CHAPTER - 1

Introduction

In the year 1953, an article entitled “What Holds the Nucleus Together?” by Hans A. Bethe was published in Scientific American (Bethe (1953)). Since the discovery of the atomic nucleus and its constituents, the question formulated by Bethe has been a primary problem in physics and remains only partially answered at present. Nevertheless, the challenge for the nuclear physicist does not end there. Once a model for the interaction between the nucleons has been assumed, the difficult programme of investigating the nuclear properties in terms of the underlying interaction arises.

Finite nuclei are complex many-body systems. Any attempt to describe these objects within a single and fundamental theory becomes, from the outset, too ambitious an undertaking. Traditionally, the description of nuclear structure has been approached by solving a many-body Schrödinger equation which involves nucleons interacting through static nucleon-nucleon (NN) potentials. This has been done within the Hartree-Fock (HF) approximation, where the many-body wave function of the system is replaced by a slater determinant of a single-particle wave function obtained in a self-consistent way from the mean field produced by the nucleons themselves.

In non-relativistic nuclear physics, semiclassical models have become very popular (Hasse and Schuck (1984); Gross and Dreizler (1985); Hasse (1987)) due to their reliability and feasibility, as they avoid the task of solving the quantal mean field HF equations.

Basically there are two different types of approximations which are called ‘semiclassical’: The first- which we prefer to call ‘quasiclassical’- is a combination of purely classical treatments of some degrees of freedom with a quantal treatment of the remaining ones, e.g. time dependent classical trajectory approximations (Child (1976)). The second category of approximations construct the rigorous limit of quantum mechanics for $\hbar \to 0$ (Froman (1980)).

Quite generally, semiclassical wave functions (and other quantities) can be written as a sum of terms arising from classically allowed trajectories,
\[ \psi^{sc} = \sum_{j} A_j e^{iS_j / \hbar} \]  

(1.1)

where amplitudes \( A_j \) are square roots of classical probabilities and phases in the exponent are determined by classical actions \( S_j \). From expression (1.1) it is clear how semiclassical approximation can account for many other quantum phenomenon, such as potential or dynamical tunneling (Davis and Heller (1981)), which are forbidden in classical mechanics.

### 1.1 Survey of Semiclassical Development

Semiclassical approximation is as old as quantum theory itself. The old quantum theory of Bohr and Sommerfeld later turned out to be a semiclassical approximation to Schrödinger wave mechanics. Before Schrödinger equation was discovered, N. Bohr (Bohr (1913)) had successfully described the hydrogen spectrum, using the quantization condition

\[ \oint p dq = 2\pi \hbar n \]  

(1.2)

This equation selects only those classical states of the system for which the action integral equals an integer multiple of Planck’s constant \( \hbar = 2\pi \hbar \). This condition was soon generalized to systems with more degrees of freedom: first by Epstein (1916) to separable systems and later by Einstein (1917) to integrable systems. A system with \( f \) degrees of freedom is integrable (in the sense of Liouville (1855) and Arnold (1978)) if there are \( f \) independent integrals of motion in involution. The phase space of integrable systems foliates into invariant tori on which the quantization condition becomes

\[ \oint_{C_j} p dq = 2\pi \hbar n \]  

(1.3)

where \( \{ C_1, ..., C_f \} \) is a basis of non-contractible loops on the torus.

Wentzel (1926), Kramers (1926), Brillouin (1926) and Jeffreys (1925) showed that the old quantization conditions follow from the Schrödinger equation in the short-wavelength limit, or now so-called WKB or JWKB approximation. It was found that the integer \( n \) in equation (1.2) has to be corrected by a small number depending on the character of motion (e.g., one half for each oscillating degree of freedom). A full derivation of the semiclassical quantization conditions was completed by Keller (1958).
who incorporated both the shift of the quantum numbers and the integrability concept of Einstein, to find

$$\oint_{C_j} pdq = 2\pi \hbar (n + \frac{\mu_j}{2})$$  \hspace{1cm} (1.4)$$

This is now known as Einstein-Brillouin-Keller (EBK) or torus quantization. The number $\mu_j$ denotes the Maslov index (Maslov (1972); Maslov and Fedoriuk (1981)).

Almost simultaneously with the derivation of the WKB approximation for the stationary solutions of the Schrodinger equation, Van Vleck (1928) derived a semiclassical form of the quantum propagator, a quantity useful in quantum dynamics. Mathematically, the propagator is the kernel $K(q,q';t)$ of the linear transformation that enables finding the wave function $\psi(q;t)$ at time $t$ if we know the wavefunction $\psi(q';0)$ at time $0$:

$$\psi(q;t) = \int dq \ K(q,q';t) \ \psi(q';0)$$

Van Vleck derived the short-wavelength approximation,

$$K_{sc}(q^*,q';t) = (2\pi i\hbar)^{-1/2} \sum_j \left[ \frac{\partial^2 S_j}{\partial q^* \partial q} \right]^{1/2} \exp[iS_j((q^*,q';t)/\hbar - iv_j\pi/2)]$$  \hspace{1cm} (1.5)$$

in which $S_j(q^*,q';t)$ denotes the action along the $j$th classical trajectory connecting $q'$ with $q^*$ in time $t$,

$$S_j(q^*,q';t) = \int dt \ L[q_j(t),\dot{q}_j(t);t]$$

and $L$ is the Lagrangian. The phase $v_j\pi/2$ was not part of Van Vleck's original derivation, it was added later by Gutzwiller (1967). Without going into details, we note two properties of the VanVleck-Gutzwiller propagator (equation (1.5)): first, it can be derived from the Feynman propagator (Morette (1951), Papadopoulos (1975)), if the integral over all trajectories (even those classically forbidden) is evaluated by the stationary-phase approximation. This process selects only the classically allowed trajectories. Second, the semiclassical propagator satisfies the “composition rule” satisfied by the exact quantum propagator,

$$K_{sc}(q^*,q';t) = \int_{S.F.} dq \ K_{sc}(q^*,q';t-t') K_{sc}(q,q';t')$$  \hspace{1cm} (1.6)$$
as long as the integral is evaluated by the method of stationary phase. This method is an underlying feature of all of semiclassics: the final result is always the same, as long as all integrals used in its derivation are evaluated by the stationary phase approximation.

1.2 Role of Semiclassics in Nuclear Physics

Nuclei display a variety of phenomena, which originate from single particle to collective motion and their interplay. A basic framework for discussing these phenomena has been provided by the shell model, which focuses upon the shell structure in a single particle motion. Evolution of shell structure with particle numbers, nuclear shapes and angular momenta has been the central theme of nuclear structure physics (Jain (1998)). It has been the main guiding factor for a number of theoretical predictions and explanations of several nuclear phenomena such as fission isomers, shape coexistence, superheavy elements and superdeformation.

We present briefly the semiclassical methods and their success, which have been already tested in nuclear structure physics. Nonlinearity and chaos are interwined subjects. If a deterministic response of a system to nonlinear forces leads to an unpredictable dynamical scenario, we call it as chaos. Integrability and ergodicity of physical systems still continue to pose questions and dilemmas requiring a deep exploration of various kind of physical systems. Nuclei stand out as unique objects, which embody the features of few body and many body systems at the same time. With a tremendous advancement in the detection of very weakly populated states and a fast approach towards the era of complete spectroscopy, it is expected that new features and phenomena will emerge which will help us to understand and resolve some of the contradictions between integrability and ergodicity. Also, it may become possible to understand many of these nuclear phenomena in an entirely different light providing us a new window to nuclear structure and reaction dynamics.

One of the most important links as well as simplifying factors is provided by the success of the mean field potential in nuclear physics and the ensuing independent particle motion of the nucleons. It allows us to view the nucleus as a particle in a container or cavity – its two dimensional counterpart being the classical billiards problem. The shape of the container may be taken to correspond to that of the mean field potential. Billiards thus provide us with models of bound state systems like atoms,
molecules, nuclei and the recently discovered mesoscopic systems like the metallic clusters.

A rectangular or, a circular billiard is an integrable system. A circle when deformed to the shape of a stadium, is highly chaotic system. A lemon shaped billiard, on the other hand, lies in between and is an example of a typical mixed dynamical system having both chaotic motion as well as quasi-periodic motion. The real physical systems are expected to be of mixed type. A nucleus, with its capability to acquire various kinds of shapes and degrees of freedom, therefore, presents a rich variety of situations where order-chaos can be meaningfully studied.

1.2.1 Orbits, tori and Poincare surfaces

The Hamiltonian of an integrable system $H(J_1, J_2, ..., J_n)$ is a function of $n$ action variables $(J_i; 1 \leq i \leq n)$ only and the respective angle variables $\theta$ obey the relation $\theta_i(t) = \nu_i t + \theta_i(0)$ with the frequencies $\nu_i = \frac{\partial H}{\partial J_i}$. The system traces its trajectory in the phase space of action-angle variables forming a $k$-torus and its intersection with any $(J, \theta)$ plane through the origin either consists of a finite set of points (if all the frequencies are commensurate) or, a closed curve. This is a Poincare surface of section. It provides one of the most useful signatures of whether the motion is chaotic or not.

Even a small perturbation may destroy the integrability and chaotic trajectories begin to occupy the phase space, the invariant tori become deformed. A quantity called the Lyapunov exponent, which expresses the rate of divergence of two trajectories starting close by, acquires a finite value greater than zero for chaotic behavior (Lichtenberg and Lieberman (1992); Gutzwiller (1990)). An increase in the strength of perturbation produces chaotic behavior first near those regions of phase space where all the frequencies are commensurate. The tori of the integrable system where the frequencies are incommensurate become deformed, but not completely destroyed (The KAM theorem). A further increase in the perturbation fills up the phase space with islands of regular motion embedded in a sea of chaotic motion. In our opinion, the superdeformed states in deformed nuclei appear to be classic examples of such regular motion in a sea of chaos. If the perturbation is increased further, chaotic trajectories almost completely fill up the phase space and the motion becomes completely chaotic.
Classically, chaotic behavior is particularly manifested in nonlinear systems driven by an external, time-dependent perturbation, such as Fermi accelerator or the Fermi-Ulam model. One of the signatures of classical chaos is the presence of a self-similar structure on a progressively finer scale (a fractal structure). The existence of fractal sets has been shown even for pseudo-integrable systems which are time dependent (Jain and Pandey (1998)).

Applications of these concepts to quantum systems in general and nuclear systems in particular have been rather limited. This is because a well-defined concept of quantum chaos still does not exist. Yet a large number of quantum systems pass through the conditions under which a classical system will exhibit an order-to-chaos transition (Swiatecki (1988); Blocki et al (1993); Mottelson Nordita (1990)). We shall highlight in the following subsection some of the recent studies related to nuclei.

1.2.2 Chaos in Nuclear Physics

There is at least one area i.e. Statistical Nuclear Spectroscopy, where chaos in nuclei has been studied for almost five decades. This approach is based on the random matrix theory first introduced by Wigner in 1958 (Wigner (1958)) and used to study the fluctuation properties such as the nearest level spacings and the level density. This approach relies upon the proposal of Bohigas, Giannoni and Schmidt (Bohigas et al. (1984) derived from the study of Sinai billiard problem. The proposal states that the fluctuation properties of generic quantum systems which in the classical limit are fully chaotic, coincides with those of the Gaussian Orthogonal Ensemble (GOE) for systems with time-reversal symmetry and Gaussian Unitary Ensemble (GUE) for systems without time-reversal symmetry. A large number of studies have relied upon this conjecture to identify chaos in nuclei (Weidenmuller (1990); Shriner et al. (1990)). This, however, is not the focus of our present thesis.

An alternative view arises from the realization that the common potential in which the nucleons move, does resemble a cavity. It provides us the starting point to look at a number of questions from a different perspective, e.g., questions related to nuclear shapes, the shell structure and their relationship as a function of particle number, excitation energy and angular momentum. In a recent study, Heiss et al. (1995a,b) studied the effect of including an octupole and a hexadecapole term in a strongly
deformed quadrupole potential, while such a system is non-integrable classically. The quantum treatment produced a general smoothing effect with regard to chaos and yielded a pronounced shell structure only for a prolate case or a certain admixture of a hexadecapole deformation to it.

Order-chaos transition of a particle contained in a rotating ellipse and a rotating stadium was studied by Frisk and Arvieu (1989), who found that for large rotational frequencies, both the ellipse (integrable) and the stadium (non-integrable) display similar phase space structure.

It would be most interesting to see how the origin of shell structure could be traced to the more fundamental concepts such as the quantization of a particular set of orbits. The effect of rotation and excitation on the shell structure could then be analyzed in terms of the behavior of periodic orbits. The periodic orbit theory (POT) of Gutzwiller (Gutzwiller (1967); Gutzwiller (1969); Gutzwiller (1970); Gutzwiller (1971)) makes an important beginning in this direction. The periodic orbit theory has now been established as one of the most important semiclassical methods to study quantum systems and promises to be of great value in connecting the classical concepts of orbits with the quantized energy levels.

1.3 Methodology

1.3.1 The Periodic Orbit Theory

A universal semiclassical approximation is provided by the trace formulae which relate quantum energy levels to classical periodic orbits. For the bound states of a single-particle Hamiltonian $H$, the level density is generally defined as a sum of delta functions,

$$g(E) = \sum_i \delta(E - E_i) \quad (1.7)$$

over all the eigen-energies $E_i$ (including degeneracies). It can be split into a smooth and an oscillating part:

$$g(E) = \tilde{g}(E) + \delta g(E) \quad (1.8)$$

The smooth part $\tilde{g}(E)$ is determined by a phase-space averaging and can be described by an extended Thomas Fermi model (Brack and Bhaduri (1997)). The
oscillating part $\delta g(E)$ is the key quantity, which periodic orbit theory relates to a sum over the actions, time periods and stability angles of all the periodic orbits. This relationship, a trace formula due to Gutzviller (1990), may be written as

$$\delta g(E) = \frac{1}{\pi\hbar} \sum_{\text{PO}} \frac{T_{\text{PO}}}{\sqrt{\det|M_{\text{PO}}|}} \cos \left( \frac{S_{\text{PO}}}{\hbar} - \sigma_{\text{PO}} \frac{\pi}{2} \right)$$

(1.9)

The left-hand side of equation (1.9) is a quantity quantum in nature whereas the right hand side contains only classical quantities. Here, $T_{\text{PO}}$ represents the time period of the primitive periodic orbit or the fundamental orbit and $M_{\text{PO}}$ is the Monodromy stability matrix. The oscillations are controlled by action $S_{\text{PO}}$ of each periodic orbit and $\sigma_{\text{PO}}$ is the Maslov index. It may be pointed out that the equation (1.9) is applicable only to classically chaotic systems which display isolated periodic orbits. Strutinsky and coworkers (Strutinsky (1975); Strutinsky and Magner (1977); Strutinsky et al. (1977)) generalized the Gutzwiller’s trace formula to systems with continuous symmetries. According to Strutinsky, the oscillating component of a single particle level density $\delta g(E)$ in cylindrical coordinates $(\rho, \phi, z)$ is given by

$$\delta g(E) = \frac{1}{\pi\hbar^2} \sum_{\beta, m} f_{\beta, m} \sin \left( \frac{S_{\beta, m}}{\hbar} + \alpha_{\beta, m} \right) \int \int d\rho dz \sqrt{|p_{\rho, \rho'} p_{\phi, \phi'} p_{z, z'}|} J(\beta; \rho, \phi, z; \rho', \phi', z')$$

(1.10)

The factor $f_{\beta, m}$ equals to 1 for the diametric orbits and 2 for other orbits like triangles, squares etc. The time-period for the path from the initial point $\vec{r}$ to the final point $\vec{r}'$ for energy $E$ is defined as

$$t_{\beta} = \frac{\partial S_{\beta}(\vec{r}, \vec{r}', E)}{\partial E}$$

(1.11)

The quantity $J$ in equation (1.10) is the Jacobian of transformation between two sets of classical quantities $(p_{\rho}, p_{\phi}, t_{\beta, m})$ and $(p'_{\rho}, z'_{\rho}, E)$, which are related by the classical equations of motion. Here, $\beta$ denotes the type of orbits and $m$ is the number of repetitions of a given type of orbit.

The periodic orbit sum in equation (1.10) does not converge in most cases. Since the maximum contribution to the gross shell structure comes from the shortest periodic orbits. It has now become customary to carry out a smooth truncation of the
contributions of the longer periodic orbits by folding the level density with a Gaussian function \((Brack \text{ and Bhaduri (1997)})\) of width \(\gamma\),

\[ \delta g_{av} = \sum_{\beta, m} \delta g(E) \exp \left( -\frac{\gamma L_{\beta, m}}{2h} \right)^2 \]  (1.12)

Here, \(L_{\beta, m}\) is the length of the periodic orbit. The averaging width \(\gamma\) is chosen to be larger than the mean spacing between the energy levels within a shell, but much smaller than the distance between the gross shells. This averaging ensures that all longer paths are strongly damped and only the shortest periodic orbits contribute to the oscillating part of the level density. Also, the effect of degeneracies is included in equation (1.10) by using the prescription outlined in Strutinsky \textit{et al.} (1977).

\subsection*{1.3.2 The Maslov Index}

The Maslov index plays a very crucial role on the periodic orbit sum (Equation (1.10)) as it decides the relative phase of the various terms in the summation. The Maslov index \(\sigma\) is the sum of two terms \(\mu\) and \(\nu\), where \(\mu\) is an index counting the number of conjugate points of a given orbit of fixed energy and \(\nu\) arises while talking the trace of the Green’s function to obtain the level density \(g(E)\). Following Creagh and Littlejohn (1990), Brack and Bhaduri (1997) presented a set of rules to obtain the Maslov index for billiards. Malik and Jain (2000) We list these rules in a more explicit form for an axially symmetric deformed cavity.

(a) \(\mu\) is determined by number of conjugate points i.e. turning and caustic points.

(b) A simple conjugate point gives a phase shift of \(-\pi/2\) corresponding to change in \(\mu\) by 1.

(c) Each reflection from the boundary produces a simple turning point (sign change of the normal component of the particle momentum) and a caustic point in the tangential direction. This gives a change \(\Delta\mu = 2\) per reflection and a change in phase by \(-\pi\).

(d) Inside the billiard, there is a conjugate point along the caustic for each reflection, giving rise to \(\Delta\mu = 1\) and a phase change of \(-\pi/2\). Thus the total contribution to \(\mu\) is \(-3mn_{\nu}\pi/2\).
(e) There is a phase phase shift of \(-(mn_n - 1)\pi\) for two dimensional periodic orbits, which is related to phase change in \(\pi\) due to rotation around the center.

(f) While \(\Delta \mu = 2\) for each reflection satisfying Dirichlet boundary conditions, \(\Delta \mu = 0\) for Neumann boundary condition.

(g) The contribution due to \(\nu\) depends on the sign of the quantity \(\omega\) defined by

\[
\omega = \frac{\text{tr} \bar{M} - 2}{b}
\]

where \(b\) is given by,

\[
b = \frac{\partial r_\perp (t = T)}{\partial p_\perp (t = 0)}.
\]

(h) For diametric orbits, there are two simple caustic points presented by two focal points.

1.4 Symmetry Breaking in the Rotating Mean Field

The phenomenon of spontaneous symmetry breaking is a leading theme of a multitude of quantum effects. For instance, it is due to the spontaneous breaking of the particle-number symmetry that superconducting condenstates appear in metals. Spontaneous symmetry breaking occurs if a system in its endeavor to attain the minimal energy chooses a symmetry-violating state even though the underlying interactions are invariant under the concerned symmetry. Nevertheless, it is the nature of the interactions that determines which symmetries are broken and under which conditions. Therefore, study of symmetry violating states brings one closer to understanding the interactions-the most fundamental goal in physics.

Rotation is a universal phenomenon. The macroscopic world is replete with the examples of objects (planets, stars and galaxies) rotating independently and coherently at the same time. The microscopic world, however, places a severe restriction on rotational motion. A molecule or a nucleus must be anisotropic or, non-spherical in nature. While no such restrictions exists for the celestial objects, it was Jacobi who concluded that ‘ellipsoids with three unequal axes can very well be figures of equilibrium’ (Chandrashekhar (1969)).

If we speak about the shape of a nucleus, we mean the shape of its density distribution. The symmetry of the density distribution –spherical or deformed- decides
if the spectrum will be irregular or show rotational bands. The density distribution is found by means of mean field approaches, like the various types of the Hartree-Fock calculations or the Strutinsky method. For large angular momentum one has to use the Cranking generalizations of these methods, which describes an uniformly rotating mean field. In these studies, one used to assume that the axis of uniform rotation coincides with one of the principal axes of the density distribution, as it’s the case for molecules. Frauendorf (1993) demonstrated that nuclei are different from molecules. Nuclear chirality is a novel manifestation of spontaneous symmetry breaking resulting from an orthogonal coupling of angular momentum vectors in triaxial nuclei (Frauendorf and Meng (1997)). Nuclei contain nucleons on orbits with large angular momentum, which is kept constant by quantization. Due to presence of these micro-gyroscopes, the axis for uniform rotation (the angular momentum vector $\vec{J}$) can take any direction with respect to the density distribution. Changing the orientation of the rotational axis leads to different discrete symmetries, which show up in the level sequence of rotational bands. The chiral symmetry is broken if $J$ does not lie in one of the three mirror planes of the triaxial density distribution. Chiral rotation manifest itself as a pair of nearly identical $\Delta I=1$- bands with the same parity. Tunneling between the left and right handed configurations cause an energy splitting between the chiral sister bands. A weaker form chirality are the chiral vibrations, which are slow oscillations of $J$ between the left and right handed configurations. They show up two $\Delta I=1$- bands, separated by the (small) vibrational energy. Figure 1.1 shows how such left and right handed solution may arise. The proton aligns its angular momentum $j_p$ with the short axis of the density distribution. This orientation maximizes the overlap of its orbital with the triaxial density, which corresponds to minimal energy, because the core-particle interaction is attractive. The neutron hole aligns its angular momentum $j_h$ with the long axis. This orientation minimizes the overlap of its orbital with the triaxial density, which corresponds to minimal energy, because the core-hole interaction is repulsive. The angular momentum of the core $\vec{R}$ is of collective nature. It likes to orient along the intermediate axis, which has the largest moment of inertia, because the density distribution deviates strongest from rotational symmetry with respect to this axis.
1.5 Layout of the thesis

Encouraged by these semiclassical outcome, we have extended semiclassical methods together with appropriate symmetries to study the many body nuclear phenomena like fusion-fission, single particle spectrum for a particle in a harmonic oscillator mean field with spin-orbit coupling, wobbling mode and chiral twins. The thesis is arranged in the following way.

Chapter 2

The expensive discoveries for the synthesis of superheavy elements demand a more substantial theoretical support, which may provide a more reasonable choice of fusing nuclei, collision energies and an estimate of fusion cross-section. In this chapter, a semiclassical treatment for the fusion process near the Coulomb barrier is carried out. The interaction barrier is calculated by using the asymmetric two center shell model and it fits nicely to a quartic double barrier. The generalized Bohr-Sommerfeld quantization
condition for quartic oscillator give an eigenvalue spectrum. The energy level in that spectrum, just below the continuum domain, refers to the maximum excitation energy $E_{\text{max}}^*$ carried by the resulting compound nucleus and fall fairly well within the observed limit. The shell structure of the colliding partners plays an important role in fixing the $E_{\text{max}}^*$. Further, the phenomenon of mass transfer between the reaction partners is studied by solving the time dependent Schrödinger equation in $\eta$ degree of freedom. These results confirm that a vigorous mass transfer occurs in case of doubly magic reaction partners and the resulting compound nucleus is formed with lesser energy. Calculations have been made for superheavy elements $^{256}_{102}$No and $^{258}_{104}$Rf.

Chapter 3

The synthesis of heavy and superheavy elements is mainly governed by the resulting compound-nucleus survival probability, which in turn depends on its fission characteristics. Therefore, the fission process will play a key role in fixing the “island of stability” in the periodic table. In this chapter, the systematic of spontaneous fission in actinides and heavy elements are investigated within the framework of the semiclassical quantization. The interaction barrier is calculated by using the asymmetric two-center shell model and its appropriate parameterization generates the analytical expression for the tunneling probability. The powerful semiclassical quantization technique lifts the degeneracy between the degenerate levels just after tunneling across the barrier. Our results clearly show that dissipation enhances the tunneling rate. It has also been seen that the sub-barrier fission from the excited state is a slow process. An appropriate condition for the fission isomeric state is also pointed out. The survival probability of heavy elements is a remarkable outcome of our model. Calculations have been made for nuclei $^{236}_{92}$U, $^{252}_{98}$Cf, $^{254}_{100}$Fm and $^{252}_{102}$No.

Chapter 4

In this chapter, we present semiclassically a single particle spectrum for a particle in a mean field of isotropic harmonic oscillator with $\tilde{l}_s$ coupling. This spectrum, without $\tilde{l}_s$ coupling, exactly matches with the quantum mechanical one (without nuclear constraints). In this case, periodicity conditions give only pendulating orbits coinciding with $l = 0$ axis, which fully support the observations reported by Bohr and
Mottelson (1975). The orbits with $l \geq 0$ are generated by reflecting the particle from the nuclear surface, $R_0$, instead of infinity, which is the usual nuclear constraint. The mean field strength is fixed by virial theorem. The resulting spectrum compares reasonably well with the quantum spectrum for a particle enclosed in a perfectly reflecting walls. The variation of particle number with energy help us to identify the significant quantum numbers 'n' and 'l' in this semiclassical method. Finally, the $\bar{I}s$ coupling splits each level and the splitting width of these level compares well with that of nuclear splitting. Thus the complete nuclear shell model (with magic numbers) is reproduced without any fitting parameter.

Chapter 5

Although most nuclei are spherical in their ground state, divergence from both spherical and axial symmetry may be a rather common phenomenon for nuclei throughout the entire available mass region, when excited states at higher angular momentum are considered. The loss of spherical and axial symmetry will influence many spectroscopic observables, and an interplay of such measured quantities has often been used as evidence for triaxiality in nuclei. In recent years, two different features both uniquely related to triaxiality have been at focus. One such feature is wobbling mode and other is chiral twins. In this chapter, a systematic analysis of the triaxial particle plus rotor model with single-$j$ shell configuration is carried out to explain the prominent features of observed wobbling excitations in odd $A$ nuclei. The resulting equations of motion for the angular momentum vector $\vec{I}$ and the single particle angular momentum vector $\vec{j}$ support two types of equilibrium states, (i) the axes aligned and (ii) the planar. The Jacobian matrix $J$ for the planar stationary states gives purely imaginary eigenvalues in conjugate pairs, whose eigenstates are not the eigenstates of $R_zT$. Also, our dynamical results show a substantial projection of angular momentum vectors on all the three principal axes, which implies that the resultant angular momentum lies outside the planes of three axes. Both these signatures confirm the spontaneous breakdown of time reversal ($T$) plus rotation by $180^\circ$ ($R_z$) i.e. $R_zT$ symmetry and as a result nearly two identical bands consisting of even and odd spins emerge. Calculations are made for the wobbling mode observed in $^{163}Lu$. 
Chapter 6

In this chapter, a systematic semiclassical analysis of the triaxial particle plus rotor model with single-$j$ shell configuration is carried out to understand the mechanism of observed chiral twins in odd-$A$ nuclei. The dynamical equations of motion develop fourth order differential equation in pseudo vector variable $(\delta j_3)$, whose pair of eigenvalues are complex conjugate of each other. This equation manifests the breakdown of $PT$ (parity plus time reversal) symmetry. Also, our dynamical formalism generate a substantial projection of the angular momentum on all the three principal axes, which implies that the resultant angular momentum lies outside the plane of three axes. These signatures support the spontaneous breakdown of chiral symmetry. The overall behavior of the calculated twin $\gamma$-ray spectra compares reasonably well with the experimental results of negative parity $\Delta I = 1$ chiral bands of $^{135}Nd$ nucleus.

Chapter 7

This chapter summarizes the thesis and presents an outlook for the future work. Finally, an analytically solvable model for fusion near the Coulomb barrier is given in Appendix A. This work has been presented by the author (Gupta et al. (2009)) in International symposium on nuclear and related techniques, Havana.