2.1 Introduction

The periodic table of elements is increasing at a very slow pace and contains so far only 118 elements (Hofmann (1998); Hofmann and Munzenberg (2000); Armbuster (2000); Hofmann et al. (2001); Dullmann et al. (2002); Ginter et al. (2003a,b); Hofmann et al. (2004); Morita et al. (2004); Oganessian et al. (2004a,b); Oganessian et al. (2006); Hofmann (2010)). Recently, the names flerovium (Fl) and livermorium (Lv) were proposed for elements 114 and 116, respectively (IUPAC (2011)). Some of these elements have not yet been assigned their official names.

Extrapolations based on the nuclear shell model performed in the 1960s led to the prediction of the existence of heavy elements in a region of the nuclear chart far away from nuclei known at that time (Myers and Swiatecki (1966)). These elements were coined “superheavy elements” (SHE) and owe their existence entirely to nuclear shell effects, which stabilize them against immediate spontaneous fission. The search for these artificial elements, situated on the “island of stability” far removed from the heaviest elements found on earth, has been a strong driving force in superheavy element research. Nuclear reactions leading to the formation of new superheavy elements and isotopes are discussed in Zagrebaev and Greiner (2008) and Zagrebaev et al. (2012).

These expensive discoveries demand a more substantial theoretical support, which may provide a more reasonable choice of fusing nuclei, collision energies and an estimate of fusion cross-section. These parameters have been already explored (Bhatia et al. (2005)) by using the asymmetric two center shell model (ATCSM) (Maruhn and Greiner (1972)). Bhatia et al (2005) proposed fusion near the Coulomb barrier as a two step process of “touching and subsequent tunneling of the fusion barrier” and applied it successfully to the compound nucleus formation with $102 \leq Z \leq 110$ in step of $\Delta Z=2$. In each case, two deep minima in the potential energy $V(\eta,\eta_Z)$ were obtained by varying both the mass-asymmetry ($\eta$) and charge-asymmetry ($\eta_Z$) coordinates at fix
relative separation $R$. These minima were referred to as the symmetric and the asymmetric channels for the compound nucleus formation. Further, the asymmetric colliding partners were first shown to be captured in the pocket behind the adiabatic touching (outer) barrier which subsequently tunnel the fusion (inner) barrier to form the resulting compound nucleus. Whereas, in case of the symmetric colliding partners, it has been seen that no such fusion barrier exists and only a (conditional) saddle is formed. These prominent features clearly ruled out the possibility of the compound nucleus formation by using the symmetric reaction partners.

It is worthwhile to mention here that Bhatia et al. (2005) have made the interaction barrier $V(\eta, R)$ calculations by using adiabatic approximation. Within this approximation, the collective coordinates $R$ and $\eta$ vary very slowly, so that during their variation no internal excitation occurs. Even though this adiabatic assumption is in doubt for heavy-ion collisions (Liran et al. (1975)), still our present calculations for the interaction barrier and the cranking masses are based on this assumption due to the following reasons:

(i) For the synthesis of SHE, we are considering either one or both the reaction partners as magic nuclei.

(ii) An excitation energy carried by the compound nucleus is extremely small. So, the shell effects in potential $V(\eta, R)$ do not vanish and hence play an important role for the synthesis of compound nucleus near the Coulomb barrier. This prominent shell structure of magic nuclei ensures that the single particle energies are in competition with that of collective excitations.

(iii) In our calculations, we have not included any type of extra frictional contribution, which is, of course, required for the fusion process (Adamian et al. (1999)). On the basis of these hypothesis, the choice of adiabatic limit is quite appropriate for the fusion near the Coulomb barrier.

Therefore, we have obtained both the potential and the cranking masses in our present work by using the adiabatic approximation. In this work, we try to understand the time evolution of mass transfer in the fusion process. We first establish the time variation of the relative motion $R(t)$ semiclassically. The generalized Bohr-Sommerfeld quantization condition generates discrete eigenvalue spectrum and nicely
predicts the excitation energy carried by the resulting compound nucleus. In the second step, the time dependent Schrödinger equation for the mass asymmetry coordinate \( \eta = (A_1 - A_2) / (A_1 + A_2) \); \( A_i (i = 1, 2) \) being the masses of two partners) is solved. The potential for mass transfer also accompanies the excitations of the system through the radial degree of freedom \( R(t) \). In case of doubly magic reaction partners, our calculations clearly show a vigorous mass transfer at an incident energy near the fusion barrier and hence leads to the formation of resulting compound nucleus.

In Section 2.2, we explain our framework for the mass transfer mechanism. Results and discussion of the dynamical calculations are presented in Section 2.3. Finally, the conclusions are summarized in Section 2.4.

### 2.2 Dynamical Theory of Mass Transfer

The compound nucleus process involves two coupled degrees of freedom, namely: the mass asymmetry \( \eta \) and the relative separation \( R \) (Bhatia et al. (2005)). By using the Pauli prescription, the Hamiltonian for such coupled system is written as

\[
H(\eta, R) = - \frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \right) - \frac{\hbar^2}{2\sqrt{B_{RR}}} \frac{\partial}{\partial R} \left( \frac{1}{\sqrt{B_{RR}}} \frac{\partial}{\partial R} \right) + V(R, \eta).
\]  

(2.1)

Here, the coupling term in the kinetic energy, which is proportional to \( \frac{\partial}{\partial \eta} \frac{\partial}{\partial R} \), is neglected, since the cranking coupling mass is very small, such that \( B_{\eta\eta} \ll \sqrt{B_{RR} B_{\eta\eta}} \) holds good (Maruhn and Greiner (1974)). Also, this inequality is confirmed explicitly from the plots of masses \( B_{RR} \), \( B_{\eta\eta} \) and \( B_{\eta\eta} \) (Zohni et al. (1975)). Further the ATCSM calculations (Zohni et al. (1975)) reveal that the radial mass parameter \( B_{RR} \) does not depend strongly on the \( \eta \) motion. Therefore, this parameter may simply be replaced by the reduced mass, \( \mu \).

The coupling potential \( V(\eta, R) \) in equation (2.1) is considered to be divided into two parts (Yamaji et al. (1976)).

\[
V(R, \eta) = V(R, \eta = \eta_i) + \Delta V(R, \eta)
\]  

(2.2)
Here, $\eta_i$ is the initial mass asymmetry, which involves the nucleon numbers of the target and the projectile.

Within these simplifications, we have developed the following Schrödinger equations in two coordinates $R$ and $\eta$, respectively, as

$$
\left[ -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + V(R,\eta_i) \right] \phi(R) = E_\phi \phi(R) 
$$

(2.3)

$$
\left[ -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \right) + \Delta V(R(t),\eta) + E_R \right] \psi(\eta,t) = i\hbar \frac{\partial \psi(\eta,t)}{\partial t}
$$

(2.4)

The relative coordinate $R$ as a function of time appears in equation (2.4) and is governed by the solution of equation (2.3) by using semiclassical quantization (SCQ) technique as discussed below.

### 2.2.1 Semiclassical treatment of relative motion

To obtain the time development of relative motion coordinate i.e. $R(t)$, we solve equation (2.3) by using the phase integral method of Lakshmanan et al. (1981).

Following the ATCSM, the adiabatic interaction barriers $V(R) (= V_{LDM} + \delta u + \delta p)$ for the symmetric and asymmetric channels have been already obtained (Bhatia et al. (2005)). The double humped barriers were clearly seen in the asymmetric target plus projectile combinations. This double hump implied that the incoming reaction partners firstly overcome the strong touching (outer) barrier before being tunneled through the fusion (inner) barrier to form a resulting compound nucleus. Such an interaction barrier for the reaction $^{48}Ca + ^{208}Pb \rightarrow ^{256}No$ is shown in Figure 2.1 (solid curve). The relative separation $R$ is varied from touching configuration to a considerable overlap of two reaction partners. Beyond touching configuration, only the Coulomb interaction between the reaction partners prevails and is not shown in this plot.

The potential within this variation of $R$ is best fitted by the quartic polynomial

$$
V(R) = a_o + a_1R + a_2R^2 + a_3R^3 + a_4R^4
$$

(2.5)

and this fit is also shown by dotted curve in Figure 2.1. Using a simple translational procedure, Equation (2.5) is transformed to the following form

$$
V(R) = V_0 + V_2R^2 + V_4R^4
$$

(2.6)
Figure 2.1. The interaction barrier $V(R)$ vs. the relative separation $R$ for the system $^{256}_{102}No$ is calculated by using the asymmetric target-projectile combination. The relative separation $R$ is varied from touching configuration $R_t$ to a considerable large overlap of reaction partners. The dotted curve show a quartic fit (equation (2.5)) and its equation is also listed.
Moreover, the coupling constant \( a_i \left( \frac{a_i}{2a_i} \right) \) is clearly related to the other parameters of potential (equation (2.5)) and it vanishes for an oscillator frequency, \( a_s = \frac{1}{4} \frac{a_i}{a_i} \). Here, \( a_i \) acts as a driving force and pushes the reaction partners towards the pocket of double well. This pocket is referred to as an equilibrium position \( R_{eq} \) (shown in Figure 2.1). ATCSM calculations for the asymmetric channels (Bhatia et al. (2005)) show an extended mononuclear shape without neck at \( R_{eq} \). This shape ensures that the reaction partners get mixed up completely before being tunnelled through the fusion barrier. Hence, a complete mixing of the target and the projectile at \( R_{eq} \) implies that the system has lost its history of formation, which is an essential constraint for the synthesis of compound nucleus. Therefore, the variation of relative separation from \( R_{eq} \) to a complete overlap of the reaction partners is of primary interest. Figures 2.2(a) and 2.2(b) show, respectively, the interaction barriers \( V(R) \) for \( ^{48}Ca + ^{208}Pb \rightarrow ^{256}_{102}No \) and \( ^{50}Ti + ^{208}Pb \rightarrow ^{258}_{104}Rf \) within \( R_{eq} \leq R < R_f \) (\( R_f \) being the fusion radius).

The parameters \( V_0 \), \( V_2 \) and \( V_4 \) for these systems are listed in Table 2.1. In this table, it is quite evident in each case that \( V_2 > 0 \) and \( V_4 < 0 \), which clearly supports the double humped barrier. However, to the best of our knowledge, the double barrier with incident energy \( < V_{max} = -\frac{V_f^2}{4V_i} \) is of utmost importance for explaining the compound nucleus formation near the Coulomb barrier.
Figure 2.2. (a) The interaction barrier $V(R)$ vs. the relative separation $R$ for the system $^{256}_{102}$No is calculated by using the asymmetric target-projectile combination. Here, $R_{eq} \leq R < R_f$. (b) Same as (a) but for the system $^{258}_{104}$Rf.
With quartic potential (equation (2.6)), we have solved equation (2.3) analytically and its solution is given by

\[
\phi(R) = \frac{1}{\sqrt{q(R)}} \exp \left[ \pm \int_R^q q(R) dR \right] \tag{2.7}
\]

The function \( q(R) \) is determined by substituting equation (2.7) into equation (2.3), i.e. from the following equation

\[
q^{-\frac{1}{4}} \frac{\partial^2}{\partial R^2} q^{-\frac{1}{4}} + \frac{2\mu}{\hbar^2} \left[ E_R - V(R) \right] q^{-2} = 0. \tag{2.8}
\]

Froman and Froman (1974) proved that the solution of equation (2.8) is expressed as

\[
q(R) = Q(R) \sum_{n=0}^{N} Y_{2n}(R) \tag{2.9}
\]

Here, \( Q(R) = \sqrt{\frac{2\mu}{\hbar^2} \left[ E_R - V(R) \right]} \) and \( Y_{2n}(R) \) are obtained by using the recursion relation (Froman and Froman (1974)). Finally, the generalized Bohr-Sommerfeld quantization condition implies

\[
\frac{1}{2} \int_{\Gamma} Q(R) \sum_{n=0}^{N} Y_{2n}(R) dR + \Delta = \left( n + \frac{1}{2} \right) \pi \tag{2.10}
\]

with \( \Delta = \frac{\pi}{2} - \text{arg} \left[ F_{12}(-\infty, 0) \right] - \text{arg} \left[ F_{22}(-\infty, 0) \right] \), \( F_{12} \) and \( F_{22} \) being elements of the F-matrix (Froman and Froman (1965)). The integration contour \( \Gamma \) is a closed loop in the complex \( R \)-plane enclosing the two classical turning points. Our earlier experiences with these semiclassical calculations (Jain et al. (1997); Malik et al. (2000)) showed that \( \Delta \) is usually negligible for energies below \( V_{\text{max}} \), so we have dropped this term in our present calculations.

The evaluation of integrals in equation (2.10) can be simplified by introducing a parameter \( \tau = \int_{\Gamma} \frac{dR}{Q(R)} \), which is equivalent to

\[
\frac{\partial^2 R}{\partial \tau^2} = \frac{1}{2} \frac{\partial Q^2}{\partial R} \tag{2.11}
\]
Equation (2.11) can be easily reduced to a well-known classical equation of motion \( \frac{\delta^2 R}{\delta t^2} = -\frac{\delta V(R)}{\delta R} \) by defining \( t = \frac{\mu \tau}{\kappa} \). The general solution of equation (2.11) in terms of \( cn \) Jacobian elliptic function is written as

\[
R(t) = A \text{cn}(\gamma t + \delta, k^2)
\]  
(2.12)

Here, \( A \) is an amplitude and \( \delta \) is a constant phase. In equation (2.12), the elliptic modulus \( k^2 = \frac{v_a e^2}{v_a^2 + 2v_a e} \) and the frequency \( \gamma = \frac{\sqrt{2\mu(V_0 + 2V_a e)}}{\hbar} \). Since \( k^2 < 0 \), therefore, we have used the transformation properties of the Jacobian elliptic functions with the negative elliptic modulus (Abramowitz and Stegun (1970)) and obtained the solution of equation (2.11) as

\[
R(t) = A \text{cd} \left( \frac{\gamma t + \delta}{\sigma'}, \sigma^2 \right).
\]  
(2.13)

Now the elliptic modulus is \( \sigma^2 = -\frac{k^2}{1-k^2} \) and its complementary part is \( \sigma' = \sqrt{1-\sigma^2} \). The periodic nature of Jacobian elliptic function implies that period of classical solution (equation (2.13)) is \( 4\sigma'K(\sigma^2) \), where the function \( K \) is the complete elliptic integral of its first kind. The remaining expressions for the integrals in equation (2.10) are same as given in Lakshmanan et al. (1981).

The resulting semiclassical quantization (SCQ) condition is carried out numerically for quantization number \( n = 0, 1, 2, \) etc. In our present calculations, we have kept only five terms \( (N = 5) \) in the series sum equation (2.10) and numerical accuracy has been tested upto five significant figures after decimal place. This SCQ condition fixes the value of the amplitude \( A \) in equation (2.13). Thus, the sequence of energy eigenvalues are given by

\[
E_n = V_0 A^2 + V_a A^4
\]  
(2.14)

Here, we have normalized the ground state of our system at \( V_0 \).

2.2.2 Quantum mechanical treatment of \( \eta \) motion

The wave function for the mass transfer is a solution of time-dependent Schrödinger equation (equation (2.4)). A continuous mesh width calculations of potential \( V(R, \eta) \)
and masses $B_\eta(R,\eta)$ as a function of $R$ and $\eta$ involve a lot of computer time. So, we have considered the coupled potential $\Delta V(R(t),\eta)$ in equation (2.4) through the following ansatz:

$$\Delta V(R(t),\eta) = \frac{V_0 + V_2 R^2(t) + V_4 R^4(t)}{V(R_{eq},\eta)} V(R_{eq},\eta)$$

(2.15)

where, $\Delta V(R_{eq},\eta)$ is the potential at an equilibrium position $R_{eq}$ with fixed initial mass asymmetry $\eta_i$. Here, the $R_{eq}$ position is fixed by the pocket in the double humped barrier. Equation (2.15) implies that at an initial time $t = 0$, we have $\Delta V(R(t = 0),\eta) \sim V(R_{eq},\eta)$. This empirical potential for the mass transfer accommodates nicely the excitations of the system. We would, however, like to make some remarks. At $R_{eq}$, an elongated mononuclear shape without neck is formed, which ensures the complete mixing up of the reaction partners. The resulting system has lost its history of formation at $R_{eq}$ and fulfills the condition of compound nucleus formation. Therefore, we have chosen the initial distribution at $R_{eq}$ rather than at the touching configuration $R_t$. Thus, the mass transfer starts at $R_{eq}$ and continues till the fusion barrier is reached and the time required for this process is defined as reaction time in our formalism.

The collective potential $V(R_{eq},\eta)$ in equation (2.15) is calculated by using the standard Strutinsky renormalization procedure (Strutinsky (1967); Strutinsky (1968)) based on an appropriate liquid drop model (Myers and Swiatecki (1967)) and single particle energies of the ATCSM.

The mass parameters $B_\eta(\eta)$ for the kinetic energy term are consistently calculated by using the ATCSM states in the adiabatic cranking formula based on the BCS formalism (Inglis (1959); Balyaev (1959)). In the present calculations, $B_\eta(\eta)$ is considered to be independent of relative motion $R$.

The initial conditions for solving equation (2.4) are based on the assumption that the dynamical effects are not important for separation $R \geq R_{eq}$. Therefore, the initial wavefunction at time $t = 0$ is calculated by solving numerically the stationary Schrödinger equation.
with potential $V(R_q, \eta)$ and mass $B_{qq}$. With these initial wavefunctions together with the potential (equation (2.15)) and the masses $B_{qq}$, we have solved the time dependent Schrödinger equation (equation (2.4)) by using the finite difference technique (Goldberg et al. (1967)). The normalized time dependent mass transfer probability is given by

$$P = |\psi(\eta, t)|^2 \sqrt{B_{qq}} \frac{4}{A_m} \quad (2.17)$$

where, $A_m$ is the mass number of the composite system. Here, the normalization has been checked numerically.

### 2.3 Results and Discussion

#### 2.3.1 The relative separation $R(t)$

First of all, we obtain an excitation energy carried by the compound nucleus by using equation (2.14). Figure 2.3 shows a plot of semiclassical eigenvalues vs. the quantum number $n$ for the SHE $^{256}_{102}$No and $^{258}_{104}$Rf. The levels designated by quantum number $n$ below the plateau region refer to the stationary states of the compound nucleus. It is quite evident that the compound nucleus may exist in any of these states with a definite probability. In addition, all of these states of a nucleus, except the ground state, have a definite lifetime. The lifetime of states with energy in the continuum domain is merely shortened due to the possibility of ejection of particles. Thus the maximum excitation energy $E^{*}_{\text{max}}$ of the resulting compound nucleus is obtained from an eigenvalue spectrum just below its continuum. The $E^{*}_{\text{max}}$ values for the compound systems $^{256}_{102}$No and $^{258}_{104}$Rf are obtained from these plots and are, respectively equal to 15.296 MeV and 27.071 MeV. Whereas the experimental $E^*$ for these systems (i.e. $^{256}_{102}$No and $^{258}_{104}$Rf) fall within a range 15.5 MeV $\leq E^* \leq 25$ MeV. Our calculated values are remarkably in agreement with the observed limit. These results when added, respectively, to the $Q$-
Figure 2.3. The variation of radial energy $E_R$ (equation (2.14)) vs. the quantum no. $n$ for SHE $^{256}$No, $^{258}$Rf obtained after SCQ. Here, the scale along ordinate is normalized to the $V_0$ values given in Table 2.1.
values 153.80 MeV and 169.69 MeV of the reactions $^{48}Ca + ^{208}Pb \rightarrow ^{256}No$ and $^{50}Ti + ^{208}Pb \rightarrow ^{258}Rf$ fix the centre of mass energy $E_{cm}$ of the reaction partners.

On comparing the two plots, it is noticed that the maximum excitation energy carried by the compound nucleus $^{256}_{102}No$ is $\sim 12$ MeV lower than that of $^{258}_{104}Rf$. This large difference arises due to shell effects of the doubly magic reaction partners in $^{48}Ca + ^{208}Pb \rightarrow ^{256}No$. These shell effects enter through our ATCSM calculations. The stationary nature of the states, below the continuum domain, in Figure 2.3 is also supported from our reaction time calculations. Here, the reaction time is obtained from the periodic nature of the Jacobian elliptic function in equation (2.13). Figures 2.4(a) and 2.4(b) show, respectively, the reaction time $t$ (in $\frac{fm}{c}$) vs. the level energy $E_R$ (in MeV) for nuclei $^{256}_{102}No$ and $^{258}_{104}Rf$. It is worth noticing here that the reaction time remains nearly constant with increase in the level energy $E_R$ and drops down suddenly in the continuum domain. Further, a comparison between the two plots reveals that the reaction time is quite large for the reaction $^{48}Ca + ^{208}Pb \rightarrow ^{256}No$ ($T = 1214.9 \frac{fm}{c}$) than that of $^{50}Ti + ^{208}Pb \rightarrow ^{258}Rf$ ($T = 920.83 \frac{fm}{c}$). It is our opinion that a large reaction time in the former case allows the doubly magic reaction partners to fuse completely and hence, forms a resulting compound nucleus. It is worthwhile to mention here that the rotational bands are seen in the isotopes of $^{102}_{102}No$ (Julin (2001)), whereas no such bands have yet been observed in $Z=104$ region. Further, this difference in the reaction time between $^{256}_{102}No$ and $^{258}_{104}Rf$ may have relevance with their shapes as suggested by Cwiok et al. (2005). Thus, the shell effects in the reaction partners play an important role in the synthesis of stable heavy elements.

2.3.2 The mass distribution

The next step of our formalism is to calculate reflection-transmission phenomena of nucleons using equation (2.17). We shall analyze the reaction $^{48}Ca + ^{208}Pb \rightarrow ^{256}No$ in detail and then give our results for the other reaction $^{50}Ti + ^{208}Pb \rightarrow ^{258}Rf$. 

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Figure 2.4. (a) The plot of the reaction time vs. the radial energy $E_R$ for SHE $^{256}_{102}$No.

(b) Same as (a) for SHE $^{258}_{104}$Rf
Figures 2.5 and 2.6, respectively, show the calculated adiabatic potential $V(R_{eq}, \eta)$ and the masses $B_{eq}(\eta)$ vs. the mass asymmetry $\eta$ for the composite system $^{256}_{102}\text{No}$. The liquid drop potential $V_{LDM}(R_{eq}, \eta)$ is also shown in Figure 2.5 as a dotted curve. We notice in these plots that the liquid drop contribution is smooth, like a simple oscillator, and the shell effects contribute to both the potential and the mass parameters.

The initial wavefunction at time $t=0$ is calculated by solving the stationary Schrödinger equation (equation (2.16)) with the potential and the masses shown, respectively, in Figures 2.5 and 2.6. The normalized initial distribution vs. the mass number $A_m$ on a semilogarithmic scale is shown as dashed curve in Figure 2.7. For comparison, an initial distribution with $V_{LDM}(R_{eq}, \eta)$ and the average masses $(B_{eq} = \int_0^\eta B_{eq}(\eta) d\eta)$ is also shown as dotted line with points in Figure 2.7. It is evident from these plots that an exact Gaussian distribution emerges if the shell effects are completely absent in both the potential and the masses. Therefore, the peaks in the nearly symmetric region and other structure arise due to shell effects in both the potential and the masses. It is worthwhile here to mention here that the nearly symmetric distribution emerges due to complete mixing up of the reaction partners at equilibrium position, $R_{eq}$. In the interaction barrier, the $R_{eq}$ configuration supports an elongated mononuclear shape without neck. Therefore, the reaction partners get mixed up and then relaxed in the pocket of double oscillator before being tunneled through the fusion barrier. An essential requirement (i.e. the loss of incoming channel) for the formation of compound nucleus is achieved through the double humped barrier. Hence, the importance of double barrier for the fusion process is justified.

In order to preserve the shell effects of the reaction partners, we have chosen the wavefunctions of our dashed curve (Figure 2.7) an an initial guess in solving equation (2.4). Further, the time dependent variation of $\Delta V(R(t), \eta)$ in equation (2.4) is considered through an ansatz (equation (2.15)). Also, the incident energy $E_R$ is included by using equation (2.14). With these choices of parameters together with the masses in Figure 2.6, we have solved equation (2.4) by the finite difference technique (Goldberg
Figure 2.5. The plot of adiabatic potential $V(R_{eq}, \eta)$ vs. the mass asymmetry $\eta$ for the complete system $^{256}_{102}No$. The dotted curve refer to $V_{LDM}$. 
Figure 2.6. The plot of adiabatic masses $B_{\eta\eta}(\eta)$ vs, the mass asymmetry $\eta$ for the composite system $^{256}_{102}$No. Here, $M$ refers to the nucleon mass.
Figure 2.7. The plot of mass distribution vs. the mass number $A_m$ for the SHE $^{256}\text{No}$. The dotted curve with circles refer to distribution at $t=0$ with liquid drop potential $V_{LDM}$ plus average mass parameter $B_{av}$. The dashed curve refers to distribution at $t=0$ with liquid drop potential plus shell effects $V(R_{eq},\eta)$ plus cranking masses $B_{\eta\eta}$. The solid curve refer to distribution at $t=T$ with potential (equation (2.15)) plus cranking masses $B_{\eta\eta}$. 
et al. (1967)). Here, two nucleon transfer is taken as an incremental step in \( \eta \) (i.e. \( \Delta \eta = \frac{\Delta}{T} \)) and the reaction time \( T = 1214.99 \frac{fm}{c} \) is divided into 1000 equal steps of \( \Delta t \). A solid curve in Figure 2.7 shows the result of time dependent distribution at \( T = 1214.99 \frac{fm}{c} \). A comparison between solid and the dashed curves clearly reveals the following observations.

(i) The shape of final distribution remains the same as that of initial distribution.

(ii) The distribution spreads with time. No reflection peak near the symmetric region is seen.

These features of time dependent curve ensure the complete transmission phenomenon. Such a complete transmission can also be understood in a following manner.

The solid curve in Figure 2.7 refers to time \( T = 1214.99 \frac{fm}{c} \), when the incident energy \( E_R \) equals the fusion barrier \( V_{\text{max}} = -\frac{\nu^2}{2\mu} \) (Figure 2.2(a)) and hence supports the phenomenon of resonance. A portion of the wave packet is captured by the barrier and remains confined to a period \( T \), which is long as compared with the usual time of transmission through the barrier. The captured part of the packet bounces to and fro between the barrier walls, and acquires a small amount of probability to escape in each collision. Eventually, the entire wave packet gets transmitted. Such a vigorous mass transfer fully supports the formation of compound nucleus \(^{256}_{102}\)No. We would like to mention here that a peak at an exact location of the compound nucleus is not possible in our present calculations as the ATCSM cannot be extended beyond \( \eta \geq 0.75 \).

In view of the above observations, we have repeated our calculations for the reaction \(^{50}\)Ti + \(^{208}\)Pb \( \rightarrow ^{258}\)Rf. Figures 2.8 and 2.9, respectively, show their calculated adiabatic potential \( V(R_{eq}, \eta) \) and the masses \( B_{\eta\eta}(\eta) \). The initial wave functions are obtained by solving the stationary Schrödinger equation (equation (2.16)) with the potential and the masses shown, respectively, in Figures 2.8 and 2.9. With these initial wavefunctions plus the potential in equation (2.15) and the masses in Figure 2.9, we have solved equation (2.4) with two nucleon transfer as an incremental step in \( \eta \) (i.e. \( \Delta \eta = \frac{\Delta}{T} \)) and
Figure 2.8. The plot of adiabatic potential $V(R_{eq}, \eta)$ vs. the mass asymmetry $\eta$ for the composite system $^{258}_{104}Rf$. 
Figure 2.9. The plot of adiabatic masses $B_{\eta\eta}(\eta)$ vs. the mass asymmetry $\eta$ for the composite system $^{258}_{104}Rf$. Here $M$ refers to the nucleon mass.
Figure 2.10. The plot of mass distribution vs. the mass number $A_m$ for the SHE $^{258}\text{Rf}$. The dashed curve refer to distribution at $t = 0$ with liquid drop potential plus shell effects $V(R_{eq}, \eta)$ plus cranking masses $B_{\eta\eta}$. The solid curve refer to distribution at $t = T$ with potential (equation (2.15)) plus cranking masses $B_{\eta\eta}$. 
1000 equal steps for the reaction time \( T = 920.83 \frac{fm}{c} \). A solid curve in Figure 2.10 shows the result of time dependent distribution at time \( T = 920.83 \frac{fm}{c} \). An initial distribution is also shown in this figure as dashed curve. A comparison of solid and the dashed curves clearly show both the reflection and the transmission phenomena. Here, the incident energy, which is equal to the barrier height, does not support the phenomenon of resonance. These results are in contrast to that of \(^{256}\text{No}\) where the complete transmission dominates. This reflection process among the nucleons slows down the process of compound nucleus formation. Another interesting result of this plot is that sharp peaks are seen at the incoming reaction partners (i.e. \(^{50}\text{Ti}\) and \(^{208}\text{Pb}\)). Such observations clearly support quasi-fusion process rather than the complete fusion.

Further, we would like to correlate this phenomenon of nucleon transfer to the quantum mechanical penetration probability through the fusion barrier by using a simplified model. These tunneling probabilities through the fusion barrier for \(^{256}\text{No}\) and \(^{258}\text{Rf}\) have already been calculated (Bhatia et al. (2005)). The salient results of this model are given below.

### 2.3.3 Simplified model

In this model, we replace the \( \eta \)-dependent part of the potential in equation (2.4) by an oscillator potential

\[
\Delta V(R(t), \eta) = \Delta V_i(R(t)) + \Delta V(R(t))\eta^2
\]  

(2.18)

Following the procedure outlined in (Bhatia et al. (2005)), the collective potential at fix relative separation \( R (= R_1; \ R_1 \) being the touching configuration) \( V(\eta, \eta_Z) \) is minimized in both \( \eta \) and \( \eta_Z \). Figures 2.11(a) and 2.11(b) show, respectively the variation of \( V(\eta) \) for the composite systems \(^{256}\text{No}\) and \(^{258}\text{Rf}\). Each curve in Figure 2.11 shows two deep minima, one refers to the symmetric or nearly symmetric combination \( (\eta_s) \), whereas the other two corresponds to the asymmetric \( (\eta_a) \) one. These minima fix the reaction partners for the fusion process and are listed in Table 2.1. The interaction barriers for their asymmetric combinations
Figure 2.11 (a) The potential $V(\eta)$ vs. the mass asymmetry $\eta$ for the system $^{256}_{102}\text{No}$ is calculated at the touching configuration. (b) Same as (a) but for the system $^{258}_{104}\text{Rf}$.
are already shown in Figure 2.2. Figures 2.12(a) and 2.12(b) show, respectively, the interaction barrier for the symmetric combination of \( ^{256}_{102} \text{No} \) and \( ^{258}_{104} \text{Rf} \). Their quartic fit (equation (2.6)) parameters are also given in Table 2.1. Here, in each case \( V_2 < 0 \) and \( V_4 > 0 \). These peculiar values of the potential parameters refer to the double well character instead of double barrier. It is interesting to remark here that this double well supports the conditional saddle seen in microscopic ATCSM calculations in Bhatia et al. (2005) for these combinations. Both these prominent features (i.e., absence of fusion barrier and existence of conditional saddle) rule out the possibility of formation of a cool compound nucleus by using symmetric or nearly symmetric combinations. The symmetric and the asymmetric barriers fix \( \Delta V_i(R(t)) \) and \( \Delta V_s(R(t)) \) as

\[
\Delta V_i(R(t)) = \frac{\Delta V(R(t), \eta_s) - \Delta V(R(t), \eta_a)}{\eta_s^2 - \eta_a^2} \\
\Delta V_s(R(t)) = \Delta V(R(t), \eta_s) - \Delta V_2(R(t))\eta_i^2
\]

where, \( \eta_i \) and \( \eta_a \) refer to the symmetric and the asymmetric combinations, respectively. Now, we solve equation (2.4) under the initial condition of a very narrow Gaussian distribution

\[
\psi(\eta, t=0) = \frac{\exp \left[ -\frac{1}{2} \left( \frac{(\eta - \eta_i)^2}{\Gamma_i} \right) \right]}{\sqrt{\int_x \exp \left[ -\frac{(\eta - \eta_i)^2}{\Gamma_i} \right] \sqrt{B_{av}} \, d\eta}}^{1/2}
\]

Here, the \( B_{av} \) is the cranking mass \( B_{\eta \eta} \) averaged over the \( \eta \) coordinate, \( \Gamma_i \) gives the initial width equal to two nucleon transfer and \( \eta_i \) is the initial mass asymmetry.

The calculated mass transfer probabilities for both \( ^{256}_{102} \text{No} \) and \( ^{258}_{104} \text{Rf} \) are shown in Figure 2.13. Both these curves show Gaussian spread around their initial mass asymmetry. A comparison of full width at half maxima in these distribution reveals that large mass transfer occurs in \( ^{256}_{102} \text{No} \) than in \( ^{258}_{104} \text{Rf} \), even though the latter system carries larger excitation energy. The calculated ratio of probabilities between them is

\[
\frac{\psi(\eta, T)^{^{256}_{102} \text{No}}}{\psi(\eta, T)^{^{258}_{104} \text{Rf}}} \sim 10.
\]

The tunneling probability through fusion barrier for these systems using WKB approximation is already calculated (Bhatia et al. (2005)) and their ratio is
Figure 2.12. (a) The interaction barrier $V(R)$ vs. the relative separation $R$ for the system $^{256}_{102}$No is calculated by using the symmetric target-projectile combination. (b) Same as (a) but for the system $^{258}_{104}$Rf.
Figure 2.13. The plot of normalized mass distribution vs. the mass number $A_m$ using simplified model calculations. Here solid curve refers to $^{256}_{102}N_0$ whereas the dotted curve refers to $^{258}_{104}Rf$. 
\[ \frac{P_{R(256\text{No})}}{P_{R(252\text{Bf})}} = \frac{8.066 \times 10^{-4}}{1.776 \times 10^{-6}}. \] Both these results are fairly consistent within an order of magnitude and hence enable us to establish a correlation between the two.

### 2.4 Conclusions

A semiclassical formalism for the synthesis of SHE is carried out. The adiabatic interaction barrier, which is calculated by using the ATCSM, is best represented by double oscillator. The SCQ of relative separation coordinate \( R \) generates an eigenvalue spectra, which helps us in fixing the maximum excitation energy of the resulting compound nucleus. We have proved that the doubly magic reaction partners need less centre of mass energy and more reaction time for the complete fusion. Our results for \(^{256}\text{No}^{102}\) are fairly consistent with the experimental observations. Further, the phenomenon of mass transfer between the reaction partners is studied by treating the mass asymmetry coordinate \( \eta \) quantum mechanically. The time dependent Schrödinger equation in \( \eta \) degree of freedom is solved by finite difference technique. A resonance phenomenon is seen in case of doubly magic reaction partners, which allows a complete fusion. Hence the shell effects of the incoming channel play a dominant role in the synthesis of SHE.