CHAPTER 5
SPECTRAL ANALYSIS FOR MODULATED CYCLOSTATIONARITY SIGNALS

5.1 Introduction
Most modulated signals have cyclostationary features due to periodicities embedded by the modulation process, such as carrier frequency, symbol rate, and pilot pattern. These parameters determine the cyclic features, including cycle frequency locations and spectral correlation amplitudes. An important approach in designing signal sensing and classification algorithms is to exploit cyclostationarity of modulated signals.

Furthermore, these cyclic features can be extracted at very low SNR with uncertain noise levels whereas conventional energy detector shows poor detection performance. Another advantage of cyclostationary approach for spectrum sensing is that it can detect and extract signal features without demodulating the incoming signal. Furthermore, a distinctive cyclic feature can be built into modulated signal to help with CR device identification. However, conventional signal analysis intentionally eliminates cyclostationary features through phase randomization to compute the power spectral density (PSD) or ignores the issue altogether [14, 104].

A cyclostationary process is a signal having statistical properties that vary cyclically with time. Man-made signals are cyclostationary due to the presence of sine wave carriers, pulse trains, repeated spreading, hopping sequence, cyclic prefixes or even pilot inserted to assist channel estimation and synchronization. However, noise is not cyclostationary process.

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Furthermore, these cyclic features can be extracted at very low SNR with uncertain noise levels whereas conventional energy detector shows poor detection performance. Another advantage of cyclostationary approach for spectrum sensing is that it can detect and extract signal features without demodulating the incoming signal.
Spectral correlation and spectral coherence of binary phase shift keying (BPSK) and Quaternary phase shift keying (QPSK) are evaluated to understand basic statistical cyclic analysis [110].

One approach to calculating the PSD of a cyclostationary signal is to randomize its phase so that it becomes stationary. This conversion does not change the PSD. However, Additional benefits from cyclostationarity will be lost with phase randomization.

5.2 Analysis of Cyclostationarity

A random process is said ergodic if the long time averages of a realization (sample function or sample path) of a random process is equal to the corresponding statistical average (ensemble average or expectation operation) with probability value of one.

Physical phenomena that involve periodicities give rise to random data for which appropriate probabilistic models exhibit periodically time-variant parameters. For instance, the periodicity in modern digital communication signals arises from embedded periodic elements such as carrier frequency and symbol rate. A process \( x(t) \) is said to be cyclostationary in the wide sense if its mean and autocorrelation are periodic[56, 57]:

\[
M_x(t + T_o) = M_x(t) \tag{5.1}
\]
\[
R_x(t + T_o, u + T_o) = R_x(t, u) \tag{5.2}
\]

The cyclostationarity in Equation (5.1) and (5.2) can be found in modulated signals. The example shown below represents a special case of modulated signal. Other modulated signal having cyclostationarity can be investigated similar approach applied following example.

Let’s consider a digital waveform \( y(t) \) that can be expressed by

\[
y(t) = \sum_{k=-\infty}^{\infty} b_k s_1(t - kT_b) + (1 - b_k) s_0(t - kT_b) \tag{5.3}
\]

where \( b_k \) is a stationary random bit sequence having values of one and zero, \( s_0(t) \) and \( s_1(t) \) are predetermined different waveforms with period \( T_b \) (bit rate \( R_b = 1/T_b \)).
In communication signal design, the binary input data sequence can be mapped to an antipodal waveform in which \( s_0(t) = -s_i(t) \).

Then, Equation (5.3) can be written as

\[
y(t) = \sum_{k=-\infty}^{\infty} d_k s(t - kT_b)
\]  

(5.4)

where \( d_k \) is the zero mean stationary random sequence having values +1 and -1.

In communication systems, transmitted signal bandwidth and power must confirm to radiation regulations. PSD is a fundamental measure of the bandwidth and the power of a signal.

To get the PSD of Equation (5.4), the Weiner-Kinchin theorem, which states that the PSD of a stationary signal can be obtained by Fourier transformation of autocorrelation function [59], is used. Thus, we have to derive the autocorrelation function which is give by

\[
E\{y(t_1)y^*(t_2)\} = E\left\{ \sum_{m=-\infty}^{\infty} d_ms(t_1-mT_b) \sum_{n=-\infty}^{\infty} d_ns(t_2-nT_b) \right\}
\]

\[
= E\left\{ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d_md_n \times s(t_1-mT_b)s(t_2-nT_b) \right\}
\]

\[
= E\left\{ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} E[d_md_n] \times s(t_1-mT_b)s(t_2-nT_b) \right\}
\]  

(5.5)

We assume a real signal and the conjugation are ignored in Equation (5.5).

Now, we need to evaluate the autocorrelation of the input data sequence, \( E[d_md_j] \).

Let’s consider a zero mean, independent, and identically distributed random bit sequence having the following probabilities,

\[
P_r\{d_m = +1\} = P \quad \text{and} \quad P_r\{d_m = -1\} = 1 - P
\]  

(5.6)

Then, the autocorrelation function of the random input sequence is

\[
\mu(m-n) = E[d_md_n] = \begin{cases} 
E[d_m^2] = 1^2P + (-1)^2(1-P) = 1 
& \text{for } m = n \\
E[d_m]E[d_n] = (E[d_m])^2 = (2P-1)^2 
& \text{for } m \neq n
\end{cases}
\]  

(5.7)
Equation (5.7) can be written as
\[
\mu(q) = \begin{cases} 
1 & \text{for } q = 0 \\
(2P - 1)^2 & \text{for } q \neq 0 
\end{cases}
\] (5.8)

Therefore, Equation (5.5) can be simplified further
\[
E\{y(t + \tau) y(t)\} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mu(m-n) x(t_1 - mT_b) s(t_2 - nT_b) 
\]

\[
= R_y(t_1, t_2) = R_y(t_1 + mT_b, t_2 + nT_b) 
\]

\[
= \hat{R}_y(t + \tau, t) 
\]

where \( t_1 - t_2 = \tau \).

Equation (5.9) is clearly periodic in \( t_1 \) and \( t_2 \) with period \( T_y \) and thus is not a stationary process. Here \( y(t) \) is called second-order cyclostationary random signal (the mean of \( y(t) \) is zero and can be considered periodic in \( t \)). In this example, the symbol generation at every symbol rate results in cyclostationarity. For other types of modulation, such as carrier frequency hopping and spreading sequence in Code Division Multiple Access (CDMA), may produce cyclostationarity as well.

One approach to calculating the PSD of a cyclostationary signal is to randomize its phase so that it becomes stationary [29]. This conversion does not change the PSD. However, we will lose the additional benefits from cyclostationarity with this phase randomization.

Let's define a stationary process from a cyclostationary one by introducing a random phase into the signal.
\[
y(t) \overset{\Delta}{=} x(t - \theta) 
\] (5.10)

The random phase variable \( \theta \) is assumed to be uniformly distributed over the period \( T \) and independent of the cyclostationary process \( x(t) \). Then \( y(t) \) can be transformed to a stationary process by averaging over the random phase. That is,
\[
m_y = E_\theta\{m_x(t, \theta)\} \quad \text{and} \quad R_y(\tau) = E_\theta\{R_x(t, \theta, \tau)\} 
\] (5.11)
where $E_\theta \{ \}$ is the expectation in the probability theory over random variable $\theta$.

With the phase randomization concept, Equation (5.5) is rewritten as,

$$R_x(\tau) = \frac{1}{T_b} \int_{0}^{T_b} R_x(t, \tau) dt$$

$$= \frac{1}{T_b} \int_{0}^{T_b} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mu(m-n) \times s(t + \tau - mT_b) s(t - nT_b) dt$$

$$= \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \int_{-nT_b}^{(1-n)T_b} \sum_{q=-\infty}^{\infty} \mu(q) \times s(u + \tau - qT_b) s(u) du$$

$$= \frac{1}{T_b} \int_{-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \mu(q) \times s(u + \tau - qT_b) s(u) du$$

$$= \frac{1}{T_b} \sum_{q=-\infty}^{\infty} \mu(q) f_x(\tau - qT_b)$$

(5.12)

where a change of variable $u = t - nT_b$ is performed and we assumes

$$f_x(\tau) = \frac{1}{T_b} \int_{-\infty}^{\infty} s(u) s(u + \tau) du.$$  

Therefore, the cyclostationary signal becomes stationary signal by averaging out the time dependency. The PSD of the stationary signal can be obtained using the Wiener-Kinchin theorem and the sampling identity for Fourier transforms and is given by

$$S_\gamma(f) = \frac{4P(1-P)}{T_b} |F_x(f)|^2 + \frac{(2P-1)^2}{T_b} \sum_n |F_x(n/T_b)|^2 \delta(f - n/T_b)$$  

(5.13)

The input signal sequence is usually modeled as an equally likely random sequence. Thus, with $P=1/2$, the PSD will not have any delta function and is given by

$$|S_\gamma(f)| = \frac{1}{T_b} |F_x(f)|^2$$  

(5.14)

This implies that the spectral shape is only determined by the PSD of user information.

The cyclostationarity in the above Equations can be found in modulated signals. Since autocorrelation function is periodic it can be expressed by applying Fourier series which is decomposed as

$$R_x(t, \tau) = \sum_a R_x^a(\tau) e^{j2\pi a t}$$

(5.15)
5.3 Cyclic Autocorrelation

Generally, autocorrelation functions are considered in signal processing because they allow PSD generation. Equation 5.2 can be rewritten as:

\[ R_x(t + T_o + \frac{\tau}{2}, t + T_o - \frac{\tau}{2}) = R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) \]  

(5.16)

Then \( R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) \), a function of two independent variables, \( t \) and \( \tau \), is periodic in \( t \) with period \( T_o \) for all values of \( \tau \). Thus, we can express Equation (5.16) in Fourier series as

\[ R_x^\alpha(t + \frac{\tau}{2}, t - \frac{\tau}{2}) = \sum_\alpha R_x^\alpha(\tau)e^{j2\pi\alpha t} \]  

(5.17)

where \( R_x^\alpha(\tau) \) are the Fourier coefficients which is given by

\[ R_x^\alpha(\tau) = \frac{1}{T_0} \int_{t=-T_o/2}^{T_o/2} R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2})e^{-j2\pi\alpha t} dt \]  

(5.18)

and \( \alpha \) ranges over all integer multiples of the fundamental frequency \( 1/T_o \). For polyperiodic or almost cyclostationary signals which have more than one fundamental frequencies such as \( (1/T_1, 1/T_2, \ldots, 1/T_n) \) the above Equations need to be generalized. This can be accomplished by having \( \alpha \) in Equation (5.18) range over all integer multiples of all fundamental frequencies of interest like \( \alpha \in (K_1/T_1, K_2/T_2, \ldots, K_n/T_n) \) with any integer number \( k_i \) so that \( R_x^\alpha(\tau) \) is not identically zero [108]. For Equation (5.18), the following modification required to include the effect of signal components having different fundamental periodicities.

\[ R_x^\alpha(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{t=-T/2}^{T/2} x(t + \frac{\tau}{2})x^*(t - \frac{\tau}{2})e^{-j2\pi\alpha t} dt \]  

(5.19)

\( R_x^\alpha(\tau) \) is called cyclic autocorrelation function (CAF) and it is the base for cyclic spectral analysis [104]. A random process is said ergodic if the long time averages of a realization of a random process is equal to the corresponding statistical average with probability one.
5.4 Cycloergodicity

A random process is said ergodic if the long time averages of a realization (sample function or sample path) of a random process is equal to the corresponding statistical average (ensemble average or expectation operation) with probability one. This can be explained mathematically as

$$\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)dt = \mathbb{E}\{X(t)\}$$

and

$$\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t+\tau)dt = \mathbb{E}\{X(t)X^*(t+\tau)\} = R_x(\tau)$$ (5.20)

Where $X(t)$ is a random process, $x(t)$ is a realization of random process $X(t)$, and $\mathbb{P}$ indicates the equality holds with probability one.

Ergodicity simplifies the analysis of random data measurement by allowing the exchange of statistical average and time average. In general, stationary signals are assumed ergodic.

The ergodic concept can be extended to the cyclostationary signal. The cyclostationary signal that holds ergodicity property is called cycloergodic cyclostationary process or simply cycloergodicity [107]. If cyclostationary signal assumed as cycloergodic, then the Equation (5.19) is express as

$$R_x^{\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2})e^{-j2\alpha \tau}dt$$

$$\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathbb{E}\left\{X(t+\frac{\tau}{2})X^*(t-\frac{\tau}{2})\right\}e^{-j2\alpha \tau}dt$$

$$= \mathbb{P} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j2\alpha \tau}dt$$ (5.21)

Thus, the CAF can be expressed as

$$R_x^{\alpha}(\tau) = \mathbb{P} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathbb{E}\left\{X(t+\frac{\tau}{2})X^*(t-\frac{\tau}{2})\right\}e^{-j2\alpha \tau}dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j2\alpha \tau}dt$$ (5.22)
5.5 Spectral Correlation Function

The dual of the CAF is the spectral correlation function. Two time functions \( u(t) = x(t)e^{-j\alpha t} \) and \( v(t) = x(t)e^{j\alpha t} \), the generalized cross correlation for \( u(t) \) and \( v(t) \) can be written as

\[
R_{uv}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t + \frac{\tau}{2}) v^*(t - \frac{\tau}{2}) e^{-j2\alpha \tau} dt
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \frac{\tau}{2}) e^{-j\alpha (t+\tau/2)} v^*(t - \frac{\tau}{2}) e^{-j2\alpha \tau} dt
\]

\[
= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-j2\alpha \tau} dt
\]

From the above

\[
R_{uv}(\tau) = R_x(\tau) \quad \text{(5.24)}
\]

Therefore from the above results we can say the CAF is simply time-averaged cross-correlation between frequency-shifted versions of the process \( x(t) \). This implies that a process exhibits cyclostationarity [105] only if correlation exists between some frequency-shifted versions of the process. The above generalized cross correlation is the conventional cross correlation of the two complex valued frequency shifted signals.

Consequently, the \( R_{uv}(\tau) \) can be considered as an inverse Fourier transform of the limit cross spectrum \( S_{uv}(f) \) of \( u(t) \) and \( v(t) \).

\[
R_x(\tau) = R_{uv}(\tau) = \text{IFT} \{ S_{uv}(f) \} = \text{IFT} \{ S_x(\alpha f) \}
\]

Where \( S_x(\alpha f) \) is called Spectral Correlation Function.

Equation (5.25) can be verified in another way by taking the time-variant finite-time Fourier transformation of \( u(t) = x(t)e^{-j\alpha t} \) and \( v(t) = x(t)e^{j\alpha t} \), then

\[
U_{\tau}(t,f) = \int_{t-T/2}^{t+T/2} u(w)e^{-j2\alpha w} dw = \int_{t-T/2}^{t+T/2} x(w)e^{-j\alpha(t+\tau/2)} e^{-j2\alpha w} = X_{\tau}[t,f + \frac{\alpha}{2}]
\]

(5.26)
and

\[
V_I(t,f) = \int_{t-T/2}^{t+T/2} v(w)e^{-j2\pi f w} dw = \int_{t-T/2}^{t+T/2} x(w)e^{-j\alpha w} e^{-j2\pi f w} = X_I[t,f - \frac{\alpha}{2}]
\] (5.27)

where the time-variant finite-time Fourier transformation is defined as

\[
X_I(t,f) = \int_{t-T/2}^{t+T/2} x(u)e^{-j2\pi fu} du
\] (5.28)

To mitigate random effect due to limited observation, time-domain smoothing is required [109]. Let us define time-domain smoothed cross correlation as

\[
S^\alpha_{xy}(t,f) = \frac{1}{N} \int_{-N/2}^{N/2} S_{xy}(u,f) du
\] (5.29)

where the time-variant cross periodogram can be expressed as

\[
S_{xy}(f) = \frac{1}{T} U_I(t,f) V_I(t,f)
\]

\[= \frac{1}{T} X_I[t,f + \frac{\alpha}{2}] X_I^*[t,f - \frac{\alpha}{2}]
\]

\[= S^\alpha_{xy}(t,f)
\] (5.30)

The above Equation is called as cyclic periodogram. It is well known that the periodogram can be converted into the cross correlogram.

\[S_{xy}(\cdot) = FT[R_{xy}(\cdot)]
\] (5.31)

where the cross correlogram is defined as

\[R_{xy}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} x^*_I[t+\tau] x^*_I[t+\frac{\tau}{2}] - x^*_I[t-\frac{\tau}{2}] dx
\] (5.32)

If the periodogram-correlogram relation is applied to the Equation (5.30), the time-variant cross periodogram will be

\[S_{xy}(t,\cdot) = FT[R_{xy}(t,\cdot)]
\] (5.33)

\[R_{xy}(t,\tau) = \frac{1}{T} \int_{-T/2}^{T/2} x^*_I[t+\tau] x^*_I[t-\tau] dw
\] (5.34)
Equation (5.30) can be rewritten as

\[ S_{xy}(t, \cdot) = FT[R_{xy}(t, \cdot)] = S_{xy}^\alpha(t, \cdot) = FT[R_{xy}^\alpha(t, \cdot)] \]  

(5.35)

Where

\[ R_{xy}^\alpha(t, \tau) = \frac{1}{T} \int_{-T/2}^{T/2} x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi w\tau}dw \]  

(5.36)

In the statistical point of view, the time-series needs to be infinite, \( T \to \infty \) to suppression any random effect. Therefore,

\[ \lim_{T \to \infty} R_{xy}^\alpha(t, \tau) = R_x^\alpha(\tau) \]  

(5.37)

Substituting Equation (5.30) in (5.33), Equation (5.33) becomes

\[ R_x^\alpha(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi w\tau}dw \]  

(5.38)

Another important aspect in statistical cyclic spectrum is that the SCF from temporal smoothing method can also be derived using spectral smoothing \([109]\) when the following condition is met.

\[ \Delta \tau \gg T \]  

(5.39)

This result can be written mathematically

\[ S_{xy}^\alpha(t, f)_{\Delta \tau} \simeq S_{xy}^\alpha(t, f)_{1/T} \quad \text{When} \ \Delta \tau \gg T \]  

(5.40)

It is clear that the observation duration \( T \) is the spectral resolution \((1/\Delta f)\). Thus, the spectrally smoothed spectral correlation can be reinterpreted as

\[ S_{xy}^\alpha(t, f)_{\Delta f} = \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} X_{\Delta f}[t, w + \frac{\alpha}{2}]X^*_{\Delta f}[t, w - \frac{\alpha}{2}]dw \]  

(5.41)

Therefore, to obtain reliable statistical spectral correlation, following condition should be met

\[ \Delta t \Delta f \gg 1 \]  

(5.42)

the ideal spectral correlation can be obtained by

\[ S_x^\alpha(f) = \lim_{T \to \infty} \lim_{\Delta \tau \to \infty} S_{xy}^\alpha(t, f)_{\Delta \tau} \]  

(5.43)

and

\[ S_x^\alpha(f) = \lim_{\Delta f \to 0} \lim_{\Delta \tau \to \infty} S_{xy}^\alpha(t, f)_{\Delta f} \]  

(5.44)
The relationship between ideal spectral correlation functions can be arrived as

\[ S_x^\alpha (f) = \int_{-\infty}^{\infty} R_x^\alpha (\tau) e^{j2\pi f \tau} d\tau \quad (5.45) \]

The above relation is called cyclic wiener relation.

### 5.6 Spectral Coherence and \( \alpha \)-Profile

SCF is a correlation of frequency components shifted by \( f + \alpha /2 \) and \( f - \alpha /2 \). It is intuitive to define Spectral Coherence (SC) as

\[ C_x^\alpha = \frac{S_x^\alpha (f)}{[S(f + \frac{\alpha}{2})S(f - \frac{\alpha}{2})]^{1/2}} \quad (5.46) \]

The magnitude of SC is always between 0 and 1. In order to reduce the computational complexity, one just uses the Cyclic Domain Profile (CDP) or \( \alpha \)-profile which is defined as

\[ I(\alpha) = \max_f \left| C_x^\alpha (f) \right| \quad (5.47) \]

The process is represented in the form of block diagram as shown in the Fig. 5.1.

![Fig 5.1: Extraction of spectral coherence function](image)

### 5.7 Primary Signal Detection

All man-made signals and modulated signals exhibit second order cyclostationary. From the CDP of the signal, important information about the signal like modulation type, keying rate, pulse shape, and carrier frequency can be obtained, [102]. When SCF is plotted, the occupancy status of the spectrum can be found out. If a primary user signal is present in the operating frequency range, the SCF gives a peak at its centre.

The peak will not be present in the case when there is no primary user signal present in the concerned frequency range. Spectrum sensing is used to determine the presence
or absence of primary users so it is needed to distinguish between these two hypotheses to determine the threshold $C_{\text{TH}}$ [12].

$$H_0 : x(t) = n(t)$$

$$H_1 : x(t) = s(t) + n(t)$$

For signal detection and when no signal is present, i.e. $x(t) = n(t)$, will use the relationship as [103]:

$$C_{\text{TH}} = \max[I(\alpha)/\sqrt{(\sum_{\alpha=0}^{N} I^2(\alpha))/N}]$$

(5.48)

where $N$ is the length of observation data.

The signal is distinguished from noise by analyzing the Spectral Correlation Density (SCD) function. Furthermore, it is possible to distinguish the signal type because different signals may have different nonzero cyclic frequencies. Cyclostationary detection block contains a FFT, AWGN, correlate, average over threshold and a feature detection block is shown in the Fig 5.2.

![Cyclostationary detection](image)

**Fig 5.2: Cyclostationary detection**

A random discrete signal is taken and modulated using different modulation schemes. The CFD basically contains filters, ADC, quantizer, encoder, and FFT blocks. In this method Fast Fourier Transform and a Noise is added by AWGN block were used. Cyclostationary feature detection method deals with the inherent cyclostationary properties or features of the signal [101]. Such features have a periodic statistics and spectral correlation that cannot be found in any interference signal or stationary noise.

It exploits this periodicity in the received primary signal to identify the presence of primary users, and that is why the cyclostationary feature detection method possesses higher noise immunity than any other spectrum sensing method. The output is taken using spectrum analyzer which displays the output in a graphical form which can be easily understandable. The output plot thus obtained is the cyclic SCF. Peak detection algorithm is used for the cyclostationary output.
The plot between probability of detection and SNR is termed as the receiver operating characteristics; using sensing algorithm the cyclostationary detection method, shows that the primary signal is present, and probability of detection increases with different SNR values.

5.8 Detection Techniques
The cyclic domain profile for BPSK and QPSK Signals are extracted. For the obtained signals the peak detection techniques were applied to minimize the noise and to improve the probability of detection. Fig 5.3(a) & Fig. 5.3(b) shows the cyclic domain profile of BPSK and QPSK respectively.

Fig 5.3(a): Cyclic domain profile for BPSK signal
5.8.1 Absolute Threshold

The absolute threshold can be compared to the difference threshold which is the measure of how two different stimuli must be for the subject to notice that they are not the same. For the cyclostationary output the absolute threshold is applied where it reduces the noise peaks to a minimum. Reduction of noise peaks at different frequencies improves accuracy, and a centre peak obtained indicates the probability of detection. The noise peaks are diminished by considering an absolute value of cyclostationary output. There are several factors which can influence the level of an absolute threshold including cognitive processes, adaptation to stimulus. Cognitive Processes are those which we use to think, remember, and reason, as well as the processing of concepts and memories. Here the output signal is compared with the absolute threshold if it is beyond the threshold then the primary signal is present and secondary user must be idle which is shown in Fig 5.4(a) & Fig. 5.4(b). if it is below the threshold value then the primary signal is absent and the secondary user can easily utilize the licensed spectrum in an unlicensed manner and if primary user needs the spectrum then the secondary user must vacate the band as soon as
primary user enters and hence secondary user must sense the spectrum faster without causing interference to the primary user.

Fig 5.4 (a): Absolute threshold detection for BPSK

Fig. 5.4 (b): Absolute threshold detection for QPSK
5.8.2 Standard Deviation

Standard Deviation (SD) is a measure that is used to determine the amount of variation or dispersion of a set of data values. The standard deviation of a random variable, probability distribution is the square root of its variance.

Let ‘x’ be a random variable with mean ‘μ’. Then Formula for standard deviation is

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$  \hspace{1cm} (5.49)

To determine the standard deviation for a set of data, First the mean of the data is calculated (σ). Secondly, each number from the set of data is subtracted by mean and square the obtained result using formula \((x_i - \mu)^2\),

Where \(x_i\) = set of individual data.

Next again calculate the mean of the squared results obtained using the formula.

$$\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$$

Finally taking the square root of the mean function we obtain the standard deviation is obtained as

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$  \hspace{1cm} (5.50)

Hence this is the description of standard deviation of the whole sample of data’s. If we need to calculate a sample of data the formulae changes \(N-1\) instead of \(N\).

The formula for sample standard deviation is

$$S = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$  \hspace{1cm} (5.51)

Here ‘\(x\)’ is the mean instead of ‘μ’ and ‘\(S\)’ is the standard deviation instead of ‘σ’.

The symbols are changed that reflect from the sample to the normal standard deviation and does not affect any calculations. \(N-1\) is used instead \(N\) which is called Bessel's correction. In the whole set of data we are considering only the limited amount of values that are assumed as samples of data which gives a approximate output. Mostly users take a sample of values for the computation because it is easier and cheaper. By considering sample amount of data the simulation time can be reduced but some accuracy is lost. In this work instead of whole spectrum particular
band is considered and their mean and standard deviation is computed by using the formula and using the obtained result the presence of primary user is analyzed and diminishes the noise peaks more than the absolute threshold based on the increased number of samples.

Fig 5.5 (a): Detection based on Standard Deviation for BPSK

Fig 5.5 (b): Detection based on Standard Deviation for QPSK
Deviation is a measure for how much the frequencies in a spectrum can deviate from the centre of gravity. For a sine wave the Standard deviation is zero and hence by increasing the number of samples the noise peaks are diminished as shown in the Fig. 5.5(a) and Fig. 5.5(b). The standard deviation diminishes the noise peaks more than the absolute threshold based on the increased number of samples.

5.8.3 Filtfilt

Filtfilt is a zero phase forward and reverse filtering, after filtering in the forward direction; the filtered sequence is then reversed and run back through the filter. The filter is described by the difference Equation:

\[ y(n) = b(1)x(n) + b(2)x(n-1) + \ldots + b(nb+1)x(n-nb) - a(2)y(n-1) - \ldots - a(na+1)y(n-na) \]  \hspace{1cm} (5.52)

Where \( y \) is the time reverse of the output of the second filtering operation. The result has precisely zero phase distortion and magnitude modified by the square of the filter's magnitude response. The length of the input \( x \) must be more than three times the filter order, defined as \( \max(\text{length}(b) - 1, \text{length}(a) - 1) \). Filtfilt should not be used with differentiator and Hilbert finite impulse response filter, since the operation of these filters depends heavily on their phase response. Filtfilt can handle n-dimensional input \( x \) in earlier versions. The output of the filtfilt command shows the noise is completely diminished by using the difference Equation. By comparing with the other two techniques filtfilt is the best method for reducing the noise peaks by filtering.

Filter order that is double the order of the filter specified by \( b \) and a filtfilt minimizes start-up and ending transients by matching initial conditions, and can be uses for both real and complex inputs. Filtfilt is not suited with differentiator and Hilbert finite impulse response filter, because the operation of these filters depends heavily on their phase response. Zero-phase filtering helps preserve features in the filtered time waveform exactly where those features occur in the unfiltered waveform. The simulations results are shown in Fig. 5.6(a) and Fig. 5.6(b) for BPSK and QPSK respectively.
Fig. 5.6(a): Filtfilt detection for BPSK

Fig. 5.6(b): Filtfilt detection for QPSK
Finally for the signal (peaks) obtained, the probability of detection for different SNR is calculated for BPSK and QPSK which were plotted in the Fig. 5.7(a) & Fig. 5.7(b) respectively. First, absolute threshold is fixed based on the value evaluated in first method and probability of detection is calculated for different SNRs ranging from -10 dB to 0 dB which is shown in Fig. 5.7(a) & Fig. 5.7(b). For both modulation techniques the same procedure is applied.

For second method, the threshold for detection probability is determined from standard deviation as per the Equation (5.50). Now, the probability of detection is considered only if CDP of the obtained signal crosses threshold. This procedure is repeated for the range -10 dB to 0 dB with 1dB variation every time for both modulation schemes.

Fig 5.7(a): Probability of detection versus SNR for BPSK
For Filtfilt technique, the threshold for detection probability is determined differently. The CDP of the obtained signal is filtered in forward direction for each sample. After completion of all samples, the filter order is reversed and the procedure is repeated. As noise peaks varies from sample to sample, the reverse order of filter makes the noise peaks removed keeping the CDP of the modulated signal unaltered. Therefore, the probability of detection is more when compared to other proposed methods. The performance of the Filtfilt is plotted in Fig. 5.7(a) & Fig. 5.7(b) for BPSK and QPSK respectively.

The performance comparison is tabulated in Table 5.1 and Table 5.2 for BPSK and QPSK respectively in terms of probability of detection versus SNR for different detection techniques.

![Graph showing Probability of Detection versus SNR for QPSK](image)

**Fig 5.7(b): Probability of detection versus SNR for QPSK**
TABLE 5.1: \( P_d \) versus SNR using different detection Techniques for BPSK

<table>
<thead>
<tr>
<th>Probability of Detection using</th>
<th>Probability of Detection at Signal– to– Noise Ratio</th>
<th>( 80% \ P_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1 dB</td>
<td>-2 dB</td>
</tr>
<tr>
<td>Absolute Threshold</td>
<td>97%</td>
<td>93%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>97%</td>
<td>95%</td>
</tr>
<tr>
<td>Filtfilt</td>
<td>98%</td>
<td>97%</td>
</tr>
</tbody>
</table>

TABLE 5.2: \( P_d \) versus SNR using different detection techniques for QPSK

<table>
<thead>
<tr>
<th>Probability of Detection using</th>
<th>Probability of Detection at Signal– to– Noise Ratio</th>
<th>( 80% \ P_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1 dB</td>
<td>-2 dB</td>
</tr>
<tr>
<td>Absolute Threshold</td>
<td>96%</td>
<td>94%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>97%</td>
<td>95%</td>
</tr>
<tr>
<td>Filtfilt</td>
<td>98%</td>
<td>97.5%</td>
</tr>
</tbody>
</table>

5.9 Conclusions
The peak detection algorithm for estimation and detection of the primary signal by applying threshold techniques to analyze the spectrum is presented. If the signal peak is present, which is in the form of CDP at the centre of the SCF, and then the presence of primary user is determined and considered absent otherwise. This method is able to evaluate CDP for digital modulation techniques and also clearly discriminates signal from noise which is does not exist in energy detection. Because of this feature, the threshold for detection can be reduced globally. Filtfilt and standard deviation techniques exhibit better probability of detection. This is achieved without prior knowledge of primary user. Therefore, this approach is said to be blind approach for detection of primary user which is suitable for all wireless environments.