Chapter 4

Convective conducting peristaltic flow of a Phan-Thien-Tanner fluid in an asymmetric channel with porous medium
4.1. INTRODUCTION

Peristaltic transport is a natural mechanism of pumping most physiological fluids induced by a progressive wave of cross-sectional area contraction or expansion along the length of the boundary of a fluid-filled distensible tube. This principle occurs in many biological and biomedical systems such as the transport of urine from kidney to the bladder, transport of spermatozoa in the ducts efferentes of the male reproductive tracts, transport of lymph in the lymphatic vessels, the movement of chyme in the gastrointestinal tract movement of ovum in the female fallopian tube, vasomotion of small blood vessels such as arterioles, venules and capillaries, and so on. Also this mechanism finds applications in blood pump machine, heart-lung machine, dialysis machine, roller and finger pumps and also noxious fluid transport in nuclear industries. The peristaltic flow of non-Newtonian fluids has gained considerable interest during the recent years because of its applications in industry and biology.


A porous medium is a material which contains a number of small pores distributed throughout the matter. There are examples of natural porous medium such as beach sand and rye bread etc. Flows through porous medium occur in filtration of fluids and seepage of water under a dam which is very important. It may be
mentioned that consideration of porosity is very much necessary to properly explain the biofluid dynamical process that occurs in different parts of the body, such as vascular beds, lungs, kidneys and tumorous vessels. The importance of consideration of Darcian drag effects in studies related to flow of blood and some other fluids through porous media has been discussed by different researchers. The physics of flow through a porous media discussed by Scheidgger (1963). Srinivas and Gayathri (2009) have studied peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. Vajravelu et al. (2011) discussed the influence of heat transfer on the peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Sreenadh et al. (2011) investigated the peristaltic transport of micropolar fluid in an asymmetric channel with permeable walls. Peristaltic transport of a Williamson fluid in asymmetric channels with permeable walls has been studied by Vajravelu et al. (2012).

Magnetohydrodynamics is the dynamics of electrically conducting fluids. It is now a well accepted that the peristaltic flows of magnetohydrodynamic fluids are important in medical sciences and bioengineering. The mutual interaction between the fluid motion and magnetic field is the essential feature of the physical situation in the MHD fluid flow problems. MHD principles are useful in the design of heat exchangers, pumps, radarsystems, power generation development of magnetic devices, cancer tumor treatment, hyperthermia and blood reduction during surgeries. It is realized that the principles of magnetohydrodynamics find extensive applications in bioengineering and medical sciences. Hence several scientists having in mind such importance extensively discussed the peristalsis with magnetic field effects. Mekheimer (2004) have studied the peristaltic flow of blood under the effect of magnetic field in a non-uniform channels. Kothandapani and Srinivas (2008) analyzed the peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Hayat and Ali (2008) investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube. The effects of slip and induced magnetic field on peristaltic flow of pseudoplastic fluid has been discussed by Noreen et al. (2011).

In the present chapter, the peristaltic transport of conducting Phan-Thien-Tanner fluid in an asymmetric channel with porous medium is investigated. The
governing equations of Phan-Thien-Tanner fluid model have been simplified and are solved by using perturbation technique. The expressions for stream function, pressure gradient and pressure rise have been obtained. The effects of various parameters on the velocity, pressure rise and trapping phenomenon are discussed through graphs.

4.2. MATHEMATICAL FORMULATION

We consider an incompressible Phan-Thien-Tanner fluid in an asymmetric channel with porous medium of width \( d_1 + d_2 \). Let \( c \) be the speed by which sinusoidal wave trains propagate along the channel walls. Consider the rectangular coordinate system \((\overline{X}, \overline{Y})\) where \( \overline{X} \) – axis and \( \overline{Y} \) – axis are taken parallel and transverse to the direction of wave propagation respectively. The wall surfaces are modeled as \((H_1 \text{ is the upper wall and } H_2 \text{ is the lower wall})\)

\[
\overline{Y} = H_1 = d_1 + a_1 \cos \left[ \frac{2\pi}{\lambda} \left( \overline{X} - \overline{c} t \right) \right], \quad \overline{Y} = H_2 = -d_2 - b_1 \cos \left[ \frac{2\pi}{\lambda} \left( \overline{X} - \overline{c} t \right) + \phi \right] \tag{4.1}
\]

where \( a_1 \) and \( b_1 \) are the amplitudes of the waves, \( \lambda \) is the wave length, \( \overline{c} \) is the time, \( \phi \) is the phase difference varying in the range \( 0 \leq \phi \leq \pi \). \( \phi = 0 \) corresponds to symmetric channel with waves are out of phase and \( \phi = \pi \) with waves are in phase, and further \( a_1, b_1, d_1, d_2 \ and \ \phi \) satisfy the condition \( a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2 \) so that walls will not intersect with each other.

The basic equations are

Continuity equation
\[
\nabla \cdot \mathbf{V} = 0 \tag{4.2}
\]

Momentum equation
\[
\rho \frac{d\mathbf{V}}{dt} = \text{div} \mathbf{T} \tag{4.3}
\]

The constitutive equations for PTT model are
\[
\mathbf{T} = -p \mathbf{I} + \mathbf{s} \tag{4.4}
\]
\[
f \left( \text{tr}(\mathbf{s}) \right) + k \mathbf{s} \cdot \mathbf{V} = 2\mu \mathbf{D} \tag{4.5}
\]
\[
\mathbf{s} \cdot \mathbf{V} = \frac{ds}{dt} - s, \mathbf{L}^* - L \cdot s \tag{4.6}
\]
\[
L = \text{grad} \cdot \mathbf{V} \tag{4.7}
\]
where \( p \) is the pressure, \( I \) is the identity tensor, \( V \) is the velocity, \( T \) is the Cauchy stress tensor, \( \mu \) is the dynamic viscosity, \( s \) is an extra-stress tensor, \( D \) is the deformation-rate tensor, \( k \) is the relaxation time, \( s^v \) denotes Oldroyd’s upper-convected derivative, \( d / dt \) the material derivative, \( tr \) is the trace and asterisk denotes the transpose.

Function \( f \) in the linearized PTT model satisfies the expression

\[
f(\text{tr}(s)) = 1 + \frac{\varepsilon k}{\mu} \text{tr}(s).
\]

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\]

We note that the PTT model reduces to an Upper Convected Maxwell (UCM) model when the extensional parameter \( \varepsilon \) is zero.

We introduce the following transformation between fixed and wave frames

\[
\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}, \quad \bar{p} = \bar{p}(\bar{X}, \bar{t})
\]

where \( (\bar{U}, \bar{V}) \) and \( (\bar{u}, \bar{v}) \) are the velocities, \( \bar{p}, \bar{p} \) are pressures in the laboratory and wave frames respectively.

Using the equation (4.9) the governing equations in the wave frame are

\[
\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]

\[
\rho \left[ \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} \right] \bar{u} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} - \frac{\mu}{k_0} (\bar{u} + c) - \sigma_e B_0^2 (\bar{u} + c)
\]

\[
\rho \left[ \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} \right] \bar{v} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{y}\bar{y}}}{\partial \bar{y}} - \frac{\mu}{k_0} \bar{v}
\]

\[
f\bar{S}_{\bar{x}\bar{x}} + k \left[ \frac{\partial \bar{S}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} - 2\frac{\partial \bar{u}}{\partial \bar{x}} \bar{S}_{\bar{x}\bar{x}} - 2\frac{\partial \bar{u}}{\partial \bar{y}} \bar{S}_{\bar{x}\bar{y}} \right] \bar{v} = 2\mu \frac{\partial \bar{u}}{\partial \bar{x}}
\]

\[
f\bar{S}_{\bar{x}\bar{y}} + k \left[ \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} - 2\frac{\partial \bar{v}}{\partial \bar{x}} \bar{S}_{\bar{x}\bar{x}} - 2\frac{\partial \bar{v}}{\partial \bar{y}} \bar{S}_{\bar{x}\bar{y}} \right] \bar{v} = 2\mu \frac{\partial \bar{v}}{\partial \bar{y}}
\]

\[
f\bar{S}_{\bar{x}\bar{y}} + k \left[ \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} \right] = 0
\]

\[
f\bar{S}_{\bar{x}\bar{y}} + k \left[ \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}} \bar{S}_{\bar{x}\bar{x}} - \frac{\partial \bar{v}}{\partial \bar{y}} \bar{S}_{\bar{x}\bar{y}} - \frac{\partial \bar{u}}{\partial \bar{x}} \bar{S}_{\bar{x}\bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} \bar{S}_{\bar{x}\bar{y}} \right] = \mu \left( \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right)
\]
\[ f = 1 + \frac{\varepsilon k}{\mu} \left( \overline{S^2} + \overline{S^2} + \overline{S^2} \right) \]  
(4.17)

\[ \rho c_p \left[ (u + c) \frac{\partial}{\partial x} + \nu \frac{\partial}{\partial y} \right] T = k_0 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T + 2\mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \]  
(4.18)

The non-dimensional quantities and the expressions for stream functions are given by

\[ We = \frac{kc}{\lambda}, x = \frac{x}{\lambda}, y = \frac{y}{\lambda}, u = \frac{u}{c}, v = \frac{v}{\delta c}, \delta = \frac{d_1}{\lambda}, p = \frac{d_1^2 p}{\mu c^2}, t = \frac{ci}{\lambda}, \]  
\( \text{Re} = \frac{\rho cd_1}{\mu}, d = \frac{d_2}{d_1}, a = \frac{a_1}{d_1}, b = \frac{h_i}{d_1}, S_y = \frac{S_y d_1^2}{\mu c}, h_i = \frac{H_i}{d_1}, = 1 + a \cos(2\pi x), \]
\( h_2 = \frac{H_2}{d_1} = -d - b \cos(2\pi x + \phi)M^2 = \frac{\sigma \varepsilon B_0 d_1^2}{c}, \sigma = \frac{d_1}{\sqrt{k}} \]  
(4.19)

\[ \theta = \frac{T - T_0}{T_f - T_0}, Pr = \frac{\mu c_p}{k_0}, Ec = \frac{c^2}{\sigma (T_f - T_0)} \text{and} u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \]

where \( We, \delta, \text{Re}, Pr, Ec \) are the Weissenberg, wave, Reynolds,Prandtl and Eckert numbers respectively, \( \sigma \) is the permeability parameter and \( M \) is the Magnetic parameter.

Using the above non-dimensional quantities and long wave length approximation the basic equations reduces to

\[ \frac{dp}{dx} = \frac{\partial S_y}{\partial y} - (\sigma^2 + M^2) \left( \frac{\partial \psi}{\partial y} + 1 \right) \]  
(4.20)

\[ \frac{\partial^2 p}{\partial y^2} = 0 \]  
(4.21)

\[ fS_{xx} = 2We \frac{\partial^2 \psi}{\partial y^2} S_{xy} \]  
(4.22)

\[ fS_{yy} = 0, fS_{zz} = 0 \]  
(4.23)

\[ fS_{xx} = -We \frac{\partial^2 \psi}{\partial y^2} S_{yy} + \frac{\partial^2 \psi}{\partial y^2} \]  
(4.24)

\[ 0 = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + E(\frac{\partial \psi}{\partial y})^2 \]  
(4.25)
Corresponding non-dimensional boundary conditions are

\[
\begin{align*}
\psi &= \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \quad \text{at} \quad y = h_1 \\
\psi &= -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \quad \text{at} \quad y = h_2 \\
\theta &= 0 \quad \text{at} \quad y = h_1 \\
\theta &= 1 \quad \text{at} \quad y = h_2
\end{align*}
\]

(4.26)

where \(F\) is mean flow rate in the wave frame.

The flux at any axial station in the fixed frame is

\[
Q = \int_{h_2}^{h_1} (u+1) dy = h_1 - h_2 + F
\]

(4.27)

The average volume flow rate over one period of the peristaltic wave is defined as

\[
\Theta = \frac{1}{T} \int_{0}^{T} Q dt = \frac{1}{T} \int_{0}^{T} (h_1 - h_2 + F) dt = F + 1 + d
\]

(4.28)

From the equation (4.23) we have \(S_{yy} = 0, \quad S_{zz} = 0\) and from equation (4.20) we get

\[
S_{xy} = y \frac{dp}{dx} + (\sigma^2 + M^2)(\psi + y)
\]

(4.28)

With the help of (4.23) and (4.24) we can write

\[
S_{xx} = 2 \text{We} S_{xy}^2
\]

(4.29)

From the equations (4.17), (4.23) and (4.29) we obtain

\[
\frac{\partial^2 \psi}{\partial y^2} = S_{yy} + 2 \epsilon \text{We}^2 S_{xy}^3
\]

(4.30)

Substituting (4.28) into (4.30) we get

\[
\frac{\partial^2 \psi}{\partial y^2} = y \frac{dp}{dx} + (\sigma^2 + M^2)(\psi + y) + 2 \epsilon \text{We}^2 \left( y \frac{dp}{dx} + (\sigma^2 + M^2)(\psi + y) \right)^3
\]

(4.31)

4.3. SOLUTION

Equation (4.31) is nonlinear, its exact solution is not possible, and hence we employ the perturbation technique to find the solution. For perturbation solution, we expand the flow quantities in a power series of the small parameter \(\text{We}^2\) as follows:
\[ \psi = \psi_0 + We^2 \phi_1 + O(We^4) \]
\[ F = F_0 + We^2 F_1 + O(We^4) \]
\[ \phi = \phi_0 + We^2 \phi_1 + O(We^4) \]
\[ \frac{dp}{dx} = \frac{dp_0}{dx} + We^2 \frac{dp_1}{dx} + O(We^4) \]
\[ \theta = \theta_0 + We^2 \theta_1 + O(We^4) \]

(4.32)

Using the above expressions in equations (4.25) - (4.31), we obtain a system of equations of different orders.

**System of order** \( We^0 \)

The governing equations and boundary conditions of the zeroth-order are

\[ \frac{\partial^2 \psi}{\partial y^2} = y \frac{dp_0}{dx} + (\sigma^2 + M^2)(\psi_0 + y) \]

(4.33)

\[ 0 = \frac{1}{Pr \frac{\partial}{\partial y} + E \left( \frac{\partial \psi_0}{\partial y} \right)^2 } \]

(4.34)

\[ \psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1 \quad \text{at} \quad y = h_1 \]
\[ \psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1 \quad \text{at} \quad y = h_2 \]

(4.35)

The stream function, axial velocity and temperature as

\[ \psi_0 = c_1 \cosh \sqrt{\sigma^2 + M^2} y + c_2 \sinh \sqrt{\sigma^2 + M^2} y - y \left( \frac{1}{(\sigma^2 + M^2)} \frac{dp_0}{dx} + 1 \right) \]

(4.36)

\[ u_0 = \sqrt{\sigma^2 + M^2} \left( c_1 \sinh \sqrt{\sigma^2 + M^2} y + c_2 \cosh \sqrt{\sigma^2 + M^2} y \right) - \frac{1}{(\sigma^2 + M^2)} \frac{dp_0}{dx} + 1 \]

(4.37)

\[ \theta_0 = -\left( \sigma^2 + M^2 \right)^2 Br \left( a_1 \cosh 2\sqrt{\sigma^2 + M^2} y + a_2 \sinh 2\sqrt{\sigma^2 + M^2} y + a_3 y^2 \right) + A_1 y + A_2 \]

(4.38)
where
\[ a_1 = \frac{c_1^2 + c_2^2}{8(\sigma^2 + M^2)}, a_2 = \frac{c_1 c_2}{4(\sigma^2 + M^2)}, a_3 = \frac{c_1^2 - c_2^2}{4}, D_1 = a_1 \left( \cosh 2\sqrt{\sigma^2 + M^2} h - \cosh 2\sqrt{\sigma^2 + M^2} h_2 \right), \]
\[ D_2 = a_2 \left( \sinh 2\sqrt{\sigma^2 + M^2} h - \sinh 2\sqrt{\sigma^2 + M^2} h_2 \right), D_3 = a_3 \left( h_1^2 - h_2^2 \right), D_4 = a_1 \cosh 2\sqrt{\sigma^2 + M^2} h, \]
\[ D_5 = a_2 \sinh 2\sqrt{\sigma^2 + M^2} h, D_6 = a_3 h_2^2, A_1 = \frac{1}{h_1 - h_2} \left[ \left( \sigma^2 + M^2 \right)^2 Br (D_4 + D_5 + D_6) - 1 \right], \]
\[ A_2 = \left( \sigma^2 + M^2 \right)^2 Br (D_4 + D_5 + D_6) - A_1 h \]

System of order \( \text{We}^2 \)

The governing equations and boundary conditions of the first-order are
\[ \frac{\partial^2 \psi_1}{\partial y^2} = y \frac{dp}{dx} + (\sigma^2 + M^2) \psi_1 + 2 \varepsilon \left( y \frac{dp_0}{dx} + (\sigma^2 + M^2) (\psi_0 + y) \right)^3 \quad (4.39) \]
\[ 0 = \frac{1}{Pr} \frac{\partial \psi_1}{\partial y} + 2 \varepsilon \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial \psi_0}{\partial y^2} \quad (4.40) \]
\[ \psi_0 = \begin{cases} \frac{F_1}{2}, \frac{\partial \psi_0}{\partial y} = 0 & \text{at } y = h_1 \\ \psi_1 = \begin{cases} \frac{F_1}{2}, \frac{\partial \psi_0}{\partial y} = 0 & \text{at } y = h_2 \end{cases} \end{cases} \quad (4.41) \]
\[ \theta_1 = 0 \quad \text{at } y = h_1 \]
\[ \theta_2 = 0 \quad \text{at } y = h_2 \]

The solution of the first-order problem is given by
\[ \psi_1 = c_3 \cosh \sqrt{\sigma^2 + M^2} y + c_4 \sinh \sqrt{\sigma^2 + M^2} y - \left( \frac{1}{(\sigma^2 + M^2)} \frac{dp_0}{dx} + 1 \right) + L_{31} y^3 - L_{45} y^2 + L_{52} y + \frac{1}{4} \left( L_5 \cosh 3\sqrt{\sigma^2 + M^2} y + L_6 \sinh 3\sqrt{\sigma^2 + M^2} y \right) + \cosh 2\sqrt{\sigma^2 + M^2} y \left( L_{34} + L_{33} y \right) + \sinh 2\sqrt{\sigma^2 + M^2} y \left( L_{36} + L_{55} y \right) + \sinh \sqrt{\sigma^2 + M^2} y \left( L_{27} + L_{57} y - L_{38} y^2 + L_{39} y^3 - L_{40} y^4 + L_{41} y^5 - L_{42} y^6 \right) + \cosh \sqrt{\sigma^2 + M^2} y \left( L_{28} + L_{43} y - L_{44} y^2 + L_{45} y^3 - L_{46} y^4 + L_{47} y^5 - L_{48} y^6 \right) \quad (4.42) \]
Equation (4.40) can be solved subject to the boundary condition (4.41) to obtain the solution for \( \theta_1 \). Since the expression for \( \theta_1 \) is large, it is not presented here.

Finally the expressions for axial velocity and temperature distribution are given by

\[
\begin{align*}
\theta &= \theta_0 + W e^{\varepsilon} \theta_1 \\
\theta &= \theta_0 + W e^{\varepsilon} \theta_1
\end{align*}
\]  

(4.44)

(4.45)

The pressure gradient is obtained as

\[
\frac{dp}{dx} = \frac{dp_0}{dx} + We^{\varepsilon} \frac{dp_1}{dx}
\]  

(4.46)

In which

\[
\frac{dp_0}{dx} = (\sigma^2 + M^2) \left( \frac{c_1L_2 + c_2L_3 - F_6}{h_1 - h_2} - 1 \right) \quad \text{and} \quad \frac{dp_1}{dx} = \frac{F_1 - L_{132}}{L_{111}}
\]  

(4.47)

The non-dimensional pressure rise per unit wave length in the wave frame are given by

\[
\Delta p = \int_0^1 \frac{dp}{dx} \, dx
\]  

(4.48)

### 4.4. RESULTS AND DISCUSSION

The expression for the velocity in terms of \( y \) is given by the equation (4.44). Velocity profiles are plotted in Figures 4.1-4.7 to study the effects of the different parameters such as the permeability parameter \( \sigma \), Magnetic parameter \( M \), Weissenberg number \( We \), phase difference \( \phi \) and amplitudes \( a \), \( b \) on the velocity...
distribution. Fig.4.1 is plotted for different \( \sigma \) with fixed \( a = 0.5, b = 0.5, d = 1.2, x = 0.1, \phi = \pi / 6, M = 0.6, \sigma = 0.4, F = 1.5, \text{We} = 0.2 \). We notice that the velocity profiles are parabolic. We observe that the velocity decreases with increasing \( \sigma \).

Fig.4.2 is drawn to study the effect of \( M \) with fixed \( a = 0.5, b = 0.5, d = 1.2, x = 0.1, \phi = \pi / 6, \sigma = 0.4, F = 1.5, \text{We} = 0.2 \). We identify that the velocity profiles are parabolic and the velocity decreases with increasing \( M \). The influence of Weissenberg number \( \text{We} \) on velocity is shown in Fig.4.3 with fixed \( a = 0.5, b = 0.5, d = 1.2, x = 0.1, \phi = \pi / 6, \sigma = 0.4, M = 0.6, F = 1.5 \). It reveals that the velocity increases with an increase in \( \text{We} \).

Fig.4.4 is drawn for different values of \( \phi \) with fixed \( a = 0.5, b = 0.5, d = 1.2, x = 0.1, M = 0.6, \phi = \pi / 6, M = 0.6, \sigma = 0.4, F = 1.5 \). We notice that the velocity increases with an increase in \( \phi \). The effects of amplitudes \( a, b \) and the channel length \( d \) are shown in Figures 4.5 - 4.7 with fixed \( a = 0.5, b = 0.5, d = 1.2, x = 0.1, M = 0.6, \phi = \pi / 6, \sigma = 0.4, F = 1.5 \). We observe that the velocity increases with increasing \( a \) and decreasing \( b \) and \( d \).

Equation (4.45) gives the expression for the temperature in terms of \( y \). Temperature profiles are plotted from Fig.4.8 - Fig.4.11 to study the effects of the permeability parameter \( \sigma \), Magnetic parameter \( M \), phase difference \( \phi \) and Brinkman number \( Br \). Fig.4.8 is drawn to study the effect of \( \sigma \) on the velocity with fixed \( a = 0.5, b = 0.5, d = 1, x = 0.1, \phi = \pi / 6, M = 0.6, Br = 2 \). It is observed that the temperature profiles are parabolic and the temperature decreases with increasing permeability parameter \( \sigma \).

The influence of magnetic parameter \( M \) on the temperature is shown in Fig.4.9 for fixed \( a = 0.5, b = 0.5, d = 1, x = 0.1, \phi = \pi / 6, M = 0.6, \sigma = 0.2, \sigma = 0.2, Br = 2 \). We notice that the temperature increases with an increase in \( M \). Fig.4.10 and Fig.4.11 are plotted for different values of \( \phi \) and \( Br \) with fixed: \( a = 0.5, b = 0.5, d = 1, x = 0.1, \phi = \pi / 6, M = 0.6, Br = 2 \).
\[ \sigma = 0.2, M = 0.6. \] It is noticed that the temperature increases with decreasing \( \phi \) and an increase in \( Br \).

We have calculated the pressure rise \( \Delta p \) in terms of the mean flow \( \Theta \) from equation (4.48). Fig.4.12 shows the effect of \( \sigma \) on \( \Delta p \) for fixed \( a = 0.4, b = 0.4, \, d = 1, \, \phi = \pi/8, \, M = 0.5, We = 0.01 \). We observe that for a given \( \Theta \), the pressure rise increases with increasing \( \sigma \). Also identified that for a fixed \( \Delta p \), the flow rate increases with increasing \( \sigma \). The effect of \( We \) is shown in Fig.4.13 for fixed \( a = 0.4, b = 0.4, \, d = 1, \, \phi = \pi/8, \, M = 0.5, \sigma = 0.1 \). It can be seen that for a fixed \( \Theta \), the pressure rise increases with an increase in \( We \) and for a given \( \Delta p \), \( \Theta \) increases with increasing \( We \).

Fig.4.14 is plotted to study the effect of \( \phi \) on \( \Delta p \), for fixed \( a = 0.4, b = 0.4, d = 1, \, We = 0.02, M = 0.5, \sigma = 0.1 \). We observe that for a fixed \( \Theta \), the pressure rise decreases with increasing \( \phi \) and for a given \( \Delta p \), the flow rate decreases with increasing \( \phi \). Fig.4.15 is drawn for different values of \( M \) with fixed \( a = 0.4, b = 0.4, d = 1, \, We = 0.02, \sigma = 0.1, \phi = \pi/8 \). We observe that for a fixed \( \Theta \), the pressure rise decreases with an increase in \( M \). Also it is observed that for a given \( \Delta p \), \( \Theta \) decreases with increasing \( M \).

**Trapping phenomena**

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead with the peristaltic wave. The effects of \( \sigma \) on stream lines is shown in Figure 4.16 for fixed \( a = 0.4, b = 0.3, \, d = 1.3, \phi = \pi/6, We = 0.01, F = 10, \, M = 0.6, \sigma = 0.8, (a) \sigma = 0.8, (b) \sigma = 0.9, (c) \sigma = 1 \). We observe that the size of the trapping bolus decreases with increasing \( \sigma \). Figures 4.17 is plotted to study influence of \( M \) with fixed \( a = 0.4, b = 0.3, d = 1.3, \phi = \pi/6, We = 0.01, F = 10, \sigma = 0.86, (a) M = 0.2, (b) M = 0.3, (c) M = 0.4 \). It is noticed that the bolus decreases with increasing \( M \). Figures 4.18 is drawn to study effect of \( \phi \) with fixed \( a = 0.4, b = 0.3, d = 1.3, \phi = \pi/6, We = 0.01, F = 10, \sigma = 0.87, M = 0.2, \sigma = 0.87, M = 0.2, (a) \phi = 0, (b) \phi = \pi/8, (c) \phi = \pi/6 \). It reveals that the bolus increases with increasing \( \phi \).
APPENDIX

EXPRESSIONS FOR THE CONSTANTS

\[ L_i = \frac{1}{\sigma^2 + M^2} \frac{dp_0}{dx} + 1, \quad L_2 = \cosh \sqrt{\sigma^2 + M^2} h_1 - \cosh \sqrt{\sigma^2 + M^2} h_2, \]

\[ L_3 = \sinh \sqrt{\sigma^2 + M^2} h_1 - \sinh \sqrt{\sigma^2 + M^2} h_2, \quad L_2' = \cosh \sqrt{\sigma^2 + M^2} h_1 + \cosh \sqrt{\sigma^2 + M^2} h_2, \]

\[ L_1' = \sinh \sqrt{\sigma^2 + M^2} h_1 + \sinh \sqrt{\sigma^2 + M^2} h_2, \quad L_4 = \left( \frac{h_1 + h_2}{L_1 L_2} \right) \left( \frac{L_1' - L_2'}{L_1' - L_2'} \right) + (h_2 - h_1) \]

\[ c_i = -\frac{c_1 L_2}{L_3}, \quad c_2 = \frac{L_i L_1 (h_2 + h_1)}{L_3 L_2 - L_1 L_2}, \quad L_4 = \left( \frac{dp_0}{dx} \right)^3 + 3 \left( \sigma^2 + M^2 \right) \left( \frac{dp_0}{dx} \right)^2, \]

\[ L_5 = \left( \frac{c_1}{2} \right)^3 + 3 \left( \frac{c_1}{2} \right) \left( \frac{c_2}{2} \right)^2 + \left( \frac{c_2}{2} \right)^3, \quad L_6 = \left( \frac{c_1}{2} \right)^3 + 3 \left( \frac{c_1}{2} \right) \left( \frac{c_2}{2} \right)^2 \left( \frac{c_2}{2} \right), \]

\[ L_7 = \left( \frac{c_1}{2} \right)^3 - \left( \frac{c_1}{2} \right) \left( \frac{c_2}{2} \right)^2, \quad L_8 = \left( \frac{c_2}{2} \right)^3 - \left( \frac{c_1}{2} \right) \left( \frac{c_2}{2} \right), \]

\[ L_9 = \left( \frac{c_1}{2} \right)^2 - \left( \frac{c_2}{2} \right)^2, \quad L_{10} = \left( \frac{c_1}{2} \right) \left( \frac{c_2}{2} \right)^2, \quad L_{11} = \frac{3L_4^2 \sqrt{\sigma^2 + M^2}}{2}, \quad L_{11} = \frac{3L_4^2 \sqrt{\sigma^2 + M^2}}{2} \left( \frac{c_1}{2} - \frac{c_2}{2} \right), \]

\[ L_{12} = \frac{L_2^2}{(\sigma^2 + M^2)}, \quad L_{13} = 2 \left( \frac{c_2}{2} \right)^2 + \left( \frac{c_1}{2} \right)^2, \quad L_{14} = \frac{L_{10} - L_{11}}{3} + L_{14}, \]

\[ L_{15} = \frac{L_{10} + L_{11}}{2\sqrt{\sigma^2 + M^2}}, \quad L_{16} = 3L_4L_9 + \frac{6L_9}{\sigma^2 + M^2} + \frac{L_{10} - L_{11}}{2(\sigma^2 + M^2)} \]

\[ L_{17} = 3(\sigma^2 + M^2)^2 \left( \sigma^2 + M^2 \right) + \frac{dp_0}{dx}, \quad L_{18} = \frac{6L_9}{\sigma^2 + M^2} + 2L_9, \]

\[ L_{19} = \frac{2L_{17}L_7}{3(\sigma^2 + M^2)}, \quad L_{20} = \frac{L_3^2 c_1 c_2}{3(\sigma^2 + M^2)}, \]

\[ L_{21} = \frac{8L_{17}L_9}{9(\sigma^2 + M^2)^{3/2}}, \quad L_{22} = \frac{4c_1 c_2 L_{17}}{9(\sigma^2 + M^2)^{3/2}}, \quad L_{23} = \frac{L_{17}L_8}{(\sigma^2 + M^2)}, \]

\[ L_{24} = \frac{L_{17}^2}{(\sigma^2 + M^2)}, \quad L_{25} = \frac{(c_1 + c_2)L_4L_{17}}{3\sqrt{\sigma^2 + M^2}}, \quad L_{26} = \frac{(c_1 + c_2) L_4L_{17}}{2(\sigma^2 + M^2)^{3/2}}, \]

\[ L_{27} = \frac{c_1 L_4L_{17}}{2(\sigma^2 + M^2)}, \quad L_{28} = \frac{c_1 L_4L_{17}}{2(\sigma^2 + M^2)}, \]

\[ L_{29} = 3\left( \sigma^2 + M^2 \right) \left( 1 + \frac{2}{(\sigma^2 + M^2)} \left( \frac{dp_0}{dx} \right)^2 \right)^2, \quad L_{30} = \frac{L_{29}L_1}{2\sqrt{\sigma^2 + M^2}}, \]

\[ L_{31} = L_{14} - L_{24} - \frac{L_4}{(\sigma^2 + M^2)}, \quad L_{32} = L_{16} - L_{23} - \frac{6L_4}{(\sigma^2 + M^2)}, \]

\[ L_{33} = L_{19} - 2L_1L_{12}, \quad L_{34} = \frac{8L_4L_{13}}{3(\sigma^2 + M^2)} - L_{22}, \quad L_{35} = L_{20} - 2L_1L_{13} \]
\[ L_{36} = \frac{8L_xL_{12}}{3\sqrt{(\sigma^2 + M^2)}} - L_{21}, \quad L_{37} = 3\sqrt{(\sigma^2 + M^2)}L_y + 15\frac{L_{36}c_2}{4(\sigma^2 + M^2)^{3/2}}, \]
\[ L_{38} = \frac{15L_{36}c_1}{4(\sigma^2 + M^2)^{3/2}}, \quad L_{39} = \frac{5L_{36}c_2}{2(\sigma^2 + M^2)^{3/2}}, \quad L_{40} = \frac{5L_{36}c_1}{4(\sigma^2 + M^2)^{3/2}}, \]
\[ L_{41} = \frac{L_{36}c_2}{2\sqrt{(\sigma^2 + M^2)}} \quad L_{42} = \frac{L_{36}c_1}{6}, \]
\[ L_{43} = \frac{15L_{36}c_1}{4(\sigma^2 + M^2)^{3/2}} - L_{26} - 3\sigma L_8, \quad L_{44} = \frac{15L_{36}c_2}{4(\sigma^2 + M^2)^{3/2}} - L_{25} - \frac{5L_{36}c_2}{4(\sigma^2 + M^2)^{3/2}}, \]
\[ L_{45} = \frac{L_{36}c_2}{2(\sigma^2 + M^2)^{3/2}} - L_{26} - 3\sigma L_8, \quad L_{46} = \frac{5L_{36}c_2}{4(\sigma^2 + M^2)^{3/2}}, \]
\[ L_{47} = \frac{L_{36}c_1}{2(\sigma^2 + M^2)^3}, \quad L_{48} = \frac{L_{36}c_2}{6}, \]
\[ L_{49} = L_{31}h_1^3 - L_{12}h_2^2 + L_{12}h_1, \]
\[ L_{50} = \frac{L_5 \cosh 3\sqrt{(\sigma^2 + M^2)}h_1 + L_5 \sinh 3\sqrt{(\sigma^2 + M^2)}h_1}{4}, \]
\[ L_{51} = \cosh 2\sqrt{(\sigma^2 + M^2)}h_1 \left( L_{34} + L_{33}h_2 \right), \]
\[ L_{52} = \sinh 2\sqrt{(\sigma^2 + M^2)}h_1 \left( L_{36} + L_{35}h_1 \right), \quad L_{53} = L_{27} + L_{31}h_1 - L_{38}h_1^2 + L_{39}h_1^3, \]
\[ L_{54} = -L_{40}h_1^4 + L_{41}h_1^5 - L_{42}h_1^6, \]
\[ L_{55} = \sinh \sqrt{(\sigma^2 + M^2)}h_1 \left( L_{53} + L_{54} \right), \quad L_{56} = L_{28} + L_{43}h_1 - L_{44}h_1^2 + L_{45}h_1^3, \]
\[ L_{57} = -L_{46}h_1^4 + L_{47}h_1^5 - L_{48}h_1^6, \]
\[ L_{58} = \cosh \sqrt{(\sigma^2 + M^2)}h_1 \left( L_{56} + L_{57} \right), \quad L_{59} = L_{49} + L_{50} + L_{51} + L_{52} + L_{55} + L_{58}, \]
\[ L_{60} = L_{32}h_2 - L_{35}h_2^2 + L_{31}h_2^3, \]
\[ L_{61} = \frac{L_5 \cosh 3\sqrt{(\sigma^2 + M^2)}h_2 + L_6 \sinh 3\sqrt{(\sigma^2 + M^2)}h_2}{4}, \]
\[ L_{62} = \cosh 2\sqrt{(\sigma^2 + M^2)}h_2 \left( L_{34} + L_{33}h_2 \right), \]
\[ L_{63} = \sinh 2\sqrt{(\sigma^2 + M^2)}h_2 \left( L_{56} + L_{53}h_2 \right), \quad L_{64} = L_{27} + L_{31}h_2 - L_{38}h_2^2 + L_{39}h_2^3, \]
\[ L_{65} = -L_{40}h_2^4 + L_{41}h_2^5 - L_{42}h_2^6, \]
\[ L_{66} = \sinh \sqrt{(\sigma^2 + M^2)}h_2 \left( L_{64} + L_{65} \right), \quad L_{67} = L_{28}L_{43}h_2 - L_{44}h_2^2 + L_{45}h_2^3, \]
\[ L_{68} = -L_{46}h_2^4 + L_{47}h_2^5 - L_{48}h_2^6 \]
\[ L_{69} = \cosh \sqrt{\sigma^2 + M^2} \int L_{67} + L_{68}, \quad L_{70} = L_{60} + L_{61} + L_{62} + L_{63} + L_{66} + L_{69}, \]
\[ L_{71} = 2 \sqrt{\sigma^2 + M^2} L_{34} + L_{35}, \]
\[ L_{72} = L_{33} + 2 \sqrt{\sigma^2 + M^2} L_{26}, \quad L_{73} = L_{37} + \sqrt{\sigma^2 + M^2} L_{28}, \]
\[ L_{74} = \sqrt{\sigma^2 + M^2} L_{43} - 2 L_{38}, \]
\[ L_{75} = 3 L_{39} - \sqrt{\sigma^2 + M^2} L_{44}, \quad L_{76} = \sqrt{\sigma^2 + M^2} L_{45} - 4 L_{40}, \]
\[ L_{77} = 5 L_{41} - \sqrt{\sigma^2 + M^2} L_{46}, \]
\[ L_{78} = \sqrt{\sigma^2 + M^2} L_{47} - 6 L_{42}, \quad L_{79} = \sqrt{\sigma^2 + M^2} L_{27} + L_{43}, \]
\[ L_{80} = 3 L_{45} - \sqrt{\sigma^2 + M^2} L_{44} L_{27}, \quad L_{81} = \sigma L_{39} - 4 L_{46}, \]
\[ L_{82} = 5 L_{47} - \sqrt{\sigma^2 + M^2} L_{40}, \]
\[ L_{83} = 4 L_{41} - 6 L_{48}, \quad L_{84} = 3 L_{11} h_1^2 - 2 L_{13} h_1 + L_{12} \]
\[ L_{85} = 3 \sigma \left( \frac{L_4 \sinh 3 \sqrt{\sigma^2 + M^2} h_1 + L_6 \cosh 3 \sqrt{\sigma^2 + M^2} h_1}{4} \right), \]
\[ L_{87} = \sinh 2 \sqrt{\sigma^2 + M^2} h_1 \left( L_{71} + 2 \sqrt{\sigma^2 + M^2} L_{33} h_1 \right), \]
\[ L_{88} = \cosh 2 \sqrt{\sigma^2 + M^2} h_1 \left( L_{72} + 2 \sqrt{\sigma^2 + M^2} L_{35} h_1 \right), \]
\[ L_{89} = L_{73} + L_{74} h_1 + L_{75} h_1^2, \quad L_{90} = L_{76} h_1^3 + L_{77} h_1^4 + L_{78} h_1^5, \]
\[ L_{91} = \sinh \sqrt{\sigma^2 + M^2} h_1 \left( L_{89} + L_{90} \right), \]
\[ L_{92} = L_{79} + L_{80} h_1^2 + L_{81} h_1^4, \quad L_{93} = L_{82} h_1^3 + L_{83} h_1^4 + L_{84} h_1^5, \]
\[ L_{94} = \cosh \sqrt{\sigma^2 + M^2} h_1 \left( L_{92} + L_{93} \right), \]
\[ L_{95} = L_{85} + L_{86} + L_{87} + L_{88} + L_{91} + L_{94}, \quad L_{96} = 3 L_{31} h_2^2 - 2 L_{15} h_2 + L_{32}, \]
\[ L_{97} = 3 \sqrt{\sigma^2 + M^2} \left( \frac{L_5 \sinh 3 \sqrt{\sigma^2 + M^2} h_2 + L_6 \cosh 3 \sqrt{\sigma^2 + M^2} h_2}{4} \right) \]
\[ L_{98} = \sinh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2 \left( L_{71} + 2 \sqrt{\left( \sigma^2 + M^2 \right)} L_{23} h_2 \right), \]
\[ L_{99} = \cosh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2 \left( L_{72} + 2 \sqrt{\left( \sigma^2 + M^2 \right)} L_{23} h_2 \right), \]
\[ L_{100} = L_{73} + L_{74} h_2 + L_{75} h_2^2, \quad L_{101} = L_{76} h_2 + L_{77} h_2^4 + L_{78} h_2^5, \]
\[ L_{102} = \sinh \sqrt{\left( \sigma^2 + M^2 \right)} h_2 \left( L_{100} + L_{101} \right), \]
\[ L_{103} = L_{79} + L_{80} h_2 + L_{81} h_2^2, \quad L_{104} = L_{82} h_2 + L_{83} h_2^4 + L_{84} h_2^5, \]
\[ L_{105} = \cosh \sqrt{\left( \sigma^2 + M^2 \right)} h_2 \left( L_{103} + L_{104} \right), \]
\[ L_{106} = L_{96} + L_{97} + L_{98} + L_{102} + L_{105}, \quad L_{107} = L_2^1, \]
\[ L_{108} = L_1^1, \quad L_{109} = L_{108} - L_{107}^2, \]
\[ L_{110} = \frac{-L_{108} \left( h_1 + h_2 \right) + \frac{2}{\left( \sigma^2 + M^2 \right)} L_{107}}{L_{108}}, \]
\[ L_{111} = \sqrt{\left( \sigma^2 + M^2 \right) L_{108}} \left( L_{59} + L_{70} \right) - L_{107} \left( L_{95} + L_{106} \right) \]
\[ c_3 = \frac{\frac{dp_0}{dx} \left( h_2 + h_1 \right) - \left( L_{59} + L_{70} \right) - c_4 L_{108}}{L_{107}}, \quad c_4 = \frac{-\left( \frac{dp_0}{dx} L_{70} + L_{111} \right)}{L_{109}}, \]
\[ L_{112} = \cosh 3 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - \cosh 3 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{113} = \sinh 3 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - \sinh 3 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{114} = \cosh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - \cosh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{115} = h_1 \cosh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - h_2 \cosh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{116} = \sinh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - \sinh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{117} = h_1 \sinh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_1 - h_2 \sinh 2 \sqrt{\left( \sigma^2 + M^2 \right)} h_2, \]
\[ L_{118} = \frac{L_{112} L_{113} + L_{113} L_{112}}{4}, \quad L_{119} = \frac{L_{114} \left( L_{71} + L_{72} - L_{116} \right)}{2 \sqrt{\left( \sigma^2 + M^2 \right)}}, \]
\[ L_{120} = \frac{L_{73} + L_{74}}{\sqrt{\left( \sigma^2 + M^2 \right)}}, \quad 2 \left( L_{75} + L_{81} \right) \frac{24 \left( L_{77} + L_{93} \right)}{\left( \sigma^2 + M^2 \right)}, \]
\[ L_{121} = \frac{L_{74} + L_{80}}{\left( \sigma^2 + M^2 \right)} + \frac{6 \left( L_{76} + L_{82} \right)}{\left( \sigma^2 + M^2 \right)} + \frac{120 \left( L_{78} + L_{84} \right)}{\left( \sigma^2 + M^2 \right)}, \]
\[ L_{122} = \frac{2 \left( L_{75} + L_{81} \right)}{\left( \sigma^2 + M^2 \right)} + \frac{24 \left( L_{77} + L_{93} \right)}{\left( \sigma^2 + M^2 \right)}.
\[ L_{123} = \frac{(L_{74} + L_{80})}{\sqrt{\left(\sigma^2 + M^2\right)}} + \frac{6(L_{76} + L_{82})}{\left(\sigma^2 + M^2\right)^{3/2}} + \frac{120(L_{78} + L_{84})}{\left(\sigma^2 + M^2\right)^{5/2}}, \]

\[ L_{124} = \frac{(L_{74} + L_{92})}{\sqrt{\left(\sigma^2 + M^2\right)}} + \frac{12(L_{77} + L_{83})}{\left(\sigma^2 + M^2\right)^{3/2}}, \]

\[ L_{125} = \frac{3(L_{76} + L_{92})}{\left(\sigma^2 + M^2\right)} + \frac{60(L_{78} + L_{84})}{\left(\sigma^2 + M^2\right)^{2}}, \]

\[ L_{126} = L_{31} + L_3 \left( \frac{(L_{76} + L_{82})}{\sqrt{\left(\sigma^2 + M^2\right)}} + \frac{20L_{94}}{\left(\sigma^2 + M^2\right)^{3/2}} \right) - \frac{4L_2 (L_{77} + L_{83})}{\left(\sigma^2 + M^2\right)} \]

\[ L_{127} = \frac{L_2 (L_{77} + L_{83})}{\sqrt{\left(\sigma^2 + M^2\right)}} - \frac{5L_2 (L_{78} + L_{84})}{\left(\sigma^2 + M^2\right)^{3/2}}, \]

\[ \begin{align*}
L_{128} &= L_{118} + L_{119} + L_3L_{120} - L_2L_{121} \\
L_{129} &= (h_1 - h_2)(L_{92} - L_2L_{122} + L_3L_{423}) + (h_1^2 - h_2^2)(L_3L_{424} - L_2L_{125} - L_{45}) \\
L_{130} &= (h_1^3 - h_2^3)L_{126} + (h_1^4 - h_2^4)L_{127} + \frac{(h_1^5 - h_2^5)}{\sqrt{\left(\sigma^2 + M^2\right)}}L_3 \\
L_{131} &= \frac{(h_1 + h_2)L_4L_{109} + (\sigma^2 + M^2)L_1L_{108} - (\sigma^2 + M^2)L_3L_{107} - (h_1 - h_2)L_{107}L_{109}}{\left(\sigma^2 + M^2\right)\sqrt{L_{107}L_{109}}} \\
L_{132} &= L_{128} + L_{129} + L_{130} + \frac{L_{417}(L_2L_{107} - L_{408}) - L_{109}L_2(L_{59} + L_{70})}{L_{107}L_{309}} \]

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Fig. 4.1  Velocity profiles for different ‘σ’ with fixed $a = 0.5, b = 0.5, d = 1.2, x = 0.1, \phi = \pi / 6, M = 0.6, F = 1.5, We = 0.2$
Fig. 4.2 Velocity profiles for different $M$ with fixed $a = 0.5, b = 0.5, d = 1.2, \phi = \pi / 6, F = 1.5, \sigma = 0.4, We = 0.2$
Fig. 4.3 Velocity profiles for different ‘We’ with fixed $a = 0.5, b = 0.5, d = 1.2, x = 0.1, \phi = \pi / 6, F = 1.5, M = 0.6, \sigma = 0.4$
Fig. 4.4 Velocity profiles for different $\phi$ with fixed $a = 0.5, b = 0.5, d = 1.2, x = 0.1, M = 0.6, We = 0.2, \sigma = 0.4, F = 1.5$
Fig. 4.5  Velocity profiles for different ‘a’ with fixed $b = 0.5, d = 1.2,$ $x = 0.1, M = 0.6, \phi = \pi / 6, We = 0.2, \sigma = 0.4, F = 1.5$
Fig. 4.6 Velocity profiles for different ‘b’ with fixed $a = 0.5$, $d = 1.2$, $x = 0.1$, $M = 0.6$, $\phi = \pi / 6$, $We = 0.2$, $\sigma = 0.4$, $F = 1.5$
Fig. 4.7  Velocity profiles for different ‘d’ with fixed $a = 0.5$, $b = 0.5$, $x = 0.1$, $M = 0.6$, $\phi = \pi / 6$, $We = 0.2$, $\sigma = 0.4$, $F = 1.5$
Fig. 4.8 Temperature profiles for different $\sigma$ with fixed $a = 0.5, b = 0.5$

d = 1, x = 0.1, $\phi = \pi / 6, M = 0.6, Br = 2,$
Fig. 4.9 Temperature profiles for different ‘M’ with fixed $a = 0.5$, $b = 0.5, d = 1, x = 0.1, \phi = \pi / 6, \sigma = 0.2, Br = 2$
Fig. 4.10 Temperature profiles for different ‘φ’ with fixed a = 0.5,
b = 0.5, d = 1, x = 0.1, M = 0.8, σ = 0.2, Br = 2
Fig. 4.11 Temperature profiles for different ‘Br’ with fixed $a = 0.5$, $b = 0.5, d = 1, x = 0.1, \phi = \pi / 6, M = 0.8, \sigma = 0.2$.
Fig. 4.12 Variation of pressure rise for different $\sigma$ with fixed $a = 0.4$, $b = 0.4$, $d = 1$, $\phi = \pi/8$, $M = 0.5$, $We = 0.01$
Fig. 4.13 Variation of pressure rise for different 'We' with fixed $a = 0.4$, $b = 0.4$, $d = 1$, $M = 0.5$, $\phi = \pi/8$, $\sigma = 0.1$
Fig.4.14 Variation of pressure rise for different $\phi$ with fixed $a = 0.4$, $b = 0.4, d = 1, We = 0.02, M = 0.5, \sigma = 0.1$
Fig. 4.15 Variation of pressure rise for different 'M' with fixed $a = 0.4$, $b = 0.4, d = 1$, $We = 0.02, \sigma = 0.1, \phi = \pi / 8$
Fig. 4.16 Stream lines for $a = 0.4, b = 0.3, d = 1.3, \phi = \pi/6, \text{We} = 0.01, F = 10, M = 0.6$

and for different values of $\sigma$: (a) $\sigma = 0.8$, (b) $\sigma = 0.9$, (c) $\sigma = 1$
Fig. 4.17 Stream lines for $a = 0.3, b = 0.3, d = 1.2, \phi = \pi/8, \sigma = 0.86, We = 0.001, F = 8$
and for different values of $M$: (a) $M = 0.2$, (b) $M = 0.3$, (c) $M = 0.4$
Fig. 4.18 Stream lines for $a = 0.4, b = 0.3, d = 1.2, \sigma = 0.87, We = 0.001, F = 8, M = 0.2$
and for different values of $\phi$ : (a)$\phi = 0,(b)\phi = \pi/8,(c)\phi = \pi/6$