CHAPTER - 7

Summary and Plan for Future Work

7.1 Summary

The field of nuclear physics can gain considerably from the developments taking place in the vast field of classical dynamics for non-linear systems. It is now realized that non-linearity in nature is the order of the day rather than an exception. Nuclear physics provides an ideal testing ground for the new ideas and results and remains largely untouched by the “new physics”.

In this thesis, we have discussed the possible impacts and a few recent applications of semiclassical techniques to understand “Nuclear Structure and Reaction Dynamics” together with appropriate symmetries. These techniques based on the periodic orbit theory (POT) have proved to be powerful tools for explaining the recently observed nuclear phenomena, which lie near the border of order and chaos.

In particular, we have carried out a semiclassical treatment for the fusion process near the Coulomb barrier. Within our formalism, we are able to predict a more reasonable choice of fusing nuclei, collision energies and an estimate of the fusion cross-section. The interaction barrier is calculated by using the asymmetric two center shell model and it fits nicely to a quartic double barrier. The generalized Bohr-Sommerfeld quantization condition for quartic oscillator gives an eigenvalue spectrum. The energy level in that spectrum, just below the continuum domain, refers to the maximum excitation energy $E_{\text{max}}^*$ carried by the resulting compound nucleus and fall fairly well within the observed limit. The shell structure of the colliding partners plays an important role in fixing the $E_{\text{max}}^*$. Further, the phenomenon of mass transfer between the reaction partners is studied by solving the time dependent Schrodinger equation in mass asymmetry ($\eta$) degree of freedom. These results confirm that a vigorous mass transfer occurs in case of doubly magic reaction partners and the resulting compound nucleus is formed with lesser energy. Calculations have been made for superheavy elements $^{256}_{102}No$ and $^{258}_{104}Rf$. 
The systematic of spontaneous fission in actinides and heavy elements are investigated within the framework of semiclassical quantization. The interaction barrier is calculated by using the asymmetric two-center shell model and its appropriate parameterization generates the analytical expression for the tunneling probability. The powerful semiclassical quantization technique lifts the degeneracy between the degenerate levels just after tunneling across the barrier. Our results clearly show that dissipation enhances the tunneling rate. It has also been seen that the sub-barrier fission from the excited state is a slow process. An appropriate condition for the fission isomeric state is also pointed out. The survival probability of heavy elements is a remarkable outcome of our model. Calculations have been made for nuclei $^{236,252,254}U$, $^{252}Fm$, and $^{252}No$.

We have presented the single particle spectrum for a particle in a mean field of isotropic harmonic oscillator with $\tilde{l}s$ coupling based on our semiclassical approach. It has been seen that this spectrum, without $\tilde{l}s$ coupling, exactly matches with the quantum mechanical one (without nuclear constraints). In this case, periodicity conditions give only pendulating orbits coinciding with $l=0$ axis, which fully support the observations reported by Bohr and Mottelson (1975). The orbits with $l \geq 0$ are generated by reflecting the particle from the nuclear surface, $R_0$, instead of infinity, which is the usual nuclear constraint. The mean field strength is fixed by the virial theorem. The resulting spectrum compares reasonably well with the quantum spectrum for a particle enclosed in a perfectly reflecting walls. The variation of particle number with energy help us to identify the significant quantum numbers 'n' and 'l' in this semiclassical method. Finally, the $\tilde{l}s$ coupling splits each level and the splitting width of these levels compares well with that of nuclear splitting. Thus the complete nuclear shell model (with magic numbers) is reproduced without any fitting parameter.

The success of semiclassical approach to the axially symmetric particle-rotor models (Jain et al. (1997); Dudeja et al. (1997); Malik et al. (2000)) has inspired us to generalize it to the triaxial particle plus rotor model (TPRM). We have extended this semiclassical treatment to the triaxial particle plus rotor model (TPRM) with single- $j$ configuration. The resulting equations of motion for the angular momentum vector $\tilde{l}$
and the single particle angular momentum vector $\vec{j}$ support two types of equilibrium states, (i) the axes aligned and (ii) the planar. Our dynamical results clearly show a breakdown of $R_T$ symmetry, which generates two spectra. The features of these spectra resemble very closely with tilted superdeformed (TSD1 and TSD2) bands observed in wobbling mode of odd-A $^{163}$Lu nucleus and chiral twins observed in odd-A $^{135}$Nd nucleus, respectively.

7.2 Plan for future work

In view of the work that we have already carried out, a number of new avenues have opened up for further study and development of this field. It is our hope and belief that we shall be able to accomplish most of these aims in near future and will be able to provide a new way of doing nuclear structure. We briefly list our ideas and aims as below:

1. Exploration of detailed shell structure of spherical and deformed cavities of various shapes and the relationship of periodic orbits with the onset of deformation in nuclei. Since the total binding energy of a nucleus with $N$ neutrons and $Z$ protons is given by

$$E_{\text{tot}}(N, Z; \text{def}) = E_{\text{LDM}}(N, Z; \text{def}) + \delta E_n(N, Z; \text{def}) + \delta E_p(N, Z; \text{def})$$

Here, $E_{\text{LDM}}$ is the liquid drop model energy, $\delta E_n$ and $\delta E_p$ are the shell correction energies of the neutrons and protons, respectively. So far, only one kind of nucleon has been treated in periodic orbit theory (POT) calculations. In order to obtain more correct information, neutrons and protons should be treated explicitly and independently. Such an approach will also be extremely useful in extending our work to nuclei far from stability.

2. To study in detail the role of non-coplanar 3-dimensional periodic orbits in superdeformation. It is our hope that such 3-dimensional orbits will throw light on what we term as the unusual scaling behavior of rotational bands. Such a connection will also help in resolving the puzzle of identical bands phenomena of superdeformed as well as normal deformed nuclei.
3. Maslov indices play a crucial role in the periodic orbit theory. Considerable attention has been paid to it in the literature. Yet some confusion always arise. It will be our endeavour to provide a safe technique of obtaining the Maslov indices.

4. A spherical billiard is a system with continuous symmetries and has found many applications in atoms, molecules and metal clusters. We hope to relate the nuclear structure of spherical or, near spherical nuclei within the framework of the periodic orbit theory of spherical cavity.

5. A knowledge of level density from periodic orbit theory can immediately be used to provide a length spectrum by a Fourier transform. Since knowledge of a large number of levels is very crucial, a windowed Fourier transform has recently been used in molecules (Gaspard et al. (1995)). We propose to extend this technique to nuclear spectra thereby obtaining information on nuclear shapes and sizes.

6. Semiclassical methods need to find extensions for weakly bound systems and resonances.