APPENDIX – 1

LIST OF PUBLICATIONS IN SUPPORT OF THE THESIS


COOPERATIVE SPECTRUM SENSING IN COGNITIVE RADIO NETWORKS AND MINIMIZATION OF ERROR PROBABILITY USING OPTIMAL DECISION VOTING RULE

By

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ABSTRACT

To solve the conflicts between spectrum scarcity and spectrum under-utilization, cognitive radio (CR) technology has been recently proposed. It can improve the spectrum utilization by allowing secondary networks (users) to make use of unused radio spectrum from primary licensed networks (users) or to share the spectrum with the primary networks (users). As an intelligent wireless communication system, a cognitive radio is sentient of the radio frequency environment. It selects the communication parameters such as carrier frequency, bandwidth, and transmission power to optimize the spectrum usage and adapts its transmission and reception accordingly.

Cooperative spectrum sensing is considered where multiple cognitive radios detect the spectrum holes collaboratively through energy detection and examine the optimality of cooperative spectrum sensing. The aim is to optimize the detection performance in an efficient way. The optimal voting rule is derived for any detector functional to cooperative spectrum sensing. Also optimize the detection threshold when energy detection is in use. To conclude, we propose a spectrum sensing algorithm for the network which requires smaller quantity than the total number of cognitive radios in cooperative spectrum sensing while fulfilling a given error hurdle.

Keywords: Spectrum Sensing, Cognitive Radio, Energy Detection, Optimizatal Detection, Total Error Rate, And Rule, Or Rule.

INTRODUCTION

Spectrum sensing is a key role of cognitive radio to prevent the harmful interference with licensed users and identify the available spectrum for improving the spectrum’s utilization. However, detection performance in practice is often compromised with multipath fading, shadowing and receiver uncertainty issues. To mitigate the impact of these issues, cooperative spectrum sensing has been shown to be an effective method to improve the detection performance by exploiting spatial diversity.

While cooperative gain such as improved detection performance and relaxed sensitivity requirement [3] can be obtained, cooperative sensing can incur cooperation overhead. The overhead refers to any extra sensing time, delay, energy, and operations devoted to cooperative sensing and any performance degradation caused by cooperative sensing. In this paper, the state-of-the-art survey of cooperative sensing is provided to address the issues of cooperation method, cooperative gain, and cooperation overhead. Specifically, the cooperation method is analyzed by the fundamental components called the elements of cooperative sensing, including cooperation models[2], sensing techniques, hypothesis testing, data fusion, control channel and reporting, user selection, and knowledge base. Moreover, the impacting factors of achievable cooperative gain and incurred cooperation overhead are presented. The factors under consideration include sensing time and delay, channel impairments.

The detection performance can be primarily determined on the basis of two metrics: probability of false alarm, which denotes the probability of a CR user declaring that a PU is present when the spectrum is actually free[1,7], and probability of detection, which denotes the probability of a CR user declaring that a PU is present when the spectrum is indeed occupied by the PU. Since a miss in the detection...
will cause the interference with the PU and a false alarm will reduce the spectral efficiency, it is usually required for optimal detection performance that the probability of detections maximized subject to the constraint of the probability of false alarm.

1. Primary Signal Detection

The process of cooperative sensing starts with spectrum sensing performed individually at each CR user called local sensing. Typically, local sensing for primary signal detection can be formulated as a binary hypothesis problem as follows.

\[ x_i(t) = \begin{cases} \frac{w_i(t)}{H_0} + h_i(t)x_i(t) + w_i(t), \\ \frac{w_i(t)}{H_1} \end{cases} \]

where \( x_i(t) \) denotes the received signal at \( i \)th CR user, \( s(t) \) is the transmitted PU signal, \( h_i(t) \) is the channel gain of the sensing channel, \( H_0 \) and \( H_1 \) denote the hypothesis of the absence and the presence, respectively[5], of the PU signal in the frequency band of interest[7]. For the evaluation of the detection performance, the probabilities of detection \( P_d \) and false alarm \( P_f \) are defined as

\[ P_d = P\{\text{decision} = H_1 | H_1\} = P\{Y > \lambda | H_1\} \]

\[ P_f = P\{\text{decision} = H_1 | H_0\} = P\{Y > \lambda | H_0\} \]

(2)

Where \( Y \) is the decision statistic and \( \lambda \) is the decision threshold. The value of \( \lambda \) is set depending on the requirements of detection performance.

To facilitate the analysis of cooperative sensing, we classify cooperative spectrum sensing into three categories [2,4] based on how cooperating CR users share the sensing data in the network i) Centralized cooperative sensing, ii) Distributed cooperative and iii) Relay-assisted cooperative sensing.

2. Perfect Reporting Channel

If the channels between the cognitive users and the common receiver are perfect, the local decisions will be reported without any error. In this case,

\[ PA = P\{H=0 | H_0, K>1\} \]

classifies the probability of the event that under hypothesis \( H_0 \), all the K users claim \( H_0 \) and other \( N-K \) users make no local decisions. We will obtain the false alarm probability \( Q_f \), the detection probability \( Q_d \), and the missing probability \( Q_m \) for cooperative spectrum sensing, respectively.

3. Imperfect Reporting Channel

It is not realistic that the reporting channel between the cognitive user and the common receiver is assumed to be perfect since it is usually subject to fading. Due to the reporting error introduced by the imperfect channel, the reported local decisions should be firstly decoded in the common receiver before the final decision is made. Let \( P_e \), denote the reporting error between the \( i \)th cognitive user and the common receiver, for \( i = 1, \cdots , K \). For simplicity, we assume that all the reporting channels are independent and identical, i.e., \( P_e, i = P_e \). At the common receiver, the local decision will be recovered as \( 0 \) in two cases: a), the cognitive user transmits \( H_0 \) and the it is decoded as \( H_0 \); b), the cognitive user transmits \( H_1 \) while the it is decoded as \( H_0 \) because of the reporting error. Due to the existence of reporting errors, the sensing performance is decreased compared with that in the perfect channel.

A Cognitive radio network is considered comprising of secondary users called CRs and a receiver to receive commonly from CRs, as shown in Figure 1. Here each CR performs spectrum sensing discretely and then the local decisions are sent to the common receiver or data fusion center is considered which can combine all on hand decision information to infer the absence or presence of the PU[6].

\[ \text{Figure 1. Spectrum sensing structure in cognitive radio network.} \]
4. Cooperative Spectrum Sensing

In cognitive radio systems, cooperative spectrum sensing has been widely used to detect the primary user with a high agility and accuracy. Every cognitive user conducts its individual spectrum sensing using some detection method and then sends a binary local decision to the common receiver. Usually, the local decision is made by comparing the observation with a pre-fixed threshold.

In cooperative spectrum sensing, each cooperative partner makes a binary decision based on its local observation and then forwards one bit of the decision (1 standing for the presence of the PU, 0 for the absence of the PU) to the common receiver through an error-free channel. At the common receiver, all 1-bit decisions are fused together according to logic rule.

Hypothesis testing is a statistical test to determine the presence or absence of a PU. This test can be performed individually by each cooperating user for local decisions or performed by the fusion centre for cooperative decision.

In the following we only consider the spectrum sensing at CRi. The sensing method is to decide between the following two hypotheses \[ H_0: \text{primary user is absent}; \quad H_1: \text{primary user is in operation}. \]

The authors assume that the sensing time is smaller than the coherence time of the channel. Then, the sensing channel \( h(t) \) can be viewed as time-invariant during the sensing process. Without loss of generality, we denote \( h(t) \) as \( h_i \). Moreover, we assume that the status of the PU remains unchanged during the spectrum sensing process.

The information of the PU signal is unknown; the energy detection method is optimal for detecting zero-mean constellation signals [9].

For the \( j \)th CR with the energy detector, over AWGN channels the average probability of false alarm is \( P_{f,j} = \frac{\Gamma(\nu_i, -\frac{\lambda_i}{2})}{\Gamma(\nu_i)} \) (3) the average probability of detection is \( P_{d,j} = Q(u, \sqrt{\nu_i \lambda_i}) \) (4) and the average probability of missed detection is [9] \( P_{m,j} = 1 - P_{d,j} \) (5)

In the above equations, \( \lambda_i \) and \( \nu_i \) denote the energy detection threshold and the instantaneous signal-to-noise ratio (SNR) at the \( j \)th CR, \( u \) is the time-bandwidth product of the energy detector.

Where \( \Gamma(a, x) \) is the incomplete gamma function given by \( \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt \)

\( \Gamma(a) \) is the gamma function, and \( Qu(a,b) \) is the generalized Marcum Q-function given by \( Qu(a,b) = \int_0^\infty \frac{t^{a-1} e^{-bt^2} I_{u-1}(at)}{t} dt \)

With \( I_u(t) \) being the modified Bessel function of the first kind and order \( u-1 \).

Each cooperative partner makes a binary decision based on its local observation and then forwards one bit of the decision \( D_i, 1 \) for presence of the PU, 0 for the absence of the PU. At the common receiver, all 1-bit decisions are fused together according to logic rule

\[ Y = \sum_{i=1}^K D_i \left\{ \begin{array}{ll} \geq n & H_1 \\ < n & H_0 \end{array} \right\} \] (6)

The threshold \( n \) is an integer, represents the “ \( n \)-out-of \( K \)” voting rule. It can be seen that the OR rule corresponds to the case of \( n=1 \) and the AND rule corresponds to the case of \( n=K \).

Compared the distance from any cognitive radio to the primary transmitter, the distance between any two cognitive radios is small, then the received signal at each cognitive radio experiences almost indistinguishable path loss. In the case of an AWGN environment, we can assume that \( \nu_i = \nu_j = \ldots = \nu_K \).

Furthermore, we assume that all cognitive radios use the same threshold \( \lambda_i \) implying \( \lambda_1 = \lambda_2 = \ldots = \lambda_K = \lambda \). This results in \( P_{f,i} \) being independent of \( i \), and we denote it as \( P_f \).

In the case of an AWGN channel, \( P_{d,i} \) is independent of \( i \) (we denote this as \( P_d \)).

\( Q_i = \text{prob} \{ H_1 | H_i \} = \sum_{l=1}^K \binom{K}{l} P_i^l (1-P_i)^{K-l} \) (7)

In a cooperative spectrum sensing the false alarm probability is given by \( Q_m = \text{prob} \{ H_0 | H_1 \} = 1 - \sum_{l=1}^K \binom{K}{l} P_i^l (1-P_i)^{K-l} \) (8)

5. Cooperative Spectrum Sensing Optimization
In this section, it is to investigate the optimality of cooperative spectrum sensing when energy detection and decision fusion are applied.

If \( K \) is fixed, then the optimal voting rule, that minimizes the total error rate \( Q_f + Q_m \),

\[
K_{\text{opt}} = \min \left( K, \frac{K}{1 + \alpha} \right)
\]  

(9)

Where \( \alpha \) is given by

\[
\alpha = \frac{\ln \left( \frac{P_f}{1 - P_m} \right)}{\ln \left( \frac{P_m}{1 - P_f} \right)}
\]

and \([\cdot]\) denotes the ceiling function.

The OR rule is optimal when \( \alpha \geq k - 1 \). This means that \( P_f \leq P_m^{-1} \), this implies that \( P_f < P_m \) for a large value \( K \). This can be achieved when the detection threshold \( \lambda \) is very large.

The AND rule is optimal when \( \alpha \to 0 \). This is achieved when \( P_m < P_f \), i.e., for a very small \( \lambda \).

Let \( G \) be a function given by

\[
G(n) = \sum_{k=n}^{K} \left( \frac{K!}{n!} \right) \left( P_f^n (1 - P_f)^{K-n} - P_m^n (1 - P_m)^{K-n} \right)
\]

(10)

We get \( Q_f + Q_m = 1 + G(n) \)

\[
\frac{\partial G(n)}{\partial n} \approx G(n+1) - G(n)
\]

\[
\left( \frac{K}{n} \right) [(1 - P_m)^n P_m^{K-n} - P_f^n (1 - P_f)^{K-n}]
\]

the optimal value of \( n \) is obtained when \( \frac{\partial G(n)}{\partial n} \) i.e., when

\[
(1 - P_m)^n P_m^{K-n} = P_f^n (1 - P_f)^{K-n}
\]

Let \( \alpha = \frac{\ln \left( \frac{1 - P_f}{P_m} \right)}{\ln \left( \frac{P_m}{1 - P_f} \right)} \)

Then we get \( n \approx \left[ \frac{K}{1 + \alpha} \right] \)

(11)

From the above the following remarks can be applied to any detector.

- \( P_f \) and \( P_m \) have the same order, therefore the optimal choice of \([\cdot]\) is \( \lceil \frac{K}{2} \rceil \).
- OR rule is optimal when \( \alpha \geq K - 1 \) which means that \( P_f \leq PK - 1 \). This implies that \( P_f < P_m \) for a large value \( K \). This can be achieved when the detection threshold \( \lambda \) is very large and
- The AND rule is optimal when \( \alpha \to 0 \). This is achieved when \( P_f < P_m \), i.e., for a very small \( \lambda \). Figure 3. shows the exact solution of \( n \) in terms of detection threshold evaluated from (7).

In the following section the discussion is on simulation results of different optimum voting rule techniques for the cooperative spectrum sensing and also gives the optimal number of cognitive radio users.

Figure 2 shows variation of the Total Error Rate in cognitive radio network for the variation in the Threshold values of the Energy Detector for Constant SNR 10 dB.

- From the above Simulation Result, for the case of \( n=1 \) it is OR rule. The Error is minimum for the largest values of the Threshold.
- For the case of \( n=10 \) it is AND rule. The Error is minimum for the smaller values of the Threshold.
- Among all the values from \( n=1 \) to 10 the error is very much Reduced for the case of \( n=5 \), it also covers the almost all the Threshold values.
- The Figure 3 shows that variation of the Total Error Rate in cognitive radio network for the variation in the Threshold values of the Energy Detector for different values of the SNR.
- From the above Simulation Result for the larger values of the Thresholds (50 to 55) the Error is coming around
the order of $10^{4}$ for the less number of Cognitive Radio Users ($n=2$).

- The error value is increasing for the smaller values of the Threshold.

Figure 4. shows optimal voting rule Vs detection threshold in cooperative spectrum sensing in AWGN channel with SNR = 0, 5, 10 dB and cognitive users are $(K) = 16$.

- The Figure 4 shows that variation of the optimal number of Cognitive Radio Users with respect to Threshold for different values of the SNR.

- From that simulation fixing the value of the Threshold for a particular value it gives the optimal number of Cognitive Radio Users. By considering only these optimal number of Cognitive Radio Users instead of taking all the Cognitive Radio Users in the network it will reduces the sensing time.

- The detection threshold is increasing when the SNR value is increasing. i.e. to achieve Optimal number of Cognitive Radio Users for various values of the SNRs, the Threshold must be directly proportional to the SNR.

- From the graph it is giving same number of optimal cognitive radio users even if we fix the Threshold very low value. So we can detect very weak signals also based on Energy Detector detection to obtain the same number of optimal cognitive radio users.

Conclusion

The performance of cooperative spectrum sensing with energy detection in cognitive radio networks is studied. It has been establish that the optimal decision voting rule to minimize the total error probability is the half-voting rule. A method of numerically obtain the optimal detection threshold has been presented. In addition, a proficient spectrum sensing algorithm has been proposed which requires smaller number than the total number of cognitive radios in cooperative spectrum sensing while satisfying a given error bound.

References


RESEARCH PAPERS


CYCLOSTATIONARY DETECTION
BASED SPECTRUM SENSING IN
COGNITIVE RADIO NETWORKS

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Abstract - With the exponential growth of wireless communication, it becomes an important to tackle the spectrum scarcity problem. Most of TV licensed spectrum band, users only utilize their chosen resources partially, thus force the need of cognitive radios (CR) which offers the capable feature of accessing the unused spectrum by dynamic spectrum. In this paper, we are presenting the cyclostationary detection method for estimation and spectral autocorrelation function technique to analyze the spectrum. We used cyclostationary feature detection under modulation scheme to detect the primary users at very low SNR and enhancing cyclostationary feature detection with peak detection algorithm for effective performance. To reduce the noise peaks in the cyclostationary output absolute threshold, standard deviation and filtfilt are the techniques used to get a better efficiency for signal detection.

Index Terms - Cyclostationary, Correlation Function, autocorrelation functions, cyclic frequency, Spectral Coherence, Cyclic domain profile, spectrum sensing.

1 INTRODUCTION

The idea of cognitive radio was first presented officially in an article by Joseph Mitola III and Gerald Q. Maguire, Jr in 1999. Different communication systems such as medical, broadband mobile telecommunication, marine communication, defense and emergency services utilize the radio spectrum. A report by the FCC Spectrum Policy Task Force in November 2002 has stated that the spectrum scarcity problem is mostly due to the insufficient utilization of the RF spectrum rather than the lack of unoccupied frequency bands. With the growth of communication users, makes the spectrum more congested even though federal communication commission (FCC) has expanded some unlicensed spectrum bands for users. The overcrowded spectrum reduces overall quality of service for users in that allotment. To overcome spectrum scarcity a potential solution to this problem is COGNITIVE RADIOS.

"Cognitive radio: A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets. [1]. Cognitive radio is one of the modern techniques for wireless communication systems to utilize the unused spread spectrum effectively. The motivation for Cognitive radio is a concept of utilizing licensed spectrum in an unlicensed manner without causing interference.

Fig 1: Representation of spectrum holes

To implement without Interference to the primary signal, the cognitive radio needs to sense the availability of the spectrum before accessing the channel [2]. So the ability of sensing an idle spectrum and the ability to temporarily utilize a spectrum without interfering with Primary Users are two essential components required for the success of cognitive radios [3]. In cognitive radio terminology, PU can be defined as the user who has license to use a specific part of the spectrum. On the other hand, secondary users (SU) do have license to use the spectrum but only when PU is absent. Cognitive radio can sense the available spectrum for the secondary users when primary user is not using the allotted
frequency spectrum, so that spectrum utilization can be improved. Cognitive radios are fully programmable wireless devices that can sense their environment and dynamically adapt their transmission waveform, channel access method, spectrum use, and networking protocols as needed for good network and application performance. Consequently, spectrum sensing performed by CR cannot be restricted to simply monitor the power in some frequency bands of interest but must include detection and identification in order to avoid interference. The main challenges with cognitive radios are that it should not interfere with the licensed users and should vacate the band when required. For this it should sense the signals faster. CR's are regarded as Transceivers that automatically detect (sense the existence of) available channels in a wireless spectrum and accordingly, change their transmission or reception parameters [3, 4]. This work focuses on the spectrum sensing techniques that are based on primary transmitter detection. Spectrum sensing plays an important role in cognitive radio (CR) systems; the fundamental problem of spectrum sensing is to discriminate an observation that contains only noise from an observation that contains a very weak signal embedded in noise. It is difficult to find vacant bands to deploy new services and enhance existing ones. To overcome this situation, we need an improved utilization of the spectrum which will create opportunities for Dynamic Spectrum Access (DSA).

An important aspect of a cognitive radio is spectrum sensing, which involves two main tasks: signal detection and modulation classification. Signal detection refers to detection of unused spectrum [4] which are called spectrum holes and it is shown in the Fig 1. This task is important so that the unlicensed users do not cause interference to licensed users. Modulation classification consists of automatically identifying the modulation scheme (PSK, FM, BPSK, etc) of a given communication system.

Spectrum sensing and estimation is the first step to implement Cognitive Radio system. Spectrum Sensing Methods are classified as Energy Detection, Matched filter detection and Cyclostationary Based Spectrum Sensing detection. Energy detection as a non coherent method is easy to implement but it cannot discriminate between the primary signal and noise, and hence makes it difficult to set the threshold used for primary user detection, especially at low SNR. Matched filter is an optimal detection technique but requires a prior knowledge of primary user signal for the detection. Cyclostationary feature detection can discriminate between the primary signal and noise, and no need of prior knowledge of primary user for spectrum sensing and produces efficient output even at low SNR Cases. Cyclostationary detector is based on the spectral redundancy present in almost every manmade signal. It is called a cyclic feature detector. The second order cyclostationary is used to extract sine-wave from the signal is introduced by Gardner [5–6]. The mathematical functions used to characterize cyclostationary signals are Cyclic Autocorrelation Function (CAF) and Cyclic Domain Profile (CDP). Cyclic Domain Profile refers to the cyclic repetition of frequency [7]. In a cognitive radio system A primary system operated in the licensed band has the highest priority to use that frequency band (e.g. 3G/4G cellular, digital TV broadcast). Other unlicensed users/systems can neither interfere with the primary system in an Intolerable way nor occupy the license band [13].

2 CYCLOSTATIONARY DETECTION

In Cyclostationary signals, the mean value and autocorrelation function have periodicity. In this paper a signal is taken which can be called as primary signal

\[ X(t) = s(t) + w(t) \]

Where x(t) is the input transmitted signal

w(t) is the noise signal (AWGN) and

s(t) is the primary user signal

Cyclic spectral analysis deals with second order transformations of a function and its spectral representation. A function x(t) is said to exhibit second order periodicity if spectral components of x(t) exhibit temporal correlation.

A. Temporal Redundancy:

A wide-sense cyclostationary signal x(t) exhibits a periodic autocorrelation function [6, 8]. It has periodic components that can be found by CR to eliminate it from noise. A cyclostationary process is a signal having statistical properties that vary cyclically with time. A cyclostationary process can be viewed as multiple interleaved stationary processes. These processes are not periodic function of time but their statistical features indicate periodicities. The following conditions are essential to be filled by a process for it to be wide sense cyclostationary. The periodicity of the mean and autocorrelation functions are expressed by the equations as follows:

\[ R_x(t, \tau) = E[x(t)x^*(t-\tau)] \quad (1) \]

Mean function is expressed as

\[ m_x = E[x(t)] = 0 \]

Since autocorrelation function is periodic it can be expressed by applying Fourier series which is decomposed as

\[ R_x(t, \tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi f_{\alpha} t} \quad (2) \]

Where the sum is over integer multiples of the fundamental frequencies. The coefficient \( R_x^{\alpha}(\tau) \) is called the cyclic autocorrelation function, and represents the Fourier coefficient of the series given by

\[ R_x^{\alpha}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} R_x(t, \tau)e^{-j2\pi f_\alpha t} dt \quad (3) \]
The autocorrelation function is replaced by its time average which is represented as
\[ R_x'(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t-\tau)e^{-j2\pi ft} dt \] (4)

The cyclic autocorrelation is therefore intuitively obtained by extracting the frequency \( \alpha \) sine-wave from the time-delay product \( x(t)x^*(t-\tau) \). The Spectral correlation density (SCD) \( S^\alpha_x(f) \) is defined as the Fourier transform of \( R^\alpha_x(\tau) \) over \( \tau \).

### B. Spectral Redundancy:

The Fourier transform of \( x(t) \) is \( X(f) \). The SCD measures the degree of spectral redundancy between the frequencies \( f - \alpha/2 \) and \( f + \alpha/2 \) (\( \alpha \) is the cyclic frequency). The Fourier transform of autocorrelation function is defined as Spectral Correlation Function (SCF) [12] and is expressed as
\[ S^\alpha_x(f) = \int_{-\infty}^{\infty} R^\alpha_x(\tau)e^{-j2\pi ft} d\tau \] (5)

It can be mathematically expressed as the correlation between two frequency bins centered on \( f - \alpha/2 \) and \( f + \alpha/2 \) when their width tends toward zero [6, 8].

\[ S^\alpha_x(f) = \lim_{\Delta f \to 0} \lim_{\Delta t \to 0} \frac{1}{\Delta f \Delta t} \int_{-\Delta t/2}^{\Delta t/2} X_T(t, f + \alpha/2)X^*_T(t, f - \alpha/2) dt \] (6)

In practice there are only a limited number of samples available and hence SCF needs to be estimated from these samples. Let us define the cyclic periodogram as [9][10].

\[ S^\alpha_{tx}(t, f) = \frac{1}{T} X_T^2(t, f, \alpha/2) \] (7)

Where \( X_T(t, f) \) is the short-time Fourier transform of signal
\[ X_T(t, f) = \int x(u)e^{-j2\pi fu} du \] (8)

SCF can be obtained by increasing the observation length \( T \) and decreasing \( \Delta t \).

\[ S^\alpha_x(f) = \lim_{T \to \infty} \lim_{\Delta t \to 0} S^\alpha_{tx}(t, f) \] (9)

### C. Spectral Coherence and \( \alpha \)-Profile:

SCF is a correlation of frequency components shifted by \( f + \alpha/2 \) and \( f - \alpha/2 \). It is intuitive to define Spectral Coherence (SC) [11] as
\[ C^\alpha_x(f) = \frac{S^\alpha_x(f)}{S(f + \alpha/2)S(f - \alpha/2)} \] (10)

The magnitude of SC is always between 0 and 1. In order to reduce the computational complexity, one just uses the Cyclic Domain Profile (CDP) or \( \alpha \)-profile which is defined as
\[ I(\alpha) = \max_{f} \left| C^\alpha_x(f) \right| \] (11)

The process is shown in the Fig.2 for better understanding.

D. Signal detection

All man-made signals and modulated signals exhibit second order cyclostationary. From the CDP of the signal, important information about the signal like modulation type, keying rate, pulse shape, and carrier frequency can be obtained, [12]. Fig. 4 shows the CDP for BPSK. When SCF is plotted, the occupancy status of the spectrum can be found out. If a primary user signal is present in the operating frequency range, the SCF gives a peak at its centre. The peak will not be present in the case when there is no primary user signal present in the concerned frequency range. Spectrum sensing is used to determine the presence or absence of primary users so we need to distinguish between these two hypotheses [14];

\[ H_0 : x(t) = n(t) \]
\[ H_1 : x(t) = s(t) + n(t) \] (12)

First we need to determine the threshold \( C_{TH} \). For signal detection and when no signal is present, i.e. \( x(t) = n(t) \), \( C_{TH} \) will use the relationship as [15];

\[ C_{TH} = \max[I(\alpha)]/\sqrt{\sum_{i=0}^{N} I^2(\alpha)/N} \] (13)

\( N \) is the length of observation data.

We can distinguish signal from noise by analyzing the SCD function. Furthermore, it is possible to distinguish the signal type because different signals may have different nonzero cyclic frequencies. Cyclostationary detection block contains a FFT, AWGN, correlate, average over threshold and a feature detection block is shown in the Fig.3.

A random discrete signal is taken and modulated using different modulation schemes. The CFD basically contains filters, ADC, quantizer, encoder, and fft blocks. In this paper we use fast Fourier transform (FFT) and a Noise is added by AWGN block. Cyclostationary feature detection method deals with the inherent cyclostationary properties or features of the signal. Such features have a periodic statistics and spectral correlation that cannot be found in any interference signal or stationary noise. It exploits this
periodicity in the received primary signal to identify the presence of primary users, and that is why the cyclostationary feature detection method possesses higher noise immunity than any other spectrum sensing method. The output is taken using spectrum analyzer which displays the output in a graphical form which can be easily understandable. The output plot thus obtained is the cyclic SCF. Peak detection algorithm is used for the Cyclostationary output. The plot between probability of detection and SNR is termed as the receiver operating characteristics; using sensing algorithm the cyclostationary detection method, shows that the primary signal is present, and probability of detection increases with Different SNR values.

Techniques applied

The Cyclic Domain Profile for BPSK Signals are extracted and for the obtained signals the following techniques are applied to minimize the noise for better probability of detection. Fig shows the Cyclic Domain Profile of BPSK signal.

I. Absolute Threshold:

In some applications we do not need to know the exact peak amplitudes and locations, rather we need to know the number or general location of peaks, in this case we use an absolute threshold function. The absolute threshold can be influenced by several different factors such as motivations, expectations, and cognitive processes, that whether the subject is adapted to the stimulus. The absolute threshold can be compared to the difference threshold which is the measure of how two different stimuli must be for the subject to notice that they are not the same. For the cyclostationary output the absolute threshold is applied where it reduces the noise peaks to a minimum. Fig 5 shows reduction of noise peaks at different frequencies, and a centre peak that indicates the probability of detection. The noise peaks are diminished by considering an absolute value of Cyclostationary output.

II. Standard deviation:

Standard deviation is a measure for how much the frequencies in a spectrum can deviate from the centre of gravity. For a sine wave the Standard deviation is zero and hence by increasing the number of samples the noise peaks are diminished as shown in the Fig 6. The standard deviation diminishes the noise peaks more than the absolute threshold based on the increased number of samples.

\[
S = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{1/2}
\]

Where \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \)

Fig. 6. Detection of peak signal using standard deviation

III. Filtfilt:

Filtfilt is a zero phase forward and Reverse filtering, after filtering in the forward direction; the filtered sequence is
then reversed and run back through the filter. The filter is described by the difference equation:

\[ y(n) = b(1)\cdot x(n) + b(2)\cdot x(n-1) + \ldots + b(nb+1)\cdot x(n-nb) - a(2)\cdot y(n-1) - \ldots - a(na+1)\cdot y(n-na). \]

Where \( y \) is the time reverse of the output of the second filtering operation. The result has precisely zero phase distortion and magnitude modified by the square of the filter's magnitude response. The length of the input \( x \) must be more than three times the filter order, defined as \( \max(\text{length}(b)-1, \text{length}(a)-1) \). FILTFILT should not be used with differentiator and Hilbert FIR filters, since the operation of these filters depends heavily on their phase response. Fig 7 shows the output of the filtfilt command where the noise is completely diminished by using the difference equation. By comparing with the other two techniques filtfilt is the best method for reducing the noise by filtering.

Finally for the obtained signal the Probability of detection for different SNR is calculated and plotted which is shown in the Fig 8.

**CONCLUSION**

In this paper, we have presented the peak detection algorithm for estimation and detection of the primary signal by applying certain threshold and techniques to analyze the spectrum. If the peak signal is present at the centre of the SCF then it is said to be that primary user is present if not the primary user is absent and the secondary user can occupy the spectrum band and if primary user needs the spectrum band the secondary user must vacate the spectrum by sensing the spectrum using spectrum sensing technique. Among the techniques the filtfilt outperforms. Thus the Cyclostationary detection is best suited for very low SNR.

**REFERENCES**


Efficient Cyclostationary Detection Based Spectrum Sensing in Cognitive Radio Networks

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Abstract- As the customers in wireless communication becoming crowdy it becomes an important to tackle the spectrum scarcity problem. Most of TV licensed spectrum band, users only utilize their chosen resources partially, thus force the need of cognitive radios (CR) which offers the capable feature of accessing the unused spectrum by dynamic spectrum.

In this paper, we are presenting the cyclostationary detection method for estimation and spectral autocorrelation function technique to analyze the spectrum. We used cyclostationary feature detection under modulation scheme to detect the primary users at very low SNR and enhancing cyclostationary feature detection with peak detection algorithm for effective performance. To reduce the noise peaks in the cyclostationary output absolute threshold, standard deviation and filtfilt are the techniques used to get a better efficiency for signal detection.

Keywords- Cyclostationary, Correlation Function, autocorrelation functions, cyclic frequency, Spectral Coherence.

I. INTRODUCTION

The idea of cognitive radio was first presented officially in an article by Joseph Mitola III and Gerald Q. Maguire, Jr in 1999. Different communication systems such as medical, broadband - mobile - telecommunication, marine communication, defense and emergency services utilize the radio spectrum. A report by the FCC

Spectrum Policy Task Force in November 2002 has stated that the spectrum scarcity problem is mostly due to the insufficient utilization of the RF spectrum rather than the lack of unoccupied frequency bands. With the growth of communication users, makes the spectrum more congested even though federal communication commission (FCC) has expanded some unlicensed spectrum bands for users. The overcrowded spectrum reduces overall quality of service for users in that allotment. To overcome spectrum scarcity a potential solution to this problem is COGNITIVE RADIOS.

“Cognitive radio: A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets. [1]. Cognitive radio is one of the modern techniques for wireless communication systems to utilize the unused spread spectrum effectively. The motivation for Cognitive radio is a concept of utilizing licensed spectrum in an unlicensed manner without causing interference.

To implement without Interference to the primary signal, the cognitive radio needs to sense the availability of the spectrum before accessing the channel [2]. So the ability of sensing an idle spectrum and the ability to temporarily utilize a spectrum without interfering with Primary Users are two essential components required for the success of cognitive radios [3]. In cognitive radio terminology, PU can be defined as the user who has license to use a specific part of the spectrum. On the other hand, secondary users (SU) do have license to use the spectrum but only when PU is absent.

Cognitive radio can sense the available spectrum for the secondary users when primary user is not using the allotted frequency spectrum, so that spectrum utilization can be improved. Cognitive radios are fully programmable wireless devices that can sense their environment and dynamically adapt their transmission waveform, channel access method, spectrum use, and networking protocols as needed for good network and application performance. Consequently, spectrum sensing performed by CR cannot be restricted to simply monitor the power in some frequency bands of interest but must include detection and identification in order to avoid interference. The main challenges with cognitive radios are that it should not interfere with the licensed users and should vacate the band when required. For this it should sense the signals faster. CR’s are regarded as Transceivers that automatically detect (sense the existence of) available channels in a wireless spectrum and accordingly, change their transmission or reception parameters [3, 4]. This work focuses on the spectrum sensing techniques that are...
based on primary transmitter detection. Spectrum sensing plays an important role in cognitive radio (CR) systems; the fundamental problem of spectrum sensing is to discriminate an observation that contains only noise from an observation that contains a very weak signal embedded in noise. It is difficult to find vacant bands to deploy new services and enhance existing ones. To overcome this situation, we need an improved utilization of the spectrum which will create opportunities for Dynamic Spectrum Access (DSA).

An important aspect of a cognitive radio is spectrum sensing, which involves two main tasks: signal detection and modulation classification. Signal detection refers to detection of unused spectrum [4] which are called spectrum holes and it is shown in the Fig 1. This task is important so that the unlicensed users do not cause interference to licensed users. Modulation classification consists of automatically identifying the modulation scheme (PSK, FM, QPSK, etc) of a given communication system.

Spectrum sensing and estimation is the first step to implement Cognitive Radio system. Spectrum Sensing Methods are classified as Energy Detection, Matched filter detection and Cyclostationary Based Sensing Spectrum detection. Energy detection as a non coherent method is easy to implement but it cannot discriminate between the primary signal and noise, and hence makes it difficult to set the threshold used for primary user detection, especially at low SNR. Matched filter is an optimal detection technique but requires a priori knowledge of primary user signal for the detection. Cyclostationary feature detection can discriminate between the primary signal and noise, and need no prior knowledge of primary user for spectrum sensing and produces efficient output even at low SNR Cases. Cyclostationary detector is based on the spectral redundancy present in almost every manmade signal. It is called a cyclic feature detector. The second order cyclostationary is used to extract sine-wave from the signal is introduced by Gardner in [5–6]. The mathematical functions used to characterize cyclostationary signals are Cyclic Autocorrelation Function (CAF) and Cyclic Domain Profile (CDP). Cyclic Domain Profile refers to the cyclic repetition of frequency [7]. In a cognitive radio system A primary system operated in the licensed band has the highest priority to use that frequency band (e.g. 3G/4G cellular, digital TV broadcast). Other unlicensed users/systems can neither interfere with the primary system in an Intolerable way nor occupy the license band [13].

II. CYCLOSTATIONARY DETECTION

In Cyclostationary signals, the mean value and autocorrelation function have periodicity. In this paper a signal is taken which can be called as primary signal

\[ X(t) = s(t) + w(t) \]

Where \( s(t) \) is the input transmitted signal \( w(t) \) is the noise signal (AWGN) and \( s(t) \) is the primary user signal

Cyclic spectral analysis deals with second order transformations of its function and its spectral representation. A function \( x(t) \) is said to exhibit second order periodicity if spectral components of \( x(t) \) exhibit temporal correlation.

A. Temporal Redundancy:

A wide-sense cyclostationary signal \( x(t) \) exhibits a periodic autocorrelation function [6, 8]. It has periodic components that can be found by CR to eliminate it from noise. A cyclostationary process is a signal having statistical properties that vary cyclically with time. A cyclostationary process can be viewed as multiple interleaved stationary processes. These processes are not periodic function of time but their statistical features indicate periodicities. The following conditions are essential to be filled by a process for it to be wide sense cyclostationary. The periodicity of the mean and autocorrelation functions are expressed by the equations are as follows:

\[ R_x(t,\tau) = E[x(t)x^*(t - \tau)] \]

Mean function is expressed as

\[ m_x = E[x(t)] = 0 \]

Since autocorrelation function is periodic it can be expressed by applying Fourier series which is decomposed as

\[ R_x(t,\tau) = \sum_{\alpha} R_x^\alpha(\tau) e^{j2\pi\alpha t} \]

Where the sum is over integer multiples of the fundamental frequencies. The coefficient \( R_x^\alpha(\tau) \) is called the cyclic autocorrelation function, and represents the Fourier coefficient of the series given by

\[ R_x^\alpha(\tau) = \frac{1}{T} \int_{T/2}^{T/2} R_x(t,\tau) e^{-j2\pi\alpha t} dt \]

The autocorrelation function is replaced by its time average which is represented as

\[ R_x^\alpha(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x(t)x^*(t - \tau) e^{-j2\pi\alpha t} dt \]

The cyclic autocorrelation is therefore intuitively obtained by extracting the frequency \( \alpha \) sine-wave from the time-delay product \( x(t)x^*(t-\tau) \). The Spectral correlation density (SCD) \( S_x^\alpha(f) \) is defined as the Fourier transform of \( R_x^\alpha(\tau) \) over \( \tau \).

B. Spectral Redundancy:

The Fourier transform of \( x(t) \) is \( X(f) \). The SCD measures the degree of spectral redundancy between the frequencies \( f - \alpha/2 \) and \( f + \alpha/2 \) (\( \alpha \) is the cyclic frequency). The Fourier transform of autocorrelation function is defined as Spectral Correlation Function (SCF) [12] and is expressed as

\[ S_x^\alpha(f) = \int R_x^\alpha(\tau) e^{-j2\pi\alpha f} d\tau \]

It can be mathematically expressed as the correlation between two frequency bins centered on \( f - \alpha/2 \) and \( f + \alpha/2 \) when their width tends toward zero [6, 8].

\[ S_x^\alpha(f) = \lim_{\Delta f \to 0} \left| \int_{-\Delta f/2}^{\Delta f/2} X_T^*(t,\frac{f + \alpha}{2}) X_T(t,\frac{f - \alpha}{2}) dt \right| \]
In practice there are only a limited number of samples available and hence SCF needs to be estimated from these samples. Let us define the cyclic periodogram as [9][10].

\[
S_x^{\alpha}(f) = \frac{1}{T} X_T(t) \frac{f - \alpha}{2} X_T^*(t) \frac{f - \alpha}{2}
\]

Where \( X_T(t) \) is the short-time Fourier transform of signal \( X_T(t, f) = \int x(u)e^{-j2\pi fu} du \)

SCF can be obtained by increasing the observation length \( T \) and decreasing \( \Delta t \).

\[
S_x^{\alpha}(f) = \lim_{T \to \infty} \lim_{\Delta \to 0} S_x^{\alpha}(t, f)
\]

C. Spectral Coherence and \( \alpha \)-Profile:

SCF is a correlation of frequency components shifted by \( f + \frac{\alpha}{2} \) and \( f - \frac{\alpha}{2} \). It is intuitive to define Spectral Coherence (SC) [11] as

\[
C_x = \frac{S_x^{\alpha}(f)}{[S(f + \frac{\alpha}{2})S(f - \frac{\alpha}{2})]^{1/2}}
\]

The magnitude of SC is always between 0 and 1. In order to reduce the computational complexity, one just uses the Cyclic Domain Profile (CDP) or \( \alpha \)-profile which is defined as

\[
I(\alpha) = \max_f |C_x(f)|
\]

The process is shown in the Fig.2 for better understanding.

D. Signal detection

All man-made signals and modulated signals exhibit second order cyclostationary. From the CDP of the signal, important information about the signal like modulation type, keying rate, pulse shape, and carrier frequency can be obtained, [12]. Fig. 4 shows the CDP for QPSK. When SCF is plotted, the occupancy status of the spectrum can be found out. If a primary user signal is present in the operating frequency range, the SCF gives a peak at its centre. The peak will not be present in the case when there is no primary user signal present in the concerned frequency range. Spectrum sensing is used to determine the presence or absence of primary users so we need to distinguish between these two hypotheses [14];

First we need to determine the threshold \( C_{TH} \). For signal detection and when no signal is present, i.e. \( x(t) = n(t) \), CTH will use the relationship as [15]:

\[
C_{TH} = \max(I(\alpha)/\sqrt{\sum_{\alpha=0}^{N} f^2(\alpha)/N})
\]

N is the length of observation data.

We can distinguish signal from noise by analyzing the SCD function. Furthermore, it is possible to distinguish the signal type because different signals may have different nonzero cyclic frequencies. Cyclostationary detection block contains a FFT, AWGN, correlate, average over threshold and a feature detection block is shown in the Fig.3.

A random discrete signal is taken and modulated using different modulation schemes. The CFD basically contains filters, ADC, quantizer, encoder, and fft blocks. In this paper we use fast Fourier transform (FFT) and a Noise is added by AWGN block. Cyclostationary feature detection method deals with the inherent cyclostationary properties or features of the signal. Such features have a periodic statistics and spectral correlation that cannot be found in any interference signal or stationary noise. It exploits this periodicity in the received primary signal to identify the presence of primary users, and that is why the cyclostationary feature detection method possesses higher noise immunity than any other spectrum sensing method. The output is taken using spectrum analyzer which displays the output in a graphical form which can be easily understandable. The output plot thus obtained is the cyclic SCF. Peak detection algorithm is used for the Cyclostationary output. The plot between probability of detection and SNR is termed as the receiver operating characteristics; using sensing algorithm the cyclostationary detection method, shows that the primary signal is present, and probability of detection increases with Different SNR values.

III. TECHNIQUES APPLIED

The Cyclic Domain Profile for QPSK Signals are extracted and for the obtained signals the following techniques are applied to minimize the noise for better probability of detection. Fig shows the Cyclic Domain Profile of QPSK signal.
1. Absolute Threshold:

In some applications we do not need to know the exact peak amplitudes and locations, rather we need to know the number or general location of peaks, in this case we use an absolute threshold function. The absolute threshold can be influenced by several different factors such as motivations, expectations, and cognitive processes, that whether the subject is adapted to the stimulus. The absolute threshold can be compared to the difference threshold which is the measure of how two different stimuli must be for the subject to notice that they are not the same. For the cyclostationary output the absolute threshold is applied where it reduces the noise peaks to a minimum. Fig 5 shows reduction of noise peaks at different frequencies, and a centre peak that indicates the probability of detection. The noise peaks are diminished by considering an absolute value of Cyclostationary output.

2. Standard deviation:

Standard deviation is a measure for how much the frequencies in a spectrum can deviate from the centre of gravity. For a sine wave the Standard deviation is zero and hence by increasing the number of samples the noise peaks are diminished as shown in the Fig 6. The standard deviation diminishes the noise peaks more than the absolute threshold based on the increased number of samples.

$$S = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{1/2}$$

Where $$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$.

3. Filtfilt:

Filtfilt is a zero phase forward and Reverse filtering, after filtering in the forward direction; the filtered sequence is then reversed and run back through the filter. The Filter is described by the difference equation:

$$y(n) = b(1)x(n) + b(2)x(n-1) + ... + b(n_b+1)x(n-n_b) - a(2)y(n-1) - ... - a(n_a+1)y(n-n_a).$$

Where $$y$$ is the time reverse of the output of the second filtering operation. The result has precisely zero phase distortion and magnitude modified by the square of the filter's magnitude response. The length of the input $$x$$ must be more than three times the filter order, defined as max(length(b)-1, length(a)-1). FILTFILT should not be used with differentiator and Hilbert FIR filters, since the operation of these filters depends heavily on their phase response. Fig 7 shows the output of the filtfilt command where the noise is completely diminished by using the difference equation. By comparing with the other two techniques filtfilt is the best method for reducing the noise by filtering.
Finally for the obtained signal the Probability of detection for different SNR is calculated and plotted which is shown in Fig 8.

Fig 8: Probability of detection for different SNR.

**IV. CONCLUSION**

In this paper, we have presented the peak detection algorithm for estimation and detection of the primary signal by applying certain threshold and techniques to analyze the spectrum. If the peak signal is present at the centre of the SCF then it is said to be that primary user is present if not the primary user is absent and the secondary user can occupy the spectrum band and if primary user needs the spectrum band the secondary user must vacate the spectrum by sensing the spectrum using spectrum sensing technique. Among the techniques the filtfilt outperforms. Thus the Cyclostationary detection is best suited for very low SNR.

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Enhanced Throughput and Energy Efficient Approach for Cognitive Radio using Cooperative Spectrum Sensing

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Abstract

Cognitive radio is a promising technology for efficient spectrum usage. Cognitive radios are proposed as a way-out to the spectrum underutilization problem and proved that spectrum efficiency increases largely. The performance of the cognitive radio based on cooperative spectrum sensing is analyzed with energy and throughput setup. In the energy efficient setup, the number of cooperating cognitive radios is minimize for a $k$-out-of-$N$ fusion rule with a restraint on the probability of detection and false alarm while in the throughput setup, we maximize the throughput of the cognitive radio network, by deriving the optimal reporting time in a sensing time frame which is proportional to the number of cognitive users, subject to a constraint on the probability of detection. The above problems can be simplified to line search problems. The simulation results show that the OR and the majority rule do better than the AND rule in terms of energy efficiency and OR rule gives a higher throughput than the AND rule with a limited number of users.

Keywords—Cognitive radio, Cooperative spectrum sensing, Throughput, Energy efficiency, Hard decision Fusion Center.

1. Introduction

Low detection reliability, fading and shadowing problems are overcome with Cooperative spectrum sensing in a single radio detection scheme [2]. In this paper, a cognitive radio network is considered, where each cognitive user makes a local decision about the presence of primary user presence sends the information to Fusion Center (FC) as shown in Fig.1, by employing a Time-Division-Multiple-Access (TDMA) approach. The decision is made at the FC. Here a hard fusion scheme [3][4], is considered due to its improved energy and bandwidth efficiency. In this scheme, the OR and AND rules are special cases of the more general $k$-out-of-$N$ rule with $k = 1$ and $k = N$, respectively. In $k$-out-of-$N$ rule, the FC decides the target presence, if at least $k$-out-of-$N$ sensors report to the FC that the target is present [3]. In the cognitive radio network throughput is optimized [1] subject to a detection rate constraint in order to find different system parameters including the detection threshold, sensing time and optimal $k$ for a fixed number of users. However, the effect of the reporting time corresponding to the number of cognitive radios on reducing the throughput of the cognitive radio network has not extensively been studied. In [6], the number of cognitive radios is minimized under a detection error probability constraint. However, the detection error probability is formulated as a weighted sum of the probability of false alarm and detection and it does not have a meaningful interpretation from a cognitive radio perception. In [7], the effect of the cooperation overhead on the throughput of the cognitive network is considered for a soft decision scheme. However, an exact problem formulation that allows for parameter optimization, such as the threshold, is not provided. In this paper we find the the optimal number of cognitive radios, $N$ involved in the spectrum sensing under two scenarios. i) An energy efficient setup, defined by minimizing the number of cognitive radios subject to a constraint on the global probability of false alarm and detection. ii) A throughput optimization setup where the throughput of the cognitive radio network is maximized subject to a constraint on the global probability of detection in order to determine the optimal number of cognitive users for a fixed $k$ and sensing duration.

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2. System Model

A network is considered with N identical cognitive radios under a cooperative spectrum sensing scheme. Each cognitive radio senses the spectrum periodically and makes a local decision about the presence of the primary user during observation. The local decisions are to be sent to the FC in slots allocated on TDMA frame.

Each cognitive radio solves a binary hypothesis testing problem to make local decisions about the presence or absence of the primary user, by choosing $H_1$ for present and $H_0$ for absent.

$$\begin{align*}
H_0 & : y[n] = w[n], \quad n = 1, \ldots, M \\
H_1 & : y[n] = x[n] + w[n], \quad n = 1, \ldots, M
\end{align*}$$

(1)

Where the noise and the signal are assumed to be i.i.d Gaussian random processes with zero mean and variance $\sigma_w^2$ and $\sigma_x^2$ respectively and received signal-to-noise ratio (SNR) is denoted by

$$\gamma = \frac{\sigma_x^2}{\sigma_w^2}$$

(2)

Each cognitive radio employs an energy detector in which the accumulated energy $M$ observation samples is to be compared with a predetermined threshold denoted by $\lambda$, as follows

$$E = \sum_{m=1}^{M} y^2[n] > \lambda$$

(3)

For a large number of samples we can employ the central limit theorem and the decision statistic is distributed as[2]

$$\begin{align*}
H_0 & : E \sim N(M\sigma_w^2, 2\sigma_w^4) \\
H_1 & : E \sim N(M(\sigma_w^2 + \sigma_x^2), 2M(\sigma_w^2 + \sigma_x^2)^2)
\end{align*}$$

(4)

Denoting $P_f$ and $P_d$ as the respectively local probabilities of false alarm and detection, $P_f = \Pr(E \geq \lambda, H_0)$ and $P_d = \Pr(E \geq \lambda, H_1)$ are given by

$$\begin{align*}
P_f & = Q \left( \frac{\lambda - M\sigma_w^2}{\sqrt{2M} \sigma_w^4} \right), \\
P_d & = Q \left( \frac{\lambda - (\sigma_w^2 + \sigma_x^2)}{\sqrt{2M(\sigma_w^2 + \sigma_x^2)^2}} \right)
\end{align*}$$

(5)

The reported local decisions are combined at the FC and the final decision regarding the presence or absence of the primary user is made according to a certain fusion rule. Several fusion schemes have been discussed in the literature [4]. Due to its simplicity in implementation, lower overhead and energy consumption, we employ a $K$-out-of-$N$ rule is combine the local binary decision sent to the FC. Thus, the resulting binary hypothesis sent to the FC is given by,

$$I = \sum_{i=1}^{N} D_i < K \quad \text{for} \quad H_0$$

$$I = \sum_{i=1}^{N} D_i \geq K \quad \text{for} \quad H_1$$

(6)

Where $D_i$ is the binary local decision of the $i$-th cognitive radio which takes a binary value 0 if the local decision supports the absence of the primary user and 1 for the presence of the primary user. Each cognitive radio employs an identical threshold $\lambda$ to make the decision. Hence, the global probability of false alarm ($Q_f$) and the ($Q_d$) at the FC is given by,

$$\begin{align*}
Q_f & = \sum_{i=k}^{N} N_i P_i^f (1 - P_i^d)^{N-i} \\
Q_d & = \sum_{i=k}^{N} N_i P_i^d (1 - P_i^f)^{N-i}
\end{align*}$$

(7)

Each cognitive radio employs periodic time frame of length $T$ for sensing and transmission. The time frame for each cognitive radio is shown in fig. 2. Each frame comprises two parts namely a sensing time required for observation and decision making and a transmission time required for energy accumulation and local decision making denoted by $T_x$ for transmission in case the primary user is absent. The sensing time can be further divided into a time required for energy accumulation and local decision making denoted by $T_a$ and a reporting time where cognitive radios send their local decisions to the FC. Here, we employ a TDMA based approach for reporting the local decision to the FC. Hence, denoting $T_r$ as the required time for each cognitive radio to report its result, as the required time for a network with $N$ cognitive radios is $NT_r$.

Considering the cognitive radio time frame, the normalized effective throughput, $R$, of the cognitive radio network is given by

$$R = \frac{T - T_a - NT_r (1 - Q_f)}{T}$$

(8)
In the next section, we derive the optimal number of cognitive radios participating in spectrum sensing from two viewpoints, an energy efficient and throughput optimization setup.

3. Problem Formulation

The cooperative sensing performance improves with the number of cognitive users. However, a large number of cooperative users leads to a higher network energy consumption and reporting time. Therefore, it is desirable to find the optimal number of users that satisfies a certain detection performance constraint defined by the probability of false alarm and detection [8]. A high probability of detection represents a low interference to the primary user and a low probability of false alarm represents high spectrum utilization. In the following subsections, first the number of cognitive radios is minimized to meet the system requirements on interference and false alarm and then we consider a setup where network throughput is maximized subject to a constraint on the interference to find the system parameters including the number of users and the probability of false alarm.

3.1. Energy efficient setup

The detection performance of a cognitive radio network is closely related to the number of cooperating cognitive radios. The larger the number of cognitive radios, higher the detection performance, which in turn increases the network energy consumption. The current standards [5] impose a lower bound on the probability of detection and an upper bound on the probability of false alarm. Therefore, as soon as these constraints are satisfied, increasing the number of cognitive users is a waste of energy which is very critical for cognitive sensor networks. Hence, it is necessary to design an efficient mechanism to reduce the network energy consumption while still maintaining the standard requirements on the interference and false alarm.

We define the energy efficiency optimization problem [6] so as to minimize the total number of cooperative cognitive users to attain the required probability of false alarm and probability of detection for the fixed $K$ as follows,

\[
\min N \quad \text{s.t. } Q_d < \alpha \text{ and } Q_f \leq \beta
\]

The optimal value of $N$ is attained by for a minimum value of $N$ in the feasible set of (9). We rewrite (7) using the binomial theorem as follows,

\[
Q_f = 1 - \psi(k - 1, P_f, N) ,
\]

\[
Q_d = 1 - \psi(k - 1, P_d, N) \tag{10}
\]

Where $\psi$ is the regularized incomplete beta function as follows,

\[
\psi(k, p, n) = I_{1-p}(n-k, k+1)
\]

\[
= (n-k) k \int_0^{1-p} t^{n-k-1}(1-t)^k dt \tag{11}
\]

Denoting $P_x$ as the local probability of detection or false alarm and $Q_x$ as the global probability of detection or false alarm, we can define $P_x = \psi^1(k, 1 - Q_x, N)$ as the inverse function of $\psi$ in the second variable. For the given $K$ and $N$, since $\psi$ and $\psi^{-1}$ are monotonic increasing functions in $P_x$ and $Q_x$, respectively, the constraints in (9) become

\[
P_f = \psi^{-1}(k - 1, 1 - Q_f, N) \leq \psi^{-1}(k - 1, 1 - \beta, N) \tag{12}
\]

\[
P_d = \psi^{-1}(k - 1, 1 - Q_d, N) \leq \psi^{-1}(k - 1, 1 - \alpha, N) \tag{13}
\]

From the $P_d$ expression in (5) we obtain

\[
\lambda = \sqrt{2M(\sigma_\omega^2 + \sigma_\gamma^2)^2 Q^{-1}(P_d) + M(\sigma_\omega^2 + \sigma_\gamma^2)^2}.
\]

Inserting $\lambda$ in $P_f$, we obtain

\[
P_f = Q \left( \frac{M \sigma_\omega^2 + Q^{-1}(P_d) \sqrt{2M(\sigma_\omega^2 + \sigma_\gamma^2)^2}}{\sqrt{2M \sigma_\omega^4}} \right).
\]

Applying this to (12), we obtain after some simplifications.

\[
P_f \geq Q \left( \frac{M \sigma_\omega^2 + Q^{-1}(\xi_x) \sqrt{2M(\sigma_\omega^2 + \sigma_\gamma^2)^2}}{\sqrt{2M \sigma_\omega^4}} \right) \tag{14}
\]

Where $\xi_x = \psi^{-1}(k - 1, 1 - \alpha, N)$.

Therefore, for any $K$, based on (11) and (12), the optimal $N$ will be the minimal solution of the following inequality,

\[
Q \left( \frac{M \sigma_\omega^2 + Q^{-1}(\xi_x) \sqrt{2M(\sigma_\omega^2 + \sigma_\gamma^2)^2}}{\sqrt{2M \sigma_\omega^4}} \right) \leq \xi \beta \tag{15}
\]
Where \( \xi_\beta = \psi^{-1}(k-1,1-\beta,N) \) and \( Q^{-1}(x) \) is the inverse Q-function. Therefore, the optimal value of \( N \) can be found by an exhaustive search over \( N \) from 1 to the first value that satisfies (14).

Based on (14), the optimal \( N \) for the AND rule is the minimum solution of the following inequality problem,

\[
Q(A + BQ^{-1}(\alpha^{1/N})) \leq \beta^{1/N},
\]

(16)

And for the OR rule, the optimal \( N \) is the minimum solution of the following inequality,

\[
Q(A + BQ^{-1}(\alpha')) \leq \beta'
\]

(17)

Where, \( \alpha' = 1 - (1 - \alpha)^{1/N} \), \( \beta' = 1 - (1 - \beta)^{1/N} \)

\[
A = \gamma \sqrt{\frac{M}{2}}, \quad B = 1 + \gamma
\]

3.2. Throughput optimization setup

Optimization of the reporting time as received less attention in the literature, although it is a necessary redundancy in the system. Reducing it leads to an increase in the throughput of the cognitive radio network. Here, we fix the sensing time, \( T_s \), and focus on optimizing the reporting time \( NT_r \) where \( T_r = \frac{1}{R_0} \), with \( R_0 \) the cognitive radio transmission bit rate.

The previous setup is focused toward reducing the number of cognitive radios while maintaining a certain false alarm rate and interference constraint mainly to reduce the energy consumption of the system. However, the energy efficient setup also increases the throughput by reducing the reporting time for a bounded probability of false alarm. Here, we explain that feature in more detailed and defined our problem as to maximize the throughput of the cognitive radio network, while maintaining the required probability of detection specified by the standard. The solution for the optimization problem determines the optimal \( N \) that maximize the throughput\cite{12} yet meeting the specified constrain. First, we present the optimization problem for an arbitrary \( K \) and then we focus on the optimization problem for two special cases: the OR and AND rule. The optimization problem is given by

\[
\max_{N,P_f} \left( \frac{T - T_s - NT_r}{T} \right) (1 - Q_f)
\]

s.t. \( Q_d \geq \xi \) and \( 1 \leq N \leq \left[ \frac{T - T_s}{T_r} \right] \)

(18)

For the given \( N \) the optimization problem reduces to,

\[
\max_{P_f} (1 - Q_f), \quad \text{s.t.} \ Q_d \geq \xi
\]

(19)

Which can be further simplified to

\[
\min_{P_f} Q_f
\]

s.t. \( P_d \geq \psi^{-1}(k-1,1-\alpha,N) \)

(20)

Since the probability of false alarm grows with the probability of detection, the solution of (19) is the \( P_f \) that satisfies \( P_d = \psi^{-1}(k-1,1-\alpha,N) \). Hence, the optimal \( P_f \) is given by,

\[
\hat{P}_f = Q \left( \frac{M\sigma^2_e + Q^{-1}(\xi,\alpha')\sqrt{2M(\sigma^2_e + \sigma^2_t)^2}}{\sqrt{2M\sigma^4_e}} \right)
\]

(21)

Inserting \( \hat{P}_f \) in (17), we obtain a line search optimization problem as follows

\[
\max_{N} \left( \frac{T - T_s - NT_r}{T} \right) (1 - \hat{Q})
\]

s.t. \( 1 \leq N \leq \left[ \frac{T - T_s}{T_r} \right] \)

(22)

Where \( \hat{Q}_f = 1 - \psi(k-1,\hat{P}_f, N) \).

Based on what we have shown for a general \( K \), denoting \( \hat{P}_{f,\text{AND}} \) as the \( P_f \) evaluated at \( P_d = \alpha^{1/N} \) for the AND rule, the optimal global probability of false alarm for a given \( N \) is \( \hat{Q}_f = \hat{P}_f, \text{AND} \), and thus the optimization problem can be rewritten as follows

\[
\max_{N} \left( \frac{T - T_s - NT_r}{T} \right) (1 - \hat{P}_f, \text{AND}^N)
\]

s.t. \( 1 \leq N \leq \left[ \frac{T - T_s}{T_r} \right] \)

(23)

Where

\[
\hat{P}_{f, \text{AND}} = Q \left( \frac{M\sigma^2_e + Q^{-1}(\alpha^{1/N})\sqrt{2M(\sigma^2_e + \sigma^2_t)^2}}{\sqrt{2M\sigma^4_e}} \right)
\]

(24)

\[
= Q(A + BQ^{-1}(\alpha^{1/N}))
\]

With \( A \) and \( B \) as in equations (15) and (16). As for the AND rule, the optimization problem for the OR rule can be simplified to a line search optimization problem as follows

\[
\max_{N} \left( \frac{T - T_s - NT_r}{T} \right) (1 - \hat{P}_{f, \text{OR}})^N
\]

s.t. \( 1 \leq N \leq \left[ \frac{T - T_s}{T_r} \right] \)

(25)

Where

\[
\hat{P}_{f, \text{OR}} = Q \left( \frac{M\sigma^2_e + Q^{-1}(\alpha')\sqrt{2M(\sigma^2_e + \sigma^2_t)^2}}{\sqrt{2M\sigma^4_e}} \right)
\]

(26)

\[
= Q(A + BQ^{-1}(\alpha'))
\]
With \( \alpha' = (1-\alpha)^{1/N} \), and A and B as in (15) and (16). The optimal value of N for both (21) and (23) can be found by a line search over N from 1 to \( \left[ \frac{T-T_s}{T_r} \right] \).

4. Simulation Results

Cognitive radio network with secondary users is considered for the simulations. Each cognitive radio accumulates \( M = 275 \) observations samples in the energy detector to make the local decision. The received SNR at each cognitive user is assumed to be \( \gamma = -7 \) dB. The simulations are performed for three different bit rates, \( R_b = 50\text{Kbps}, 75\text{Kbps and 100Kbps} \) with sampling frequency assumed \( f_s = 1/T_s = 6\text{MHz} \). Fig. 3 shows the optimal N versus the probability of false alarm constraint \( \beta \), for the energy efficient set up using the OR, AND and majority rules for two fixed values of the probability of detection constrain \( \alpha = \{0.9, 0.95\} \), while the probability of false alarm constraint varies in the range \( 0.01 \leq \beta \leq 0.1 \). In different scenarios, the OR rule outperforms the AND rule in terms of energy efficiency which requires a limited number of cognitive users[10] for better detection performance, while the OR rule doesn’t outperforms the majority rule for the whole \( \beta \) range.

Fig. 3. Optimal N versus the probability of false alarm

In fig. 4, shows the performance of Energy efficient setup when the probability of detection constraint changes from 0.9 to 0.97 for the two fixed values of the probability of false alarm constraint \( \beta = \{0.05, 0.1\} \). We can see that similar to the previous scenario, the OR rule performance better than the AND rule over the whole \( \alpha \) range. However, it is shown that the OR rule is not always dominant to the majority rule.

Fig. 4. Optimal N versus the probability of detection

In fig. 5, The optimal number of cognitive users N that maximizes the throughput is considered for a probability of detection constraint \( 0.9 \leq \alpha \leq 0.94 \) while its corresponding throughput is shown in fig. 6. We can see that different bit rates \( R_b = \{50\text{Kbps}, 75\text{Kbps}, 100\text{Kbps}\} \), the OR rule performs better then the AND rule by achieving the same detection reliability with less cognitive radios. Furthermore, it is shown in Fig. 6 that the OR rule gives a higher throughput for the same probability of detection constraint with less users.

Fig. 5. Optimal N versus the probability of detection.

In fig. 7, shows the throughput versus the number of cognitive users for two fixed values of the probability of detection constraint, \( \alpha = \{0.98, 0.99\} \), for the AND and OR rule. It is shown that there is an optimal N [9]that maximizes the network throughput. Further, we can see that for the whole N range, the OR rule gives a better performance than the AND rule for a fixed \( \alpha \).

Fig. 7. Performance analysis with throughput optimization setup

Fig. 6. Performance analysis with energy efficient setup
Conclusions

Optimization of cooperative spectrum sensing for cognitive radio network was considered. The optimal number of cognitive users required to satisfy the constraints defined by the standards was derived under two different setups. In the energy efficient setup, we reduced the network energy consumption by minimizing the number of cognitive users subject to a constraint on the probability of detection and false alarm while in the throughput optimization setup; the network throughput is maximized subject to a detection rate constraint. It is shown that the OR and the majority rule are more energy efficient than the AND rule. Furthermore, we have shown that the OR rule outperforms the AND rule in the throughput achieved by the network, and this optimal throughput is achieved exploiting less cognitive radios.

References