_5}{D_1}, \]

where
\[ N_5 = p_{01} p_{14} \mu_4 + (p_{01} p_{14} p_{45} + p_{01} p_{15}) \mu_5 + (p_{01} p_{14} p_{45} + p_{01} p_{15} + p_{01} p_{16}) \mu_6 + p_{01} p_{17} \mu_7 + p_{02} p_{28} \mu_8 + (p_{02} p_{28} p_{89} + p_{02} p_{29}) \mu_9 + (p_{02} p_{28} p_{89} + p_{02} p_{29} + p_{02} p_{2,10}) \mu_{10} \]

and \( D_1 \) is as already specified.

**Busy Period of Repair Team (Replacement Time Only)**

Using arguments for regenerative process, the following recursive relations are obtained:
\[ BRP_0(t) = q_{01}(t)\circ BRP_1(t) + q_{02}(t)\circ BRP_2(t) + q_{03}(t)\circ BRP_3(t) \]
\[ BRP_1(t) = q_{14}(t)\circ BRP_4(t) + q_{15}(t)\circ BRP_5(t) + q_{16}(t)\circ BRP_6(t) + q_{17}(t)\circ BRP_7(t) \]
\[ BRP_2(t) = q_{28}(t)\circ BRP_9(t) + q_{29}(t)\circ BRP_{10}(t) + q_{2,10}(t)\circ BRP_{10}(t) \]
\[ \begin{align*} 
\text{BRP}_1(t) &= q_{30}(t) \odot \text{BRP}_0(t) + q_{3,13}(t) \odot \text{BRP}_{13}(t) \\
\text{BRP}_2(t) &= q_{40}(t) \odot \text{BRP}_0(t) + q_{4,5}(t) \odot \text{BRP}_5(t) \\
\text{BRP}_3(t) &= q_{56}(t) \odot \text{BRP}_6(t) \\
\text{BRP}_4(t) &= q_{60}(t) \odot \text{BRP}_0(t) + q_{6,11}(t) \odot \text{BRP}_{11}(t) \\
\text{BRP}_5(t) &= q_{70}(t) \odot \text{BRP}_6(t) \\
\text{BRP}_6(t) &= q_{80}(t) \odot \text{BRP}_0(t) + q_{8,9}(t) \odot \text{BRP}_9(t) \\
\text{BRP}_7(t) &= q_{9,10}(t) \odot \text{BRP}_{10}(t) \\
\text{BRP}_8(t) &= q_{10,12}(t) \odot \text{BRP}_0(t) + q_{10,12}(t) \odot \text{BRP}_{12}(t) \\
\text{BRP}_{11}(t) &= W_{11}(t) + q_{11,0}(t) \odot \text{BRP}_0(t) \\
\text{BRP}_{12}(t) &= W_{12}(t) + q_{12,6}(t) \odot \text{BRP}_6(t) \\
\text{BRP}_{13}(t) &= q_{13,14}(t) \odot \text{BRP}_{14}(t) + q_{13,15}(t) \odot \text{BRP}_{15}(t) \\
\text{BRP}_{14}(t) &= q_{14,0}(t) \odot \text{BRP}_0(t) \\
\text{BRP}_{15}(t) &= q_{15,0}(t) \odot \text{BRP}_0(t)
\end{align*} \]

where

\[ W_{11}(t) = \overline{H_{b_1}}(t); \quad W_{12}(t) = \overline{H_{b_2}}(t) \]

Using L.T., the above recursive relations are solved in terms of \( \text{BRP}_0^*(s) \), we get

\[ \text{BRP}_0^*(s) = \frac{N_6(s)}{D_1(s)}, \]

where

\[ N_6(s) = q_{01}^*(s)[q_{14}^*(s)q_{45}^*(s)q_{56}^*(s)q_{6,11}^*(s) + q_{15}^*(s)q_{56}^*(s)q_{6,11}^*(s) + q_{16}^*(s)q_{6,11}^*(s)]W_{11}^*(s) + q_{02}^*(s)[q_{28}^*(s)q_{89}^*(s)q_{9,10}^*(s)q_{10,12}^*(s) + q_{29}^*(s)q_{9,10}^*(s)q_{10,12}^*(s)]W_{12}^*(s) + q_{29}^*(s)q_{9,10}^*(s)q_{10,12}^*(s) + q_{29}^*(s)q_{9,10}^*(s)q_{10,12}^*(s)]W_{12}^*(s) \]

and \( D_1(s) \) is as already given.

In steady-state, the total fraction of time for which the system is under replacement (BRR\(_{52}\)) is given by

\[ \text{BRP}_{52} = \lim_{s \to 0} \left( s \text{BRP}_0^*(s) \right) = \frac{N_6}{D_1}, \]

where

\[ N_6 = (p_{01}p_{14}p_{45}p_{6,11} + p_{01}p_{15}p_{6,11} + p_{01}p_{16}p_{6,11}) \mu_{11} + (p_{02}p_{28}p_{89}p_{10,12} + p_{02}p_{29}p_{10,12} + p_{02}p_{2,10}p_{10,12}) \mu_{12} \]

and \( D_1 \) is as already specified.
Expected Number of Hardware Expansions

Using arguments for regenerative process, the following recursive relations are obtained:

\[
\begin{align*}
\text{HE}_0(t) &= Q_{01}(t) \cdot \text{HE}_1(t) + Q_{02}(t) \cdot \text{HE}_2(t) + Q_{03}(t) \cdot \text{HE}_3(t) \\
\text{HE}_1(t) &= Q_{14}(t) \cdot \text{HE}_4(t) + Q_{15}(t) \cdot \text{HE}_5(t) + Q_{16}(t) \cdot \text{HE}_6(t) + Q_{17}(t) \cdot \text{HE}_7(t) \\
\text{HE}_2(t) &= Q_{28}(t) \cdot \text{HE}_8(t) + Q_{29}(t) \cdot \text{HE}_9(t) + Q_{2,10}(t) \cdot \text{HE}_{10}(t) \\
\text{HE}_3(t) &= Q_{30}(t) \cdot \text{HE}_9(t) + Q_{3,13}(t) \cdot \text{HE}_{13}(t) \\
\text{HE}_4(t) &= Q_{40}(t) \cdot \text{HE}_9(t) + Q_{45}(t) \cdot \text{HE}_5(t) \\
\text{HE}_5(t) &= Q_{55}(t) \cdot \text{HE}_6(t) \\
\text{HE}_6(t) &= Q_{60}(t) \cdot \text{HE}_9(t) + Q_{6,11}(t) \cdot \text{HE}_{11}(t) \\
\text{HE}_7(t) &= Q_{70}(t) \cdot \text{HE}_9(t) \\
\text{HE}_8(t) &= Q_{80}(t) \cdot \text{HE}_9(t) + Q_{89}(t) \cdot \text{HE}_9(t) \\
\text{HE}_9(t) &= Q_{9,10}(t) \cdot \text{HE}_{10}(t) \\
\text{HE}_{10}(t) &= Q_{10,0}(t) \cdot \text{HE}_9(t) + Q_{10,12}(t) \cdot \text{HE}_{12}(t) \\
\text{HE}_{11}(t) &= Q_{11,0}(t) \cdot \text{HE}_9(t) \\
\text{HE}_{12}(t) &= Q_{12,0}(t) \cdot \text{HE}_9(t) \\
\text{HE}_{13}(t) &= Q_{13,14}(t) \cdot \text{HE}_9(t) + Q_{13,15}(t) \cdot \text{HE}_{15}(t) \\
\text{HE}_{14}(t) &= Q_{14,0}(t) \cdot [1 + \text{HE}_9(t)] \\
\text{HE}_{15}(t) &= Q_{15,0}(t) \cdot \text{HE}_9(t)
\end{align*}
\]

Using L.S.T, the above recursive relations are solved in terms of \(\text{HE}_0^{**}(s)\), we get

\[
\text{HE}_0^{**}(s) = \frac{N_f(s)}{D_1(s)}
\]

where

\[
N_f(s) = Q_{03}^{**}(s)Q_{3,13}^{**}(s)Q_{13,14}^{**}(s)Q_{14,0}^{**}(s)
\]

and

\(D_1(s)\) is as already given.
In steady-state, the number of hardware expansions per unit time of the system (HE
52) is given by:

\[
\text{HE}_{52} = \lim_{s \to 0} \left( s \text{HE}^*_{0}(s) \right) = \frac{N_7}{D_1},
\]

where

\[N_7 = p_{03} p_{13,14}\]

and \(D_1\) is as already specified.

**Expected Number of Software Expansions**

Using arguments for regenerative process, the following recursive relations are obtained:

\[
\begin{align*}
\text{SE}_0(t) &= Q_{01}(t) \; \text{s: SE}_1(t) + Q_{02}(t) \; \text{s: SE}_2(t) + Q_{03}(t) \; \text{s: SE}_3(t) \\
\text{SE}_1(t) &= Q_{13}(t) \; \text{s: SE}_4(t) + Q_{15}(t) \; \text{s: SE}_5(t) + Q_{16}(t) \; \text{s: SE}_6(t) \\
&\quad + Q_{17}(t) \; \text{s: SE}_7(t) \\
\text{SE}_2(t) &= Q_{28}(t) \; \text{s: SE}_8(t) + Q_{29}(t) \; \text{s: SE}_9(t) + Q_{2,10}(t) \; \text{s: SE}_{10}(t) \\
\text{SE}_3(t) &= Q_{30}(t) \; \text{s: SE}_0(t) + Q_{3,13}(t) \; \text{s: SE}_{13}(t) \\
\text{SE}_4(t) &= Q_{40}(t) \; \text{s: SE}_0(t) + Q_{45}(t) \; \text{s: SE}_5(t) \\
\text{SE}_5(t) &= Q_{56}(t) \; \text{s: SE}_6(t) \\
\text{SE}_6(t) &= Q_{60}(t) \; \text{s: SE}_0(t) + Q_{6,11}(t) \; \text{s: SE}_1(t) \\
\text{SE}_7(t) &= Q_{70}(t) \; \text{s: SE}_0(t) \\
\text{SE}_8(t) &= Q_{80}(t) \; \text{s: SE}_0(t) + Q_{8,9}(t) \; \text{s: SE}_9(t) \\
\text{SE}_9(t) &= Q_{9,10}(t) \; \text{s: SE}_{10}(t) \\
\text{SE}_{10}(t) &= Q_{10,6}(t) \; \text{s: SE}_0(t) + Q_{10,12}(t) \; \text{s: SE}_{12}(t) \\
\text{SE}_{11}(t) &= Q_{11,0}(t) \; \text{s: SE}_0(t) \\
\text{SE}_{12}(t) &= Q_{12,0}(t) \; \text{s: SE}_0(t) \\
\text{SE}_{13}(t) &= Q_{13,14}(t) \; \text{s: SE}_{14}(t) + Q_{13,15}(t) \; \text{s: SE}_{15}(t) \\
\text{SE}_{14}(t) &= Q_{14,0}(t) \; \text{s: SE}_0(t) \\
\text{SE}_{15}(t) &= Q_{15,0}(t) \; \text{s: } [1 + \text{SE}_0(t)]
\end{align*}
\]
Using L.S.T, the above recursive relations are solved in terms of $SE_0^{**}(s)$, we get

$$SE_0^{**}(s) = \frac{N_8(s)}{D_1(s)},$$

$$N_8(s) = Q_{03}^{**}(s)Q_{3,13}^{**}(s)Q_{13,15}^{**}(s)Q_{15,0}^{**}(s),$$

and

$$D_1(s)$$ is as already given.

In steady-state, the number of software expansions per unit time of the system ($SE_{52}$) is given by:

$$SE_{52} = \lim_{s \to 0} (sSE_0^{**}(s)) = \frac{N_8}{D_1},$$

where

$$N_8 = p_{03} p_{3,13} p_{13,15}$$ and $D_1$ is as already specified.

**Profit Analysis**

The expected profit incurred from the system in steady state is given by

$$P_{52} = C_0 UT_{52} + C_1 DT_{52} + C_2 CT_{52} + C_3 BI_{52} + C_4 BR_{52} + C_5 BRP_{52} - C_6 HE_{52} - C_7 SE_{52} - C$$

where

$C_0$ = revenue per unit uptime of the system

$C_1$ = revenue per unit degradation time of the system

$C_2$ = revenue per unit congestion time of the system

$C_3$ = cost per unit time of inspection

$C_4$ = cost per unit time of repair

$C_5$ = cost per unit time of replacement

$C_6$ = cost per unit hardware expansion

$C_7$ = cost per unit software expansion

$C$ = cost of installation of the system.
**Particular Case**

For graphical analysis purpose, following particular case is considered:

\[
\begin{align*}
    i_1(t) &= \alpha_1 e^{-\alpha_1 t}; \\
    i_2(t) &= \alpha_2 e^{-\alpha_2 t}; \\
    i_3(t) &= \alpha_3 e^{-\alpha_3 t}; \\
    g_{b_1}(t) &= \beta_{b_1} e^{-\beta_{b_1} t}; \\
    g_{b_2}(t) &= \beta_{b_2} e^{-\beta_{b_2} t}; \\
    g_{b_3}(t) &= \beta_{b_3} e^{-\beta_{b_3} t}; \\
    g_{b_4}(t) &= \beta_{b_4} e^{-\beta_{b_4} t}; \\
    h_{b_1}(t) &= \gamma_{b_1} e^{-\gamma_{b_1} t}; \\
    h_{b_2}(t) &= \gamma_{b_2} e^{-\gamma_{b_2} t}; \\
    h_{b_3}(t) &= \gamma_{b_3} e^{-\gamma_{b_3} t}; \\
    u_{b_1}(t) &= \xi_{b_1} e^{-\xi_{b_1} t}; \\
    u_{b_2}(t) &= \xi_{b_2} e^{-\xi_{b_2} t};
\end{align*}
\]

Then, transition probabilities and mean sojourn times are given by:

\[
\begin{align*}
    p_{01} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \eta} \\
    p_{02} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \eta} \\
    p_{03} &= \frac{\eta}{\lambda_1 + \lambda_2 + \eta} \\
    p_{14} &= a_1 \\
    p_{15} &= c_1 \\
    p_{16} &= b_1 \\
    p_{17} &= d_1 \\
    p_{28} &= a_2 \\
    p_{29} &= c_2 \\
    p_{2,10} &= b_2 \\
    p_{30} &= p_1 \\
    p_{3,13} &= q_1 \\
    p_{40} &= \frac{\beta_{b_1}}{\lambda_3 + \beta_{b_1}} \\
    p_{45} &= \frac{\lambda_3}{\lambda_3 + \beta_{b_1}} \\
    p_{60} &= \frac{\beta_{b_3}}{\lambda_4 + \beta_{b_3}} \\
    p_{45} &= \frac{\lambda_4}{\lambda_4 + \beta_{b_3}} \\
    p_{61} &= \frac{\beta_{b_3}}{\lambda_4 + \beta_{b_3}} \\
    p_{62} &= \frac{\lambda_4}{\lambda_4 + \beta_{b_3}} \\
    p_{10,0} &= \frac{\beta_{s_2}}{\lambda_6 + \beta_{s_2}} \\
    p_{10,12} &= \frac{\lambda_6}{\lambda_6 + \beta_{s_2}} \\
    p_{13,14} &= p_2 \\
    p_{13,15} &= q_2 \\
    p_{14,0} &= p_{15,0} = 1 \\
    p_{30} &= p_{50} = p_{70} = 1 \\
    p_{9,10} &= p_{11,0} = p_{12,0} = 1 \\
    \mu_0 &= \frac{1}{\lambda_1 + \lambda_2 + \eta} \\
    \mu_1 &= \frac{1}{\alpha_1} \\
    \mu_2 &= \frac{1}{\alpha_2} \\
    \mu_3 &= \frac{\delta_1}{\delta_1 + \delta_2} \\
    \mu_4 &= \frac{1}{\lambda_3 + \beta_{b_1}} \\
    \mu_5 &= \frac{1}{\beta_{b_1}} \\
    \mu_6 &= \frac{1}{\lambda_5 + \beta_{b_1}} \\
    \mu_7 &= \frac{1}{\beta_{c_1}} \\
    \mu_8 &= \frac{1}{\beta_{c_1}}
\end{align*}
\]
Using the above particular case and the estimated values of the various parameters from the collected data, as given in Chapter 2, the values of the measures of the system performance are obtained as under:

Mean time to system failure (T<sub>52</sub>) = 839.99
Expected up time of the system (UT<sub>52</sub>) = 0.7271
Expected degradation time of the system (DT<sub>52</sub>) = 0.006
Expected congestion time of the system (CT<sub>52</sub>) = 0.006
Busy period of a repair team (Inspection time only) (BI<sub>52</sub>) = 0.1244
Busy period of a repair team (Repair time only) (BR<sub>52</sub>) = 0.0046
Busy period of a repair team (Replacement time only) (BRP<sub>52</sub>) = 5.13x10<sup>-7</sup>
Expected number of hardware expansions (HE<sub>52</sub>) = 0.0182
Expected number of software expansions (SE<sub>52</sub>) = 0.0727

**Graphical Interpretation**

Various graphs for measures of system performance viz. MTSF, expected uptime, expected degradation time, expected congestion time and profit are plotted for different values of rates of faults (λ<sub>1</sub>, λ<sub>2</sub>, λ<sub>3</sub>, λ<sub>4</sub>, λ<sub>5</sub>, λ<sub>6</sub>), probabilities of hardware/software/hardware based software/common cause failure (a<sub>1</sub>, a<sub>2</sub>, b<sub>1</sub>, b<sub>2</sub>, c<sub>1</sub>, c<sub>2</sub>, d<sub>1</sub>), probabilities of automatic/manual network restoration (p<sub>1</sub>, q<sub>1</sub>), probabilities of hardware/software expansion (p<sub>2</sub>, q<sub>2</sub>), inspection rates (α<sub>1</sub>, α<sub>2</sub>, α<sub>3</sub>), hardware/software/hardware based software/common cause repair rates (β<sub>1</sub>, β<sub>2</sub>, β<sub>3</sub>, β<sub>4</sub>, β<sub>5</sub>, β<sub>6</sub>, β<sub>7</sub>), software based hardware replacement rates (γ<sub>h1</sub>, γ<sub>h2</sub>), network traffic congestion rate (η), automatic and manual network restoration rates (δ<sub>1</sub>, δ<sub>2</sub>), rates of hardware/software expansion (ξ<sub>h1</sub>, ξ<sub>h2</sub>).
Fig. 5.13 gives the graph between MTSF ($T_{52}$) and rate of minor hardware based software faults ($\lambda_4$) for different values of rate of major faults ($\lambda_1$). The graph reveals that MTSF decreases with increase in values of the rate of minor hardware based software faults. Further it can be observed that MTSF has lower values for higher values of the rate of major faults.

![MTSF VERSUS RATE OF MINOR HBS/W FAULTS FOR DIFFERENT VALUES OF RATE OF MAJOR FAULTS](image1)

Fig. 5.13

![EXPECTED UPTIME V/S RATE OF MINOR SBH/W FAULTS FOR DIFFERENT VALUES OF RATE OF MINOR HBS/W FAULTS](image2)

Fig. 5.14 presents the graph between expected uptime of the system ($UT_{52}$) and rate of minor software based hardware faults ($\lambda_6$) for different values of rate of minor hardware based software faults ($\lambda_4$). The graph indicates that expected uptime
of the system decreases with increase in the values of rate of minor software based hardware faults and has smaller values for higher values of the rate of minor hardware based software faults.

![Graph](image)

**Fig. 5.15**

**Fig. 5.15** shows the graph between expected degradation time \( DT_{s2} \) and repair rate of minor software faults \( \beta_{s2} \) for different values of repair rate of minor hardware faults \( \beta_{h2} \).
It is concluded from the graph that expected degradation time of the system decreases with increase in the values of the repair rate of minor software faults and has lower values for higher values of the repair rate of minor hardware faults.

**Fig. 5.16** presents the graph between expected congestion time ($CT_{52}$) of the system and network traffic congestion rate ($\eta$) for different values of rate of hardware expansion ($\xi_{h}$). The graph indicates that expected congestion time increases with increase in values of the network traffic congestion rate and has smaller values for higher values of the rate of hardware expansion.

**Fig. 5.17** depicts the behaviour of the profit ($P_{52}$) with respect to the rate of minor hardware based software faults ($\lambda_4$) for different values of rate of minor software based hardware faults ($\lambda_6$).

Following is concluded from the graph:

(i) The profit incurred to the system decreases with the increase in the values of the rate of minor hardware based software and has lower values for higher values of the rate of minor software based hardware faults.

(ii) For $\lambda_6 = 0.01$, the profit is positive or zero or negative according as $\lambda_4$ is $< \text{ or } = \text{ or } > 0.00412$. Thus, in this case, the system is profitable to the company whenever $\lambda_4$ is less than 0.00412.
(iii) For $\lambda_6 = 0.014$, the profit is positive or zero or negative according as $\lambda_4$ is $< \text{or} = \text{or} > 0.00393$. Thus, in this case, the system is profitable to the company whenever $\lambda_4$ is less than 0.00393.

(iv) For $\lambda_6 = 0.018$, the profit is positive or zero or negative according as $\lambda_4$ is $< \text{or} = \text{or} > 0.00304$. Thus, in this case, the system is profitable to the company whenever $\lambda_4$ is less than 0.00304.

The curves in the fig. 5.18 show the behaviour of profit ($P_s$) with respect to the rate of minor faults ($\lambda_2$) for different values of probability of hardware expansion ($p_2$).

Following is concluded from the graph:

(i) The profit of the system decreases with the increase in values of the rate of minor faults and has lower values for higher values of the probability of hardware expansion.

(ii) For $p_2 = 0.1$, the profit is positive or zero or negative according as $\lambda_2$ is $< \text{or} = \text{or} > 3.88$. Hence, in this case, the system is profitable to the company whenever $\lambda_2$ is less than 3.88.

(iii) For $p_2 = 0.5$, the profit is positive or zero or negative according as $\lambda_2$ is $< \text{or} = \text{or} > 3.79$. Hence, in this case, the system is profitable to the company whenever $\lambda_2$ is less than 3.79.
(iv) For $p_2 = 0.9$, the profit is positive or zero or negative according as $\lambda_2$ is $< \text{or} = \text{or} > 3.71$. Hence, in this case, the system is profitable to the company whenever $\lambda_2$ is less than 3.71.

The graph in the fig. 5.19 shows the pattern of profit ($P_{52}$) with respect to the revenue per unit uptime ($C_0$) for different values of cost per unit software expansion ($C_7$) of the system.

From the graph following conclusions are drawn:

(i) The profit incurred to the system increases with the increase in the values of the revenue per unit uptime of the system and has lower values for higher values of the cost per unit software expansion of the system.

(ii) For $C_7 = Rs. 200$, the profit is positive or zero or negative according as $C_0$ is $> \text{or} = \text{or} < Rs. 453.99$ and hence, in this case, for the system to be profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 453.99.

(iii) For $C_7 = Rs. 1300$, the profit is positive or zero or negative according as $C_0$ is $> \text{or} = \text{or} < Rs. 563.99$ and hence, in this case, for the system to be profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 563.99.

![Fig. 5.19](image-url)

**PROFIT VERSUS REVENUE PER UNIT UPTIME OF THE SYSTEM FOR DIFFERENT VALUES OF COST PER UNIT S/W EXPANSION**

- $\delta_1=1, \delta_2=1, \alpha_1=0.78, \alpha_2=0.72, \alpha_3=0.76, \beta_1=0.54, \beta_2=0.98, \beta_3=0.82, \beta_4=0.57, \beta_5=0.83, \beta_6=0.52,$
- $\beta_7=0.39, \beta_8=0.3, \xi_1=0.88, \eta_1=0.5, \alpha_1=0.71, \beta_1=0.06, \beta_2=0.11, \beta_3=0.12, \alpha_2=0.75, \beta_2=0.09, \beta_3=0.161,$
- $\lambda_1=0.0017, \lambda_2=0.0032, \lambda_3=0.00036, \lambda_4=0.000412, \lambda_5=0.00018, \lambda_6=0.00039, \gamma_1=0.84, \gamma_2=0.81,$
- $\mu_1=0.75, \mu_2=0.2, \mu_3=600, \mu_4=300, \mu_5=100, \mu_6=150, \mu_7=200, \mu_8=350$
(iv) For \( C_7 = \text{Rs. 2400} \), the profit is positive or zero or negative according as \( C_0 \) is \( > \) or \( = \) or \( < \) Rs. 673.99 and hence, in this case, for the system to be profitable, the revenue per unit uptime of the system should be fixed greater than Rs. 673.99.

The graph in the **fig. 5.20** depicts the behaviour of the profit (\( P_{52} \)) with respect to the cost per unit hardware expansion of the system (\( C_6 \)) for different values of rate of hardware expansion (\( \xi_{h_1} \)).

**PROFIT VERSUS RATE OF H/W EXPANSION FOR DIFFERENT VALUES OF COST PER UNIT H/W EXPANSION**

\( \delta_1=1.5, \delta_2=1.5, a_1=0.78, a_2=0.72, a_3=0.76, b_1=0.54, b_2=0.98, \beta_0=0.82, \beta_1=0.57, \beta_2=0.83, \beta_3=0.52, \beta_t=0.39, \xi_t=0.88, \eta=0.5, a_1=0.71, c_1=0.06, b_1=0.11, d_1=0.12, a_2=0.75, c_2=0.09, b_2=0.16, \lambda_1=0.0017, \lambda_2=0.00032, \lambda_3=0.00036, \lambda_4=0.00041, \lambda_5=0.00018, \lambda_6=0.00039, \gamma_1=0.84, \gamma_2=0.81, p_1=0.75, p_2=0.2, c_1=600, c_2=300, c_3=100, c_4=150, c_5=200, c_6=225, c_7=1200 \)

**Fig. 5.20**

From the graph following conclusions are made:

(i) The profit of the system decreases with the increase in the values of the cost per unit hardware expansion and has higher values for higher values of the rate of hardware expansion.

(ii) For \( \xi_{h_1} = 0.3 \), the profit is positive or zero or negative according as \( C_6 \) is \( < \) or \( = \) or \( > \) Rs. 30090.34. Hence, in this case, the system is profitable to the company whenever \( C_6 \) is less than Rs. 30090.34.

(iii) For \( \xi_{h_1} = 0.5 \), the profit is positive or zero or negative according as \( C_6 \) is \( < \) or \( = \) or \( > \) Rs. 30357.00. Hence, in this case, the system is profitable to the company whenever \( C_6 \) is less than Rs. 30357.00.
(iv) For $\xi_{bh} = 0.7$, the profit is positive or zero or negative according as $C_6$ is $< or = or >$ Rs. 30471.28. Hence, in this case, the system is profitable to the company whenever $C_6$ is less than Rs. 30471.28.

**Fig. 5.21** reveals the behaviour of the profit ($P_{52}$) with respect to the probability of manual network restoration ($q_1$) for different values of network traffic congestion rate ($\eta$).

**PROFIT VERSUS PROBABILITY OF MANUAL RESTORATION FOR DIFFERENT VALUES OF TRAFFIC CONGESTION RATE**

\[
\begin{align*}
\delta_1=1, \delta_2=1, \alpha_1=0.78, \alpha_2=0.72, \alpha_3=0.76, \beta_1=0.98, \beta_2=0.98, \beta_3=0.83, \beta_4=0.52, \\
\beta_5=0.39, \beta_6=0.3, \beta_7=0.3, \beta_8=0.71, c_1=0.06, c_2=0.11, d_1=0.12, d_2=0.75, d_3=0.09, b_1=0.16, b_2=0.0017, \\
\lambda_{h1}=0.00035, \lambda_{s1}=0.000041, \lambda_{h2}=0.000018, \lambda_{s2}=0.00039, \lambda_{h3}=0.0002, \Lambda_{h1}=0.81, \Lambda_{s1}=0.75, p_1=0.2, \\
C_0=1200, C_1=600, C_2=300, C_3=100, C_4=150, C_5=200, C_6=350, C_7=225
\end{align*}
\]

Fig. 5.21

From the graph that following conclusions are drawn:

(i) The profit of the system decreases with the increase in values of the probability of manual network restoration and has lower values for higher values of the network traffic congestion rate.

(ii) For $\eta = 3$, the profit is positive or zero or negative according as $q_1$ is $< or = or >$ 0.515. Hence, in this case, the system is profitable whenever $q_1$ is less than 0.515.

(iii) For $\eta = 5$, the profit is positive or zero or negative according as $q_1$ is $< or = or >$ 0.426. Hence, in this case, the system is profitable whenever $q_1$ is less than 0.426.

(iv) For $\eta = 7$, the profit is positive or zero or negative according as $q_1$ is $< or = or >$ 0.387. Hence, in this case, the system is profitable whenever $q_1$ is less than 0.387.
The graph in the fig. 5.22 shows the pattern of profit \( (P_{52}) \) with respect to the revenue per unit congestion time of the system \( (C_2) \) for different values of probability of automatic network restoration \( (p_1) \).

From the graph following conclusions are made:

(i) The profit of the system increases with the increase in values of the revenue per unit congestion time and has higher values for higher values of probability of automatic network restoration.

(ii) For \( p_1 = 0.1 \), the profit is positive or zero or negative according as \( C_2 \) is > or = or < Rs. 378.61. Hence, in this case, the system is profitable to the company whenever \( C_2 \) is greater than Rs. 378.61.

(iii) For \( p_1 = 0.3 \), the profit is positive or zero or negative according as \( C_2 \) is > or = or < Rs. 325.20. Hence, in this case, the system is profitable to the company whenever \( C_2 \) is greater than Rs. 325.20.

(iv) For \( p_1 = 0.5 \), the profit is positive or zero or negative according as \( C_2 \) is > or = or < Rs. 246.54. Hence, in this case, the system is profitable to the company whenever \( C_2 \) is greater than Rs. 246.54.