APPENDIX- A

Analytically solvable model for fusion near the Coulomb barrier

The recent excitement in the study of superheavy elements (SHE) stems from the predictions of new magic numbers for both protons and neutrons. In a spherical relativistic mean field calculation, Rutz et al. (1997) scanned a wide range of nuclei in superheavy region by using different sets of parameters and predicted \( Z=120 \) and \( N=172 \) as the next spherical magic shells. The recently reported isotopes of elements with \( Z=114 \) (Oganessian et al. (1999)) and \( Z=116 \) (Oganessian et al. (2001)) are still on the neutron deficient side for a neutron gap at \( N=184 \) and hence firmly support the prediction of \( Z=120 \) and \( N=172 \).

A theoretical description for the synthesis of heavy and superheavy nuclei near the coulomb barrier was proposed by Bhatia et al. (2005) and a method was given for selecting out an optimum target+projectile combination. The nuclei were considered to be formed preferably by those asymmetric target+projectile combinations that lie at the bottom of potential energy minima. An interesting aspect of this study is that all these reaction channels have already been successfully tested by GSI and Dubna groups for the synthesis of heavy and superheavy nuclei with \( 102 \leq Z \leq 110 \) in step of \( \Delta Z = 2 \). This complete theory is based on decoupling of the radial \( (R) \) and the mass asymmetry \( (\eta) \) degrees of freedom. Within this approximation, the stationary Schrodinger equations in both these coordinates were solved separately to obtain the reaction cross-section.

In the present work, our primary motive is to study the dynamics of both \( R \) and \( \eta \) motions. Our semiclassical quantization (SCQ) results in the radial degree of freedom, show discrete eigenvalue spectrum of the resulting compound nucleus and fix nicely the maximum excitation energy carried by it. These calculations also predict the upcoming spherical magic numbers both in the protons and the neutrons. Further, we have solved the time dependent Schrodinger equation analytically to obtain an estimate of mass.
transfer for the fusion process. These calculations are made for SHE $^{256}_{102}$No and $^{258}_{104}$Rf and support a large mass transfer in case of doubly magic target-projectile combination even at low excitation energy.

**Dynamics of $R$ and $\eta$ motions**

The compound nucleus process involves two coupled degrees of freedom, namely: the mass asymmetry $\eta$ and the relative separation $R$ (Bhatia et al. (2005)). By using the Pauli prescription, the Hamiltonian for such coupled system is written as

$$H(\eta, R) = -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \right) - \frac{\hbar^2}{2\sqrt{B_{RR}}} \frac{\partial}{\partial R} \left( \frac{1}{\sqrt{B_{RR}}} \frac{\partial}{\partial R} \right) + V(R, \eta). \quad (A.1)$$

Here, the coupling term in the kinetic energy, which is proportional to $\frac{\partial}{\partial \eta} \frac{\partial}{\partial R}$, is neglected since the cranking coupling mass is very small, such that $B_{RR} << \sqrt{B_{RR}B_{\eta\eta}}$ holds good (Maruhn and Greiner (1974); Zohni et al. (1975)).

The coupling potential $V(\eta, R)$ in equation (A.1) is considered to be divided into two parts (Yamaji et al. (1976)).

$$V(R, \eta) = V(R, \eta = \eta_i) + \Delta V(R, \eta) \quad (A.2)$$

Here, $\eta_i$ is the initial mass asymmetry, which involves the nucleon numbers of the target and the projectile. The asymmetric two center shell model (ATCSM) calculations of Zohni et al. (1975) reveal that the radial mass parameter $B_{RR}$ does not depend strongly on the $\eta$ motion. Therefore, this parameter may simply be replaced by the reduced mass $\mu$.

Within these simplifications, we have developed the following Schrödinger equations in two coordinates $R$ and $\eta$, respectively, as

$$\left[ -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + V(R, \eta_i) \right] \phi(R) = E_R \phi(R) \quad (A.3)$$

$$\left[ -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \right) + \Delta V(R(t), \eta) + E_R \right] \psi(\eta, t) = i\hbar \frac{\partial \psi(\eta, t)}{\partial t} \quad (A.4)$$
The relative coordinate $R$ as a function of time appears in equation (A.4) and is generated from the solution of equation (A.3) by using SCQ technique as discussed below.

**Solution of radial equation**

Following the ATCSM procedure outlined in Bhatia et al. (2005), the adiabatic interaction barriers $V(R) = V_{LDM} + \tilde{\delta}u + \tilde{\delta}p$ for the reactions $^{48}Ca + ^{208}Pb \to ^{256}No$ and $^{50}Ti + ^{208}Pb \to ^{258}Rf$ are shown in Figure A.1(a) and A.1(b), respectively. It is worthwhile to mention here that the adiabatic assumption is in doubt for heavy ion collisions. Still, our potential calculations are based on this assumption due to the following reasons:

(i) An excitation energy carried by the compound nucleus is extremely small (as discussed in the next section). So, the shell effects in potential do not vanish and hence play an important role for the synthesis of compound nucleus near the Coulomb barrier.

(ii) In our calculations, we have not included any type of extra frictional contribution, which is of course required for the fusion process. Therefore, the choice of adiabatic potential is quite reasonable in the absence of frictional terms.

In each reaction, a double-humped barrier is seen in Figure A.1. This double-hump implies that the incoming reactions partners firstly overcome the strong touching (outer) barrier before being tunneled though the fusion (inner) barrier to form a resulting compound nucleus. Thus, the variation of $R$ from touching barrier to a considerable overlap of the two reaction partners is of particular interest. The potential within this region of $R$ can be best fitted by the quartic polynomial.

$$V(R) = a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4$$  \hspace{1cm} (A.5)

and this fit is also shown in Figure A.1. Five fitted a's are listed in Table A.1. Similar fits has also been seen for the potential of other reactions (Bhatia et al. (2005)) i.e. $^{54}Cr + ^{206}Pb \to ^{260}Sg$, $^{58}Fe + ^{208}Pb \to ^{266}Hs$ and $^{64}Ni + ^{208}Pb \to ^{272}Ds$ and their respective parameters are also included in this Table.
Figure A.1 (a). The interaction barrier $V(R)$ vs. the relative separation $R$ for the system $^{256}_{102}$No is calculated by using the asymmetric target-projectile combination. (b) Same as (a) but for the system $^{258}_{104}$Rf. The dotted curve in each case refers to quartic fit.
Table A.1. Fitted potential parameters of equation (A.5)

<table>
<thead>
<tr>
<th>Fissioning nuclei</th>
<th>$a_0$ (MeV)</th>
<th>$a_1$ ($\text{MeV}^2$ fm$^{-1}$)</th>
<th>$a_2$ ($\text{MeV}^3$ fm$^{-2}$)</th>
<th>$a_3$ ($\text{MeV}^4$ fm$^{-3}$)</th>
<th>$a_4$ ($\text{MeV}^5$ fm$^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{256}_{102}$No</td>
<td>-822.5118</td>
<td>1084.9600</td>
<td>-164.4990</td>
<td>10.8511</td>
<td>-0.2622</td>
</tr>
<tr>
<td>$^{258}_{104}$Rf</td>
<td>-378.5831</td>
<td>908.5621</td>
<td>-134.8638</td>
<td>8.6576</td>
<td>-0.2022</td>
</tr>
<tr>
<td>$^{260}_{106}$Sg</td>
<td>-292.6529</td>
<td>895.1665</td>
<td>-132.6883</td>
<td>8.4967</td>
<td>-0.1980</td>
</tr>
<tr>
<td>$^{266}_{108}$Hs</td>
<td>-10.0237</td>
<td>775.0227</td>
<td>-110.8675</td>
<td>6.8007</td>
<td>-0.1503</td>
</tr>
<tr>
<td>$^{272}_{110}$Ds</td>
<td>242.2163</td>
<td>679.2477</td>
<td>-94.9117</td>
<td>5.6656</td>
<td>-0.1214</td>
</tr>
</tbody>
</table>

Using a simple translational procedure, equation (A.5) is transformed to the following form

$$V(R) = V_0 + V_2 \left( R + \frac{a_2}{a_4} \right)^2 + V_4 \left( R + \frac{a_3}{4a_4} \right)^4$$

(A.6)

with $V_2 = a_2 - \frac{3}{8} \frac{a_4^2}{a_4}$, $V_4 = a_4$ and $V_0 = a_0 \left( \frac{a_4}{4a_4} \right)^2 \left( a_2 - \frac{2}{10} \frac{a_4^2}{a_4} \right)$. Moreover, the coupling constant of the external field i.e., $a_1 = \frac{a_0}{2a_4} (a_2 - \frac{a_4^2}{4a_4})$ is clearly related to the other parameter of the potential (equation (A.5)). It is interesting enough to notice here that the effect of external field (i.e., $a_1$) vanishes for an oscillator frequency, $a_2 = \frac{1}{4} \frac{a_4^2}{a_4}$ and hence a free system is developed for a particular value of quadratic coupling.

By using the values of $a_1$'s from Table A.1, we have calculated $V_2$ and $V_4$. It is evident that in each case $V_2 > 0$ and $V_4 < 0$. These peculiar values of the potential parameters support the double barrier character of the potential (equation (A.6)). However, to the best of our knowledge, the double barrier with incident energy $< V_{\text{max}} = \frac{V_2^2}{4V_4}$ is a primary key for explaining the compound nuclear formation near the Coulomb barrier.

With quartic potential (equation (A.6)), we have solved equation (A.3) analytically by using the phase integral method (Lakshmanan et al. (1981)) and its solution is given by
\[
\phi(R) = \frac{1}{\sqrt{q(R)}} \exp \left[ \pm \int_{R}^{\infty} q(R) dR \right] \tag{A.7}
\]

The function \(q(R)\) is determined by substituting equation (A.7) into equation (A.3), i.e. from the following equation

\[
q^{-\frac{1}{2}} \frac{\partial^2}{\partial R^2} q^{-\frac{1}{2}} + \frac{2\mu}{h^2} \left[ E_R - V(R) \right] \frac{1}{q^2} = 0. \tag{A.8}
\]

Froman and Froman (1974) proved that the solution of equation (A.8) is expressed as

\[
q(R) = Q(R) \sum_{n=0}^{N} Y_{2n}(R) \tag{A.9}
\]

Here, \(Q(R) = \sqrt{\frac{2\mu}{h^2} \left[ E_R - V(R) \right]}\) and \(Y_{2n}(R)\) are obtained by using the recursion relation (Froman and Froman (1974)). Finally the Bohr-Sommerfeld quantization condition implies

\[
\frac{1}{2} \int_{\Gamma} Q(R) \sum_{n=0}^{N} Y_{2n}(R) dR + \Delta = \left( n + \frac{1}{2} \right) \pi \tag{A.10}
\]

with \(\Delta = \frac{\pi}{2} - \arg \left[ F_{12}(-\infty, 0) \right] - \arg \left[ F_{22}(-\infty, 0) \right]\), \(F_{12}\) and \(F_{22}\) being elements of the F-matrix (Froman and Froman (1965)). The integration contour \(\Gamma\) is a closed loop in the complex \(R\)-plane enclosing the two classical turning points. Our earlier experience with these semiclassical calculations (Jain et al. (1997); Malik et al. (2000)) showed that \(\Delta\) is usually negligible for energies below \(V_{\text{max}}\), so we have dropped this term in our present calculations.

The evaluation of integrals in equation (A.10) can be simplified by introducing a parameter \(\tau = r \int_{\Gamma}^{R} \frac{dR}{Q(R)}\) which is equivalent to

\[
\frac{\partial^2 R}{\partial \tau^2} = \frac{1}{2} \frac{\partial^2 Q^2}{\partial R} \tag{A.11}
\]

Equation (A.11) can be easily reduced to a well known classical equation of motion \(\frac{d^2 R}{d\tau^2} = -\frac{\partial V(R)}{\partial R}\) by defining \(t = \frac{\tau}{\pi}\). The general solution of equation (A.11) in terms of \(cn\) Jacobian elliptic function is written as

\[
R(t) = Acn \left( \sqrt{\gamma \tau + \delta}, k^2 \right) \tag{A.12}
\]
Here, \( A \) is an amplitude and \( \delta \) is a constant phase. In equation (A.12), the elliptic modulus \( k^2 = \frac{V_2 A^2}{V_2 + 2V_4} \) and the frequency \( \gamma = \frac{\sqrt{\mu(V_2 + 2V_4)}}{\hbar} \). Since \( k^2 < 0 \), therefore, we have used the transformation properties of the Jacobian elliptic functions with the negative elliptic modulus (Abramowitz and Stegun (1970)) and obtained the solution of equation (A.11) as

\[
R(t) = Acd\left(\frac{\gamma t + \delta}{\sigma'}, \sigma^2\right).
\]  
(A.13)

Now the elliptic modulus is \( \sigma^2 = -\frac{k^2}{1-k^2} \) and its complementary part is \( \sigma' = \sqrt{1-\sigma^2} \).

The periodic nature of Jacobian elliptic function implies that period of classical solution (equation (A.13)) is \( 4\sigma' K(\sigma^2) \), where the function \( K \) is the complete elliptic integral of its first kind. The remaining expressions for the integrals in equation (A.10) are same as given in Lakshmanan et al. (1981).

The resulting SCQ condition is carried out numerically for quantization number \( n = 0,1,2, \) etc. In our present calculations, we have kept only five terms \( (N = 5) \) in the series sum equation (A.10) and numerical accuracy has been tested upto five significant figures after decimal place. This SCQ condition fixes the value of the amplitude \( A \) in equation (A.13). Thus, the sequence of energy eigenvalues are given by

\[
E_n = V_2 A^n + V_4 A^4
\]  
(A.14)

Here, we have normalized the ground state of our system at \( V_0 \).

**Solution of \( \eta \) in equation (A.4)**

The nucleon transfer probability is obtained by solving the time dependent Schrodinger equation (equation (A.4)). Here, we would like to understand this phenomenon of mass transfer by using a simplified model. The model calculations are carried out with constant mass \( B_{\eta\eta} \) and as oscillator potential \( \Delta V(R, \eta) = V_{\eta\eta} + k\eta^2 \).

We solve equation (A.4) analytically, under the initial condition of very narrow Gaussian distribution, given by
Here, $B_{av}$ is the cranking mass $B_{av}$ averaged over the $\eta$ coordinate, $\Gamma_i$ gives the initial width equal to two nucleon transfer and $\eta_i$ is the initial mass asymmetry. Also, the wave function $\psi(\eta,t)$ is expanded in terms of the stationary harmonic oscillator wave functions $\phi_n$,

$$\psi(\eta,t) = \sum a_n \phi_n(\eta) \text{exp} \left[ -i(n + \frac{1}{2}) \omega t \right]$$  \hspace{1cm} (A.16)$$

with $n$ varies from 0 to $\infty$; $a_n = \int \phi_n^*(\eta) \psi(\eta,0) d\eta$ and $\omega = \sqrt{\frac{k}{\hbar}}$.

It may be noted here that the wave function is normalized in the interval $-\infty$ to $\infty$, though the range of definition of our physical coordinate is $-1 \leq \eta \leq 1$. This is possible since in our model the initial width $\Gamma_j$ is very small, such that

$$\int_{-\infty}^{\infty} \text{exp} \left( -\frac{x^2}{\Gamma_j} \right) dx = \sqrt{\Gamma_j} \int_{-\infty}^{\infty} \text{exp} \left( -\frac{y^2}{\Gamma_j} \right) dy = \int_{-\infty}^{\infty} \text{exp} \left( -\frac{x^2}{\Gamma_j} \right) dx_j$$  \hspace{1cm} (A.17)$$

where $x = \eta - \eta_i$.

Hence we obtain

$$|\psi(\eta,t)|^2 = \frac{1}{\sqrt{B_{av} \pi \Gamma(t)}} \exp \left[ -\frac{(\eta - \eta_i + \eta_i \cos \omega t)^2}{\Gamma(t)} \right]$$  \hspace{1cm} (A.18)$$

which is Gaussian function with the half width, in terms of $\xi = \sqrt{\frac{\Gamma \omega \hbar}{k}}$

$$\Gamma(t) = \Gamma \sqrt{1 + \xi^4 - (1 - \xi^4) \cos 2\omega t} \frac{2\omega}{\sqrt{2\xi^2}}$$  \hspace{1cm} (A.19)$$

Equation (A.19) shows that the half-width $\Gamma(t)$ oscillates periodically with frequency $2\omega$ between the maximum ($= \frac{\Gamma}{\xi}$) and minimum ($-\Gamma_j$), respectively at times $t = \frac{n(\pi + \frac{1}{2})}{2\omega}$ and $\frac{m\pi}{\omega}$; $n = 0,1,2,3$ etc.
Results and discussions

It is apparent from tabulated values (Table A.1) that the coupling constant of external field \( (a_i) \) decreases as the charge \( (Z) \) of SHE increases and its variations as a function of \( Z \) is shown in Figure A.2. This smooth decrease is best fitted by a straight line. This straight line fit when extrapolated to zero ordinate, meets the abscissa at \( Z = 122 \). The absence of this coupling constant leads to free system for a particular value of quadratic coupling. Such free systems prefer to attain spherical shape. Similarly, the variation of \( a_i \) as a function of mass number \( A \) shows a linear behavior and this straight line meets the abscissa at \( A=300 \) corresponding to zero ordinate. Thus, our results indicate \( ^{300}_{122}X \) as a next doubly magic nucleus and is fairly close to the predictions of Rutz et al. (1997). Figure A.3 shows the plot of semiclassical eigenvalues vs. the quantum number \( n \) for the SHE \(^{256}_{102}No\) and \(^{258}_{104}Rf\). The levels designated by quantum number \( n \) below the plateau region refer to the stationary states of the compound nucleus. It is quite evident that the compound nucleus may exist in any of these states with a definite probability. In addition, all these states of a nucleus, except the ground state, have a definite lifetime. The lifetime of the states with energy in the continuum domain is merely shortened due to the possibility of ejection of particles. Thus the maximum excitation energy \( E_{\text{max}} \) of the resulting compound nucleus is obtained from an eigenvalue just below its continuum. The \( E_{\text{max}} \) value for the compound systems \(^{256}_{102}No\) and \(^{258}_{104}Rf\) obtained from these plots and are, respectively, equal to 15.962 MeV and 25.071 MeV. These results when added, respectively, to the Q-value 153.80 MeV and 169.69 MeV of the reactions \(^{48}Ca + ^{208}Pb \rightarrow ^{256}_{102}No\) and \(^{50}Ti + ^{208}Pb \rightarrow ^{258}_{104}Rf\) fix the center of mass energy \( E_{\text{cm}} \) of the reaction partners. These are remarkably in agreement with the experimental values.

On comparing the two plots, it is noticed that the maximum excitation energy carried by the compound nucleus \(^{256}_{102}No\) is \( \sim 9 \) MeV lower than that of \(^{258}_{104}Rf\). This large difference arises due to shell effects of the doubly magic reaction partners in \(^{48}Ca + ^{208}Pb \rightarrow ^{256}_{102}No\). These shell effects enter through our ATCSM calculations.
Figure A.2. The variation of coupling constant $a_1$ vs. the charge number $Z$ of SHE. The dotted curve show its linear line behaviour.
Figure A.3. The variation of radial energy $E_R$ (equation (A.14)) vs. the quantum no. $n$ for SHE $^{256}_{102}No$, $^{258}_{104}Rf$ obtained after SCQ.
The stationary nature of the states, below the continuum domain, in Figure A.3 is also supported from our reaction time calculations. Here, the reaction time is obtained from the periodic nature of the Jacobian elliptic function in equation (A.13). Figures A.4(a) and A.4(b) show, respectively, the reaction time $t$ (in $\text{fm}/c$) vs. the level energy $E_R$ (in MeV) for nuclei $^{256}_{102}\text{No}$ and $^{258}_{104}\text{Rf}$. It is worth noticing here that the reaction time remains nearly constant with increase in the level energy $E_R$ and drop suddenly near the continuum domain. Further, a comparison between the two plots reveals that the reaction time is quite large for the reaction $^{48}\text{Ca} + ^{208}\text{Pb} \rightarrow ^{256}_{102}\text{No}$ ($T = 1214.99 \text{ fm}/c$) than that of $^{50}\text{Ti} + ^{208}\text{Pb} \rightarrow ^{258}_{104}\text{Rf}$ ($T = 920.83 \text{ fm}$). A large reaction time in the former case allows the reaction partners to fuse completely and hence form the compound nucleus. It is worthwhile to mention here that the rotational bands are seen in the isotopes of $^{256}_{102}\text{No}$ (Julin (2001)), whereas no such bands have yet been seen in Z=104 region.

Further, this difference in the reaction time between $^{256}_{102}\text{No}$ and $^{258}_{104}\text{Rf}$ may have relevance with their shapes as suggested in Cwiok et al. (2005). Thus the shell effects of the reaction partners play an important role in the synthesis of SHE.

The next step of our formalism is to calculate the nucleon transfer probability by using equation (A.18). The potential $V(\eta)$ at fixed $R$ (i.e., near the touching configuration) and fixed $\eta_Z$ is obtained in the Strutinsky manner from the ATCSM using the adiabatic approximation for the shape parameters. Figures A.5(a) and A.5(b) show, respectively, the calculated potential $V(\eta)$ vs. the mass asymmetry $\eta$ for the composite system $^{256}_{102}\text{No}$ and $^{258}_{104}\text{Rf}$. These potentials are approximated nicely by the harmonic oscillator potential $V(\eta) = V_{\text{eq}} + k\eta^2$ as shown in this plot and their $V_{\text{eq}}$ and $k$ values in MeV are $V_{\text{eq}}(^{256}_{102}\text{No}) = 1800.13$, $k(^{256}_{102}\text{No}) = 11.1132$, $V_{\text{eq}}(^{258}_{104}\text{Rf}) = 1851.24$, $k(^{258}_{104}\text{Rf}) = 11.8733$. The average mass parameter values are taken as $B_{av}(^{256}_{102}\text{No}) = 3527.69 \text{ fm}/m$, $B_{av}(^{258}_{104}\text{Rf}) = 5015.64 \text{ fm}/m$, with $m$ the nucleon mass.

Using these parameters, we have calculated the nucleon transfer probabilities for both $^{256}_{102}\text{No}$ and $^{258}_{104}\text{Rf}$ at their respective reaction times and are shown in Figures
Figure A.4. (a) The plot of the reaction time vs. the radial energy $E_R$ for SHE $^{256}_{102}$No.

(b) Same as (a) for SHE $^{258}_{104}$Rf
Figure A.5. (a) The plot of $V(\eta)$ vs. the mass asymmetry $\eta$ for the system $^{256}_{102}No$. (b) Same as (a) but for the system $^{258}_{104}Rf$. The dotted curve in each case refers to parabolic fit.
Figure A.6. (a) The plot of nucleon transfer probability vs the mass asymmetry $\eta$ on a semilog scale for system $^{256}_{102}No$. (b). Same as (a) but for the system $^{258}_{104}Rf$. 
A.6(a) and A.6(b), respectively. Here, we have not normalized these probabilities. Following features are clearly emerged from the comparison between Figures A.6(a) and A.6(b):

(i) In both the cases, Gaussian spread prevails with an increase of time.

(ii) The nucleon transfer is quite fast in the reaction for the synthesis of $^{256}_{102}$No than that of $^{258}_{104}$Rf.

(ii) The reaction $^{48}_{20}Ca + ^{208}_{82}Pb \rightarrow ^{256}_{102}No$ clearly show transmission phenomenon and hence favours the complete fusion. Whereas the reaction $^{50}_{22}Ti + ^{208}_{82}Pb \rightarrow ^{258}_{104}Rf$ shows both the reflection and transmission phenomena. The reflection process among the nucleons slows down the process of compound nucleus formation.

These prominent features point out that large mass transfer occur only in case of a doubly magic+projectile combination even at low excitation energy.

**Conclusions**

We have shown that an analytical dynamical approach explains various prominent features of the fusion process near the coulomb barrier. Our SCQ results in radial coordinate generate a discrete eigenvalue spectrum for the compound nucleus and fix the maximum excitation energy carried by it. These $E_{\text{max}}$ values are in good agreement with the observed values. The nucleus $Z=122, A=300$ is shown to be next doubly magic. Our Calculations support large mass transfer between the doubly magic reaction partners even at low excitation energy.