CHAPTER 3
COOPERATIVE SPECTRUM SENSING OPTIMIZATION WITH ENERGY DETECTOR IN COGNITIVE RADIO NETWORKS

3.1 Introduction

In order to solve the conflict between spectrum scarcity and spectrum under-utilization, cognitive radio technology has been recently proposed. It can improve the spectrum utilization by allowing secondary users to borrow unused radio spectrum from primary licensed users or to share the spectrum with the primary networks (users) [2]. As an intelligent wireless communication system, cognitive radio is aware of the radio frequency environment. It selects the communication parameters (such as carrier frequency, bandwidth and transmission power) to optimize the spectrum usage and adapts its transmission and reception accordingly. One of the most critical components of cognitive radio technology is spectrum sensing. By sensing and adapting to the environment, a cognitive radio is able to fill in spectrum holes and serve its users without causing harmful interference to the licensed user. One of the great challenges of implementing spectrum sensing is the hidden terminal problem, which occurs when the cognitive radio is shadowed, in severe multipath fading or inside buildings with high penetration loss, while a primary user is operating in the vicinity [3]. Due to the hidden terminal problem, a cognitive radio may fail to notice the presence of the PU and then will access the licensed channel and cause interference to the licensed system. In order to deal with the hidden terminal problem in cognitive radio networks, multiple cognitive users can cooperate to conduct spectrum sensing.

It has been shown that spectrum sensing performance can be greatly improved with an increase of the number of cooperative partners [4 - 8]. In this work, the optimization of cooperative spectrum sensing with energy detection to minimize the total error rate is considered. It is mentioned that optimal spectrum sensing under data fusion was investigated in [93], where the optimal linear function of weighted data fusion has been obtained. In other recent works [88, 92], optimal sensing throughput tradeoff was studied. Optimal centralized signal detection with likelihood ratio test using reporting channels from the CRs to the fusion center has been dealt [12]. Here the optimality of cooperative spectrum sensing using the sensing channels between the
primary transmitter and the CRs when energy detection and centralized decision fusion are applied to a cognitive radio network is investigated. Specifically, we derive the optimal voting rule, i.e., the optimal value of \( n \) for the “\( n \)-out-of-\( K \)” rule. We also determine the optimal detection threshold to minimize the error rate. We further proposed a fast spectrum sensing algorithm for large cognitive networks which requires only a few, not all, cognitive radios in cooperative spectrum sensing to get a target error bound.

The conventional energy detector, which is also called the radiometer, measures the received energy and compares it to a threshold. The basic energy detector is given in by [38]

\[
T_{ed} = \frac{2}{N_0} \sum_{t=1}^{M} |x(t)|^2
\]  

(3.1)

where \( N_0 \) is the noise spectral density. Factor 2 comes from the fact that under circularity assumption the complex noise power is equally divided between the real and imaginary parts. For the case of detecting a deterministic signal in the presence of zero mean independent and identically distributed (iid) complex Gaussian noise, the energy detector test statistic obeys the following distribution [38]

\[
H_0 = T_{ed} \approx X^2_{2M}
\]

\[
H_1 = T_{ed} \approx X^2_{2M} (2\gamma)
\]

(3.2)

where \( \gamma \) is the signal-energy-to-noise-spectral-density defined as \( \gamma = E_S/N_0 \).

In this case, \( E_s \sum_{t=1}^{M} |s(t)|^2 \) is the signal energy. Therefore the test statistic follows central chi-square distribution with 2M degrees of freedom under \( H_0 \) and non-central chi-square distribution with 2M degrees of freedom and non-centrality parameter \( 2\gamma \) under \( H_1 \). Equation (3.7) is applicable in all such cases provided that the probability of detection is considered a conditional probability of detection where the condition is a given amount of signal energy [38]. The main advantages of the energy detectors are that they are simple to implement and can be applied to detect any signal, known or unknown, deterministic or random. In case of independent identical distribution Gaussian noise with known noise power the energy detector is the optimum detector for a random uncorrelated Gaussian signal and at least a GLRT for completely
unknown random signals [78]. On the other hand, energy detectors cannot distinguish among different signals (PUs, SUs and interferences) and are not able to exploit the detailed information regarding the PU which is available generally. Also in case the noise statistics are not explicitly known it is difficult to maintain specified false alarm or missed detection probabilities. In fact, the presence of uncertainty in the noise statistics results in severe performance limitation in the form of the SNR wall phenomenon [79]: In the presence of uncertainty, it is impossible to robustly distinguish the signal from noise at SNR values lower than the SNR wall even if the sensing time tends to infinity. A review of energy based detection literature has been provided in [80]. In addition, constant false alarm rate (CFAR) strategies for the channelized radiometer have been considered.

CFAR detectors adaptively adjust their thresholds to maintain a constant false alarm property if the additive noise is non-stationary. Recent performance analyses of energy detection in fading channels have been carried out [81-84]. Experimental measurements of energy detection performance with noise uncertainty have been provided in [49, 85]. Energy detection of WiMAX systems for ultra-wideband / WiMAX coexistence has been considered [86]. The detection of wireless microphone signals using the maximum of the frequency domain energy measurements has been proposed in [87]. Energy detectors have been proposed for colored Gaussian [38], independent non-Gaussian [77, 88], and colored non-Gaussian noise [88] as well.

In implementing an energy detector, the received signal $x(t)$ is filtered by a band pass filter, followed by a square law device. The band pass filter serves to reduce the noise bandwidth. Hence, noise at the input to the squaring device has a band-limited, at spectral density. The output of the integrator is the energy of the input to the squaring device over the time interval $T$. Next, the output signal from the integrator (the decision statistic) is compared with a threshold, to decide whether a primary (licensed) user is present or not. Decision regarding the usage of the band is made by comparing the detection statistic to a threshold value. The complete process is explained in Fig 3.1.

Fig 3.1: Block diagram of an energy detector.
3.2 Primary Signal Detection

The process of cooperative sensing starts with spectrum sensing performed individually at each CR user called local sensing. Typically, local sensing for primary signal detection can be formulated as a binary hypothesis problem as follows.

\[
x_i(t) = \begin{cases} 
  w_i(t) & H_0 \\
  h_i(t)s(t) + w_i(t) & H_1 
\end{cases}
\]

where \( x_i(t) \) denotes the received signal at the CR user, \( s(t) \) is the transmitted PU signal, \( h_i(t) \) is the channel gain of the sensing channel, \( w_i(t) \) is the zero-mean Additive White Gaussian Noise, \( H_0 \) and \( H_1 \) denote the hypothesis of the absence and the presence, respectively, of the PU signal in the frequency band of interest.

For the evaluation of the detection performance, the probabilities of detection \( P_d \) and false alarm \( P_f \) are defined as

\[
P_d = P \{ \text{decision} = H_1 | H_1 \} = P \{ Y > \lambda | H_1 \}
\]

\[
P_f = P \{ \text{decision} = H_1 | H_0 \} = P \{ Y > \lambda | H_0 \}
\]

where \( Y \) is the decision statistic and \( \lambda \) is the decision threshold. The value of \( \lambda \) is set depending on the requirements of detection performance.

**Perfect Reporting Channel:**

If the channels between the cognitive users and the common receiver are perfect, the local decisions will be reported without any error. In this case,

\[
P_A = P \{ H=0 | H_0, K \geq 1 \}
\]

characterizes the probability of the event that under hypothesis \( H_0 \), all the \( K \) users claim \( H_0 \) and other \( n-K \) users make no local decisions. We will obtain the false alarm probability \( Q_f \), the detection probability \( Q_d \) and the missing probability \( Q_m \) for cooperative spectrum sensing, respectively.
Imperfect Reporting Channel

It is not realistic that the reporting channel between the cognitive user and the common receiver is assumed to be perfect since it is usually subject to fading. Due to the reporting error introduced by the imperfect channel, the reported local decisions should be first decoded in the common receiver before the final decision is made.

Let \( P_e \) denote the reporting error between the \( i^{th} \) cognitive user and the common receiver, for \( i = 1, \ldots, K \). For simplicity, we assume that all the reporting channels are independent and identical, i.e., \( P_{e,i} = P_e \). At the common receiver, the local decision will be recovered as 0 in two cases:

a) The cognitive user transmits \( H_0 \) and it is decoded as \( H_0 \);  
b) The cognitive user transmits \( H_1 \) while it is decoded as \( H_0 \) because of the reporting error. Due to the existence of reporting errors, the sensing performance is decreased compared with that in the perfect channel.

3.3 Performance Metrics

The correctness of the spectral availability information is defined using sensing quality parameters. This feature makes up the performance metrics. Sensing the Performance of the energy detector is specified by the following general metrics:

1. The probability of detection (\( P_d \))
2. The probability of false alarm (\( P_f \))
3. The probability of missed detection (\( P_m \))

In opportunistic spectrum sensing, the probability of detection specifies that a detector makes a correct decision that a channel is occupied (\( H_1 \)). The \( P_d \) is an indicator of the level of interference protection provided to the primary user. Hence, a large \( P_d \) denotes exact sensing; which translate to chance of interference.

A false alarm event occurs when the detector assumes \( H_1 \); when the right decision is \( H_0 \). The probability of this occurrence is specified as a probability of false alarm. When a false alarm event occurs, the SU would not exploit the free spectrum, thus missing a chance to utilize the free channel. \( P_f \) should be kept as small as possible in order to prevent underutilization of transmission opportunities. The performance of the spectrum sensing technique is usually influenced by the probability of false alarm, since this is the most influential metric [15].
The probability of declaring the spectrum space vacant $H_0$, when it is indeed occupied $H_1$, is referred to as the probability of miss-detection or missed detection ($P_m$). A high $P_m$ implies an increase in the chance of interference between the PU and the SU. If the detection fails, or a “miss detection” occurs, the SU initiates a transmission, resulting in interference with the PU signal; contravening the opportunistic access concept. In essence, the spectrum sensing method should record a high probability of detection (low miss detection probability) and low probability of false alarm.

### 3.4 Performance Measurement

The receiver performance is quantified by depicting the Receiver Operating Characteristics (ROC) curves. These curves serve as a tool to select and study the performance of a sensing scheme. ROC graphs are preferred as a performance measure, since simple classification accuracy do not contain much detail, hence is a poor metric for measuring performance [66]. ROC graphs are employed to show trade-offs between detection probability and false alarm rates, (i.e. $P_d$ versus $P_f$), thus allowing the determination of an optimal threshold. Complementary ROC curves depict plots of probability of miss-detection ($P_m= 1-P_d$) versus the probability of false-alarm.

These curves enable exploration of the relationship between sensitivity (probability of detection) and specificity (false alarm rate) [83]. To plot ROC curves, one parameter is varied while the other is fixed. This enables the study of various scenarios of interest.

### 3.5 Cooperative Spectrum Sensing

A Cognitive radio network is considered comprising of secondary users called CRs and a receiver to receive decisions from CRs, as shown in Fig. 3.2. Here each CR performs spectrum sensing discretely and then the local decisions are sent to the common receiver or fusion centre is considered which can combine all local decisions using hard fusion rule to infer the status absence or presence of the PU.
Hard fusion combining rule is preferred because, single bit is enough to act as overhead in conveying the status to FC. However, for other fusion rules minimum two bits are required to convey the status to FC. A binary ‘1’ indicates presence of primary user whereas binary ‘0’ indicates its absence. The local decision taken by the secondary user is sent to the fusion center. At the fusion centre it fuses based on hard combining fusion rule to the received decision from SUs and global decision is taken which states the presence or absence of PU finally. At the fusion centre, choice from a group of hard fusion rules is applied to the received decision. Some of the popular rules are OR, AND and MAJORITY. This rule allows single bit decision which decreases the overhead bits that increases sensing time when compared to other fusion rules. This technique requires single threshold.

For this work, a CR network is considered with $K$ CRs. The CRs perform sensing the spectrum independently using binary hypothesis-testing problem in this network which is represented by

$$H_0: \text{primary user is absent and}$$
$$H_1: \text{primary user is present.}$$

The spectrum sensing at $i^{th}$ CR is considered for discussion

$$x_i(t) = \begin{cases} w_i(t), & H_0, \\ h_i(t) \cdot s(t) + w_i(t), & H_1, \end{cases} \quad (3.4)$$

The sensing method is to choose between the above two hypotheses where $x_i(t)$ is the received signal at the $i^{th}$ CR in time slot $t$, $s(t)$ is the PU transmitted signal, $w_i(t)$ is the
Additive White Gaussian Noise, and $h_i(t)$ denotes the complex channel gain of the sensing channel between the PU and the $i^{th}$ CR. Here the sensing time is assumed to be smaller than the coherence time of the channel. Then, $h_i(t)$ can be viewed as time-invariant during the sensing process. We denote $h_i(t)$ as $h_i$. Furthermore, we assume that the status of the PU remains unchanged during the spectrum sensing process. If prior knowledge of the PU signal is unknown, the energy detection method is optimal for detecting zero-mean constellation signals [95].

For the $i^{th}$ CR with the energy detector, the average probability of false alarm, the average probability of detection and the average probability of miss-detection over AWGN channels are given by $P_f, P_d$ and $P_m$, respectively [96].

$$P_{f,i} = \frac{\Gamma(u, \frac{\lambda_i}{2})}{\Gamma(u)} \quad (3.5)$$

$$P_{d,i} = Q_u\left(\sqrt{2\gamma_i}, \sqrt{\lambda_i}\right) \quad (3.6)$$

$$P_{m,i} = 1 - P_{d,i} \quad (3.7)$$

In Equations (3.5), (3.6) and (3.7), $\lambda_i$ and $\gamma_i$ denote the energy detection threshold and the instantaneous signal-to-noise ratio (SNR) at the $i^{th}$ CR, respectively, $u$ is the time-bandwidth product of the energy detector, $\Gamma(a, x)$ is the incomplete gamma function which is given by $\Gamma(a, x)$, a gamma function, and $Q_u(a, x)$ is the generalized Marcum Q-function with $I_{u-1}(\cdot)$ being the modified Bessel function of the first kind and order $u - 1$.

$$\Gamma(a, x) = \int_x^{\infty} t^{a-1}e^{-t}dt, \Gamma(a) \quad (3.8)$$

and

$$Q_u(a, x) = \frac{1}{a^{u-1}}\int_x^{\infty} r^{u-1}e^{-\frac{r^2}{2}}I_{u-1}(at)dt \quad (3.9)$$

The received energy is measured and compares it to a threshold. The test statistics of basic energy detection is given by

$$E = \frac{2}{N_o} \sum_{t=1}^{M} |x(t)|^2 < \lambda \quad (H_0)$$

$$E = \frac{2}{N_o} \sum_{t=1}^{M} |x(t)|^2 \geq \lambda \quad (H_1) \quad (3.10)$$
Where, $N_0$ is the noise spectral density. Factor 2 comes from the fact that under circularity assumption the complex noise power is equally divided between the real and imaginary parts.

For the case of detecting a deterministic signal in the presence of zero mean iid complex Gaussian noise, the energy detector test statistic obeys the following distribution which is similar to Equation (3.2)

$$H_0 : E \sim X_{2M}^2$$
$$H_1 : E \sim X_{2M}^2 (2\gamma)$$

(3.11)

where $\gamma$ is the signal energy-to-noise-spectral density defined as $\gamma = \frac{E_s}{N_0}$.

Similarly the energy of signal is calculated from K CRs independently and checked with the threshold $\lambda$. The Energy which crosses the threshold is considered for local decision 1 i.e., presence of PU otherwise the local decision is 0 implies PU is absent. These local decisions are sent to FC through reporting channel [89].

In cooperative spectrum sensing, each cooperative partner makes a binary decision based on its local observation and then forwards corresponding bit of the decision $D_i$ to the common receiver through an error-free channel as 1 and 0 for presence and absence respectively [90, 91].

At the FC (common receiver), all 1-bit decisions are compound together according to logic rule and the test statistics are given by

$$Y = \sum_{i=1}^{K} D_i \left\{ \begin{array}{ll}
\geq n, & H_1, \\
< n, & H_0,
\end{array} \right.$$  

(3.12)

The inferences drawn by the FC that the PU signal is whether transmitted or not, respectively is denoted by $\mathcal{H}_0$ and $\mathcal{H}_1$. Here the decision threshold ‘$n$’ is an integer, representing the “$n$-out-of-$k$” voting rule. It can be seen that the

- OR rule corresponds to the case of $n = 1$
- AND rule corresponds to the case of $n = K$.

The received signal at each cognitive radio experiences almost identical path loss if the distance from any two cognitive users is small. Therefore, in the case of an AWGN environment, we can assume that $\gamma_1 = \cdots = \gamma_K = \bar{\gamma}$.
For Rayleigh fading with the instantaneous SNRs $\gamma_1, \ldots, \gamma_K$ being autonomous are exponentially distributed random variables with the same mean $\gamma$. Furthermore, we assume that all cognitive radios use the same threshold $\lambda$, implying $\lambda_1 = \lambda_K = \lambda$. This results in $P_{f,i}$ being independent of $i$, and we denote it as $P_f$. In the case of an AWGN channel, $P_d$ is independent of $i$ (we denote this as $P_d$). In the case of a Rayleigh fading channel [96], let $P_d$ be $P_{d,i}$ averaged over the statistics of $\gamma_i$. For both kinds of channels,

Now, Probability of missing detection is $P_m = 1 - P_d$.

Therefore, the false alarm probability of cooperative spectrum sensing is given by

$$Q_f = \Pr\{H_1|H_0\} = \sum_{l=n}^{K} \binom{K}{l} P_f^l (1 - P_f)^{K-l}$$ \hspace{1cm} (3.13)

The missed detection probability of cooperative spectrum sensing is given by

$$Q_m = \Pr\{H_0|H_1\} = 1 - \sum_{l=n}^{K} \binom{K}{l} P_d^l (1 - P_d)^{K-l}$$ \hspace{1cm} (3.14)

### 3.6 Optimization of Cooperative Spectrum Sensing

Objective is to investigate the optimality of cooperative spectrum sensing when energy detection and decision fusion are applied.

#### 3.6.1 Optimal Voting Rule

For fixed $K$, what is the optimal $n$, denote as $n_{opt}$ that minimizes the total error rate ($Q_f + Q_m$).

$$Q_f + Q_m = \sum_{l=n}^{k} \binom{k}{l} P_f^l (1 - P_f)^{k-l} + 1 - \sum_{l=n}^{k} \binom{k}{l} P_d^l (1 - P_d)^{k-l}$$ \hspace{1cm} (3.15)

The insightful results shown in Fig.3.4, which shows the total error rate in terms of the detection threshold for various voting rules from $n = 1$ to 10 in a cognitive network with 10 users before probing into pursuing the optimal solution of $n$ [93]. From Fig. 3.4 it can be observed that the optimal voting rule over all the examined range of detection thresholds is $n = 5$. However, for a fixed very small threshold, the optimal rule is the AND rule, i.e., $n = 10$. Meanwhile, for a fixed very large threshold, the OR rule, i.e., $n = 1$, tends to be optimal.
Given $K$, the optimal voting rule for cooperative spectrum sensing that minimizes $Q_f + Q_m$ is to be derived. In $n$-out-of $K$ rule, when $K$ is fixed and optimal $n$ has to determine that minimizes the total error rate $Q_f + Q_m$.

In order to find the optimal number of users we minimize the error probability by differentiating with respect to ‘$n$’ and equating it to zero for finding the value [92, 94]

Let $G(n) = \sum_{l=n}^{k} \binom{k}{l} P_f^l (1 - P_f)^{k-l} + 1 - \sum_{d=n}^{k} \binom{k}{l} P_d^l (1 - P_d)^{k-l}$

\[
\frac{\partial G(n)}{\partial n} = G(n + 1) - G(n) = \binom{K}{n} [(1 - P_m)^n P_m^{K-n} - P_f^n (1 - P_f)^{K-n}]
\]

Equating the Equation (3.17) to zero to get optimal solution

\[
\frac{\partial G(n)}{\partial n} = 0
\]

\[
(1 - P_m)^n P_m^{K-n} = P_f^n (1 - P_f)^{K-n}
\]

Therefore,

\[
n_{opt} = \min \left( K, \left\lfloor \frac{K}{1 + \alpha} \right\rfloor \right)
\]

Where $\alpha$ is given by

\[
\alpha = \frac{ln \frac{p_f}{1 - p_m}}{ln \frac{p_m}{1 - p_f}}
\]

The following remarks are arrived from the above and shall be applied to any detector.

i) Generally $P_f$ and $P_m$ have the same order, therefore the optimal choice of $n$ is $K/2$ for even and for odd $n$ is $(k+1)/2$.

ii) OR rule is optimal when $\alpha \geq K - 1$ which means that $P_f \leq P_m^{K-1}$ This implies that $P_f << P_m$ for a large value $K$. This can be achieved when the detection threshold $\lambda$ is very large.

iii) The AND rule is optimal when $\alpha \rightarrow 0$. This is achieved when $P_m << P_f$, i.e., for a very small $\lambda$. 

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Fig. 3.3. shows the exact solution of \( n \) in terms of detection threshold evaluated from Equation (3.20). Also it shows the optimal voting rule of cooperative spectrum sensing in AWGN channel with SNR =0, 5, 10 dB for fixed value of \( K = 16 \).

The inferences from the simulation results (Fig. 3.3) are

- The exact solution of \( n \) in terms of threshold is evaluated. This in turn reduces sensing duration.
- For very low threshold the number of optimal cognitive radio users is same for different SNRs.
- The threshold should not be fixed very low because the optimal number of users is more with higher probability of error.
- Optimal number users for different thresholds are tabulated in Table 3.1
It is observed from Fig. 3.4 that the voting rule is applied for entire range of threshold from \( n=1 \) to \( 10 \) with \( K=10 \). The total error rate \( Q_f+Q_m \) is calculated and tabulated in Table 3.2. Fig 3.4 shows that variation of the total error rate in cognitive radio network for the variation in the Threshold values of the energy detector in a 10 dB AWGN channel.

The inferences are

- From the above Simulation Result, for the case of \( n=1 \) it is OR rule.

The Error is evaluated for the largest values of the Threshold. Here the amount of \( P_m \) is large compared to \( P_f \).
✓ For the case of \( n=10 \) it is \textit{AND rule}. The Error is evaluated for the smaller values of the Threshold.

✓ Among all the values from \( n=1 \) to 10 the error is very much reduced for the case of \( n=5 \), which is said to be \textit{optimal} and also covers all the Threshold values.

From the above statements it is concluded that optimal choice of \( n \) is \( K/2 \) for even and for odd \( n \) is \( (K+1)/2 \).

### 3.6.2 Optimal Energy Detection Threshold

For Known values of \( K, n \) and \( \text{SNR} \), the optimal threshold \( \lambda^* \) is

\[
\lambda^* = \arg \min_\lambda (Q_f + Q_m)
\]

(3.22)

The total error rate curve of \( Q_f + Q_m \) in terms of \( \lambda \) [95] has a comprehensive minimum for any given \( n \) which is observed in Fig.3.4. It is achieved when

\[
\frac{\partial Q_f}{\partial \lambda} + \frac{\partial Q_m}{\partial \lambda} = 0
\]

(3.23)

Now expressions are derived for \( \frac{\partial Q_f}{\partial \lambda} \) and \( \frac{\partial Q_m}{\partial \lambda} \)

\[
\frac{\partial Q_f}{\partial \lambda} = \sum_{i=1}^{K} \binom{K}{l} l P_f^{l-1} \frac{\partial P_f}{\partial \lambda} (1 - P_f)^{K-l} \\
- \sum_{i=1}^{K} \binom{K}{l} P_f (K - l) (1 - P_f)^{K-l-1} \frac{\partial P_f}{\partial \lambda}
\]

\[
= \frac{\partial P_f}{\partial \lambda} \sum_{i=1}^{K} \binom{K}{l} P_f^{l-1} (1 - P_f)^{K-l} \times \left[ 1 - (K - l) \frac{P_f}{1 - P_f} \right]
\]

(3.24)

Where \( P_f \) is given by Equation (3.5) and

\[
\frac{\partial P_f}{\partial \lambda} = -\frac{1}{(u-1)!} \lambda^{u-1} e^{-\lambda/2}
\]

(3.25)
From Equation (3.14) the derivative of missing probability is 3.14

$$\frac{\partial Q_m}{\partial \lambda} = -\sum_{l=n}^{K} \left(P_{d}^{l-1} (1-P_{d})^{K-l} \frac{\partial P_{d}}{\partial \lambda} + \sum_{l=1}^{K} \left(P_{d}^{l} (K-l) (1-P_{d})^{K-l-1} \frac{\partial P_{d}}{\partial \lambda}\right) \right)$$

$$= -\frac{\partial P_{d}}{\partial \lambda} \sum_{l=1}^{K} \left(P_{d}^{l-1} (1-P_{d})^{K-l} \times \left[ l-(K-l) \frac{P_{d}}{1-P_{d}} \right]\right) \tag{3.26}$$

Since AWGN channel is selected, the probability of detection is

$$P_{d} = Q_{u\left(\sqrt{2}\gamma, \sqrt{\lambda}\right)}$$

$$= \frac{1}{\left(\sqrt{2}\gamma, \sqrt{\lambda}\right)} \int_{\sqrt{\lambda}}^{\infty} \left(-\frac{\gamma^2 + 2\gamma}{2}\right) I_{u-1}\left(\sqrt{2}\gamma k x\right) dx \tag{3.27}$$

Then,

$$\frac{\partial P_{d}}{\partial \lambda} = -\frac{\lambda^{-\frac{u-1}{2}}}{2(2\gamma)^{-\frac{u-1}{2}}} \exp\left(-\frac{\lambda + 2\gamma}{2}\right) I_{u-1}\left(\sqrt{2}\gamma k x\right) \tag{3.26}$$

Using Equations (3.24) and (3.26) the solution to $\frac{\partial Q_l}{\partial \lambda}$ and $\frac{\partial Q_m}{\partial \lambda}$ is evaluated numerically for threshold $\lambda$. The obtained solution is the optimal detection threshold.

### 3.6.3 Adapting threshold using Gradient Descent Algorithm

The gradient descent algorithm is used to adapt the threshold for detection. The algorithm is an iterative one and goes on till the difference between the required quantities is less than the tolerance value for the algorithm. The basic requirement of the algorithm [95] is that the function on which it is applied must be differentiable. The gradient update equation is given by

$$\lambda(n+1) = \lambda(n) - \mu \nabla\epsilon(n) \tag{3.29}$$

where ‘$\lambda$’ is the threshold, ‘$\mu$’ is the step size, ‘$\epsilon$’ is the sensing error.

$$\nabla\epsilon(n) = \nabla(-Q_{d} + Q_{in}) \tag{3.30}$$

$$= \nabla\left[\exp\left(-\frac{(2N+1)\lambda^2}{2}\right) + 1 - Q_{\lambda}\left(\frac{2\gamma}{\sqrt{2}}, \frac{\lambda}{\delta}\right)\right]$$
Here \( I_0(.) \) is the modified Bessel function of Zero\(^{th}\) order and first kind [92]. The approximation for this function is

\[
I_0(x) \approx 1
\]

Thus the gradient equation becomes

\[
\lambda(n+1) = \lambda(n) - \mu \left[ -\frac{(2N+1)\lambda}{\delta^2} \exp\left(-\frac{(2N+1)\lambda^2}{2\delta^4}\right) + \frac{\lambda}{\delta} \exp\left(-\frac{2\gamma}{\delta^2} + \frac{\lambda^2}{\delta^4}\right) \right]
\]

Where, \( \delta_B = \frac{(2\gamma+1)\delta^4}{2N+1} \)

‘\( \delta \)’ is the variance of the received signal, ‘\( N \)’ is the number of samples values of the signal and ‘\( \gamma \)’ is the SNR.

The performance of the energy detector is evaluated for existing optimal energy detection threshold and proposed gradient descent algorithm in terms of optimal number of users and total error rate which were tabulated in Table 3.1 and Table 3.2 respectively. Table 3.1 is evaluated for optimal voting rule versus detection threshold of cooperative spectrum sensing in AWGN channel with SNR = 0, 5, 10 dB and \( K=16 \) in the cognitive radio network. The reason for choosing \( K=16 \) is to cover the entire range of threshold as specified by the literatures.

Table 3.2 is evaluated for total error rate of cooperative spectrum sensing in 10 dB AWGN channel versus threshold with voting rule ranges from 1 to 10 with \( K=10 \). Here \( K=10 \) refers to maximum number of cognitive users in spectrum sensing which is the optimal value for limited bandwidth usage and sensing duration.

After evaluating the total error rate and optimal \( n \) with respect to threshold, the probability of detection is determined for different SNR in an AWGN channel which is illustrated in Fig 3.7. Fig. 3.5 and Fig 3.6 shows the plots for optimal \( n \) and total error rate versus threshold respectively using optimal energy detection threshold with same voting rule & number of users in the network.

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Table 3.1 and Table 3.2 are used to compare the performance of energy detector using optimal energy detection threshold (OEDT) and gradient descent algorithm (GDA). The parameters are total error rate and optimal $n$ with threshold.
Table 3.1: Comparison of optimal $n$ versus decision threshold for optimal energy detection threshold and gradient descent algorithm

<table>
<thead>
<tr>
<th>Decision Threshold $\lambda$</th>
<th>Optimal $n$ at SNR (for $K = 16$)</th>
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<tbody>
<tr>
<td></td>
<td>$0 \text{ dB}$</td>
</tr>
<tr>
<td></td>
<td>$OEDT$</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
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<td>25</td>
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</tr>
<tr>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>35</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of total error rate versus decision threshold for optimal energy detection threshold and gradient descent algorithm

<table>
<thead>
<tr>
<th>Decision Threshold $n$</th>
<th>Total error rate ($Q_r + Q_m$)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$OEDT$</td>
<td>$GDA$</td>
</tr>
<tr>
<td>n = 1</td>
<td>$-1.68$</td>
<td>$-1.89$</td>
</tr>
<tr>
<td>n = 2</td>
<td>$-2.2$</td>
<td>$-2.6$</td>
</tr>
<tr>
<td>n = 3</td>
<td>$-2.6$</td>
<td>$-2.89$</td>
</tr>
<tr>
<td>n = 4</td>
<td>$-2.71$</td>
<td>$-2.95$</td>
</tr>
<tr>
<td>n = 5</td>
<td>$-2.72$</td>
<td>$-2.98$</td>
</tr>
<tr>
<td>n = 6</td>
<td>$-2.71$</td>
<td>$-2.94$</td>
</tr>
<tr>
<td>n = 7</td>
<td>$-2.6$</td>
<td>$-2.89$</td>
</tr>
<tr>
<td>n = 8</td>
<td>$-2.71$</td>
<td>$-2.6$</td>
</tr>
<tr>
<td>n = 9</td>
<td>$-1.84$</td>
<td>$-2.$</td>
</tr>
<tr>
<td>n = 10</td>
<td>$-1.4$</td>
<td>$-1.6$</td>
</tr>
</tbody>
</table>
Fig. 3.7: Probability of detection versus SNR

Fig. 3.7 shows the probability of detection versus SNR for 125 observations with sampling frequency of 6 MHz. The gradient descent algorithm is outperforming compared to optimal energy detection threshold and it exhibits 80% probability of detection at -3.9 dB and 90% probability of detection at -3.5 dB which is above the SNR wall specified in the literature.

3.7 Conclusions

A method of obtaining the optimal energy detection threshold has been proposed. In addition, an efficient spectrum sensing algorithm has been proposed which requires less number of cognitive radios in cooperative spectrum sensing while satisfying a given error bound. The performance of cooperative spectrum sensing with energy detection in cognitive radio networks is improved. Using the optimal decision voting rule the total error probability is minimized, number of CR users with the half-voting rule is $k/2$ for even $n$ and $(k+1)/2$ for odd $n$. The probability of detection in the lower SNR is improved to 80% at -3.9 dB and 90% at -3.5 dB.