CHAPTER I

GENERAL INTRODUCTION
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1.1 STATISTICAL QUALITY CONTROL

Quality control is the engineering and management activity to measure the quality characteristics of the product, to compare it with specifications or requirements, and to take appropriate remedial action whenever there is a difference between actual performance and the standard. More specifically, quality control is the regulatory process to measure actual quality performance, to compare it with standards, and to act on the difference. In other words, quality control is the operational techniques and activities which sustain a quality of product or service that will satisfy given needs. The statistical control of quality is the application of principles and techniques in all stages of design, production, maintenance and service, directed towards the economic satisfaction of demand. The statistical quality control techniques are successfully applied to maintain substantial quality, cost and productivity improvements in large industrial concerns for the last eighty years.

In any industrial process, regardless of how well designed or carefully maintained the quality of manufactured product is always subject to a certain amount of variation. Some stable system of chance causes is inherent in a particular scheme of production and inspection. The variation within this stable pattern is inevitable. The reasons for variations out side this stable pattern may be discovered and corrected. The primary objective of Statistical Quality Control is, thus, the systematic reduction of variability in quality characteristics of the products. To keep the variation under allowable limits, checks are made
at two stages. The first stage of check is offered through control chart during production, and keeps the manufacturing process stable and capable of operation such that virtually all of the items produced conform to the specification. However, control chart does not resort to screening and elimination of defective items. The second stage of check is done to ensure that lots of manufactured items ready for shipment do not contain excessively large proportion of defectives. This check is done through sampling inspection. Acceptance sampling prescribes a procedure that, if applied to a series of lots, will give a specific risk of accepting lots of given quality. Acceptance sampling, in fact, yields quality assurance. The classical work of Shewhart (1926, 31) on control chart and that of Dodge and Romig (1929, 37) on acceptance sampling plans may be considered as the beginning of Statistical Quality Control methods.

The research development in the field of statistical quality control has mainly been confined to the above two directions, namely, development of control charts (as a measure of process control) and that of acceptance sampling plans (as a measure of product control). A good deal of literature developed on classical line of Shewhart and Dodge and Romig has already appeared in the text form in a series of books by Shewhart (1931), Dudding and Jennett (1942), Smith (1947), Rice (1947), Peach (1947), Freeman et. al (1948), Kennedy (1948), Statistical Research Group (1945, 47, 48 ), Bowker and Goode (1952), Grant (1952), Burr (1953), Cowden (1957), Dodge and Romig (1959), MIL- STD-105 D (1963), Ekambaram (1963a, b), Wetherill


The development of researches in the field of Statistical Quality Control has mainly been made through two approaches: (1) the classical approach based on pioneering work of Shewhart (1926) and Dodge and Romig (1929, 37), and (2) the economic approach introduced among others by Duncan (1956) and Cowden (1957). Since then various modifications and improvements have been made by several workers in the above directions.

Graphical Evaluation and Review Technique (GERT) originally introduced for systems design and analysis has been used successfully for studying a few types of quality control problems. The study of quality control system through GERT has been the subject of considerable attention during the last four decades. A brief account of researches in quality control through GERT methods has been summarised by Shankar (1993). An introduction to the GERT methodology has been given in the next section.
1.2 BRIEF REVIEW OF GERT

Graphical Evaluation and Review Technique (GERF) initiated by Pritsker and Happ (1966), Pritsker and Whitehouse (1966) and Whitehouse and Pritsker (1969) is a new graphical procedure which combines the discipline of flowgraph theory, moment generating function and PERT (Program Evaluation and Review Technique) to obtain the solution of stochastic problems. It represents the most general network where both the graphs and functions are probabilistic. A node in a stochastic network consists of an input (receiving, contributive) function and an output (omitting, distributive) function. Three logical relations Exclusive-OR, Inclusive-OR and AND exist for the input side. Two relations deterministic and probabilistic exist for the output side depending on whether activity omitted from the node has probability of realization equal to or less than one. The input and output symbols are combined to give six possible types of nodes as described in Fig. (1.1).

An Exclusive-OR node is one which is realized if one and only one activity incident on it is realized. However, such a node is also realized if feedback activity is realized. An Inclusive-OR node is realized when any activity leading into the node causes the node to be realized. An AND node is realized if all the activity incident on it are realized. A deterministic node is one which is realized if all the branches emanating from it are taken, i.e. all the branches emanating from this node have a probability parameter equal to one. In the case of probability node, at most one branch emanating from the node is taken if the
node is realized. The detailed discussion on the node structure of the GERT has been presented by Whitehouse (1973).

GERT is developed as a technique for the analysis of stochastic networks having the following characteristics:

1. Each network consists of nodes denoting logical operation and transmittances (or directed branches).

2. A branch associated with it has a probability that the activity represented by the network will be performed.

3. Other parameters describe the activities which the branches represent. These parameters may be additive such as time and cost or multiplicative like probability and reliability. However, a moment generating function (m.g.f.) of such additive parameters convert them into multiplicative ones.

4. A realization of a network is a particular set of transmittances and nodes which describe the network for one experiment.

5. If time associated with branch is a random variable, then a realization also implies that a fixed time has been selected for each branch.

In order to include the probability and the parameter (say, time parameter), which could be treated uniformly throughout the network, the
creation of a new function is required. The duration \( t \) associated with a branch characterized by moment generating function of the form

\[
M_t(\theta) = \begin{cases} 
\int_\tau e^{\theta t} f(t) \, dt, & \text{if } t \text{ is continuous variable} \\
\sum_i e^{\theta t} f(t), & \text{if } t \text{ is discrete variable}
\end{cases}
\]

where \( f(t) \) denotes the density function associated with \( t \) and \( \theta \) is any real variable. The probability \( \phi \) that the branch is realized is multiplied by the moment generating function to yield a \( W \)-function such that

\[
W(\theta) = \phi M_t(\theta)
\]

(1.2.1)

The \( W \)-function is used to obtain information on a relationship which exists between the nodes.

Pritskar and Happ (1966) and Whitehouse (1973) have shown that a GERT network with Exclusive-OR node is equivalent to a flow graph and that \( W \)-function of GERT network is equivalent to the transmittances in flow graph. Therefore, Mason's (1953) rule used to evaluate transmittance in a flow graph is also applicable to GERT network. Once the equivalent \( W \)-function is known, the equivalent probability of the network is given by

\[
P = [W(\theta)]_{0=0} = W(0)
\]

(1.2.2)

Furthermore, the equivalent m.g.f. of the network is obtained by

\[
M_t(\theta) = W(\theta) / P
\]

(1.2.3)
And finally, mean (say, time) of realization of the network can be found as

$$E(t) = \left[ \frac{d}{d\theta} M_t(\theta) \right]_{\theta = 0} \quad (1.2.4)$$

The steps employed in applying GERT to the analysis of a stochastic network may be summarized as follows:

1. Convert a qualitative description of a system or problem to a model in network form.
2. Collect the necessary data to describe the branches of the network.
3. Determine the equivalent function or functions of the network.
4. Convert the equivalent function into two performance measures of the network as
   (a) The probability that a specific node is realized.
   (b) The moment generating function of the time associated with an equivalent network.
5. Make inferences concerning the system under study from the information obtained in step (4).

1.3 PROCESS QUALITY CONTROL

Process Quality Control had its genesis in the conviction of Shewhart (1931) that constant system of chance causes do exist in nature and that assignable causes of variation may be found and eliminated. Thus, process quality control may be thought in terms of a continuing effort to keep processes centered at their target value, while maintaining the spread at prescribed value. The control chart techniques and its various modifications have been found
useful in analyzing the process quality control problems since last seventy years. The most important aspect of process control is not the control, but the conquest, that is, control must be implemented so that the problems do not re-occur. Ott and Schilling (1990) described the process quality control in terms of following three aspects:

(1) **Process Control.** Maintaining the process on target with respect to centering and spread.

(2) **Process Capability.** Determining the inherent spread of a controlled process for establishing realistic specifications, use for comparative purposes, and so forth.

(3) **Process Change.** Implementing process modifications as a part of process improvement and troubleshooting.

Ott and Schilling (1990; pp. 173) further emphasized that *there are many ways to control a process. One way is through experience, but takes too long. Another is through intuition, but that is too risky. A third approach (all too common) is to assume the process is well-behaved and not to bother with it, but that may lead to a rude awakening. All these have their place but should be used judiciously in support of a scientific approach to achieving and maintaining statistical control through control charts.*

However, Burr (1976; pp. 26) remarked that *common sense simply is not a reliable guide for avoiding two types of errors: (a) taking action when none is desirable, and (b) missing opportunities for taking appropriate action or
learning what makes the process behaves as it does. In order to minimize the chances of making these two types of errors, we need to use the control chart and other statistical tools.

Recently, Shankar (1999) and Shankar and Sahu (1999) developed corrective action plans based on the above remarks to maintain statistical control for the production processes which are not well-behaved and/or subject to some deterioration. The above studies have specific viewpoint. It claims that the function of sampling inspection can be translated into the measure of process control beyond its original purpose. Furthermore, the above plans require no direct inspection of machines and equipment of the process. A few types of corrective action schemes for the maintenance of Markovian production processes, based on sampling inspection of the output items, have also been studied by Lave (1966, 69) and Kase and Ohta (1974, 76).

1.4 PROBLEMS UNDER CONSIDERATION

From the references quoted in the previous sections, it may be observed that the Statistical Process Control problems have mainly been studied by classical control charts techniques. Some authors [Wetherill and Brown (1991)] contrast control charts, as preventive method, with sampling inspection as a screening procedure. This distinction is not correct. Control chart can be used as a screening mechanism, and sampling inspection can be used in a preventive manner. Consequently, some authors have contributed the investigation of process control by developing corrective action models. The purpose of present investigation is to model and analyze through classical as well as GERT
methodology the properties of statistical decision procedure for the process control. This thesis is divided into seven chapters. Chapter I is introductory with brief review of GERT methodology and an outline of literature survey on statistical quality control procedure. Chapter II to VI have been devoted to the development of process control models based on corrective action plan. Chapter VII develops process control procedure where life time is the quality characteristics.

In Chapter II, we propose a corrective action plan for the production processes which are not well-behaved and/or subject to some deterioration. This chapter has been devoted to the study of three-class corrective action plans by means of classifying three categories of quality, say, good, marginal and bad. In this plan, the decision of corrective action is deferred until i sample in succession alarm a state of corrective measure. The basis of development of the plan is the differentiation of the causes of variation in quality, viz. variation due to deterioration of machines or variation produced by any potential assignable factor causing abrupt changes in product quality. The formulae for \( OC \) and other performance characteristics of the plan have been derived by applying Mason's (1953) rule on the GERT network representation of the inspection system. Poisson unity values have been tabulated to facilitate the operation and construction of the plan.

In Chapter III, a two-phase inspection process control plan has extensively been studied using different sample sizes at normal and tightened phases of inspection to achieve ultimate control over the process. The OC,
ASN and Average Length of Inspection function of the plan have been derived through GERT. In order to facilitate the operation and construction of the plan, Poisson unity values have been tabulated. Numerical examples have been included to illustrate the mathematical findings.

In Chapter IV, we propose a conditional three-class corrective action plan as a generalization of Shankar and Chandrakar’s (2004) model in which a step of corrective action is dependent on the previous performance of the process. The OC and other performance characteristics of the plan have been derived through GERT. Numerical examples have been included to illustrate the mathematical findings. Poisson unity values have been tabulated to facilitate the operation and construction of the plan. Lastly, explanation of the proposed model has been illustrated numerically.

Chapter V deals with a corrective action plan utilizing the notion of chain sampling procedure due to Dodge (1955). The process control model developed in this chapter postulates that a production process with marginal quality performance can still be considered to be under control if its previous performance show in-control state as determined by the choice of \( i \). The formulae for OC and other performance characteristics of the plan have been derived by GERT network representation of the inspection system. Poisson unity values have been tabulated to facilitate the operation and construction of the plan. Lastly, explanation of the proposed model has been illustrated numerically.
Chapter VI develops a corrective action model based on the notion of Link sampling plan due to Harishchandra and Sriveankataramana (1982). In this plan, the decision of corrective action plan is performed on the basis of inspection results of the preceding as well as succeeding samples on the production line. The formulae for OC and other performance characteristics of the plan have been derived through GERT methodology. The concept of Average Outgoing Deterioration (AOD) function as a protection characteristic of the plan has also been introduced and illustrated graphically. Poisson unity values have been tabulated to facilitate the operation and construction of the plan.

Chapter VII proposes process control models where life time is the quality characteristic. This chapter is divided into two parts. The first part deals with a life test corrective action plan for Generalized Exponential Failure model. The OC and other performance functions of the plan have been derived and illustrated through numerical examples. In order to facilitate the operation and construction of the plan, Poisson unity values have been tabulated.

The second part of this Chapter develops a cumulative sum control charts to control processes for Generalized Exponential Failure model. The Johnson's technique has been utilized to determine parameters of the V-mask. The expression for Average Run Length (ARL) has been derived and illustrated numerically.
The realization of any branch leading into the node causes the node to be realized; however one and only one of the branches leading into this node can be realized at a given time.

The realization of any branch leading into the node causes the node to be realized. The time of the realization is smallest of the completion times of activities leading into the Inclusive-Or node.

The node will be realized only if all the branches leading into the node to be realized. The time of realization is the largest of the completion times of the activities leading into the AND node.

All branches emanating from this node are taken if the node is realized.

At most one branch emanating from this node is taken if the node is realized.

Fig. (1.1): Node characteristics and Symbols of GERT Networks