Complementary Tree Domination Number of Circular-Arc Graphs

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Abstract—In this present paper, we concentrated on the theory of complementary tree domination in graphs and focused on resolving the complementary tree domination number of circular-arc graphs. Some categorized circular-arc graphs are chosen in this course of study.

Index Terms—Circular-Arc Graph, Dominating Set, Complementary Tree Dominating Set, Complementary Tree Domination Number.

1. INTRODUCTION

Complementary tree dominating set problem is the problem of finding whether the given graph has a complementary tree dominating set of a specified size. A subset S of the vertex set V of the graph G = (V, E) is a dominating set of the graph G if every vertex in the vertex set V is either a member of S or is adjacent to some vertex of S. The domination number of the graph G is denoted by γ(G) and is the minimum cardinality of a dominating set of G. Dominating sets play an imperative role in algorithmics and in combinatorics. Allan and Laskar [1] derived many results pertaining to dominating sets. Hedetniemi and Laskar in [2] listed over 300 papers related to domination in graphs. In complexity theory, dominating set was one of the first problems recognized as NP-complete. In [3] - [4] Haynes et al. discussed dominating sets in detail. A dominating set S ⊆ V of a graph G with vertex set V(G) and edge set E(G) is a complementary tree dominating set if the induced sub graph < V - S > is a tree. Complementary tree domination number is the minimum cardinality of a complementary tree dominating set of G. It is denoted by Y_{ctd}(G). The notion of complementary tree dominating set is due to S. Muttmami et al. [5]. Some results pertaining to the bounds of Complementary Tree domination number is obtained by them. Circular-arc graphs are a new class of intersection graphs, defined for a set of arcs on a circle. A graph is a circular - arc graph, if it is the intersection graph of a finite set of arcs on a circle. That is, there exists one arc for each vertex of G and two vertices in G are adjacent in G, if and only if the corresponding arcs intersect. A vertex is said to dominate another vertex if there is an edge between the two vertices. Let A = {A₁, A₂, A₃, ..., Aₙ} be a circular – arc family on a circle, where all the arcs together cover the entire circle. An arc Aᵢ that begins at endpoint p, and ends at endpoint q, considered in the clockwise direction is denoted by (pᵢ, qᵢ). Two arcs Aᵢ and Aⱼ are said to intersect each other if they have non-empty intersection. A representation of a graph with arcs helps in the solving of combinatorial problems on the graph. Neighborhood of an arc Aᵢ is defined as the set of all arcs belonging to A that intersect the arc Aᵢ.

Graphs considered in this paper are all undirected, connected and simple graphs. Throughout this paper, for the graph G = (V, E) and for S ⊆ V, the sub graph of G induced by the vertices in S is denoted by < S >. A vertex of degree one is called a support.

In section 2, preliminary information regarding the bounds of a complementary tree domination number and the standpoint information regarding the relation between the domination number and complementary tree domination number are outlined.

A. Our Results:

In our recent paper [6], we put forth the findings related to complementary tree domination number of interval graphs. The exact value of complementary tree domination number and minimal complementary tree domination sets of some particular classes of interval graphs are obtained. In this paper, we extended our study on complementary tree domination number and we put forward some results regarding the complementary tree domination number of circular-arc graphs.

II. SOME RESULTS

The following results are obtained by S. Muttamai et al. [5] and these results characterize ctd-sets.

Result 2.1: Bounds of complementary tree domination number Let G be a connected graph of order k ≥ 2. Then Y_{ctd}(G) ≤ k-1.

Result 2.2: Relation between domination number and complementary tree domination number Let G be a connected interval graph of order n ≥ 2. Then γ(G) ≤ Y_{ctd}(G). For any graph G, every complementary tree dominating set is a dominating set. But every dominating set need not be a complementary tree dominating set. Hence the result follows.

Result 2.3: Every pendant vertex is a member of all ctd-sets.

III. COMPLEMENTARY TREE DOMINATION NUMBER OF SOME CIRCULAR-ARC GRAPHS

Theorem 3.1: Let A = {A₁, A₂, A₃ ... Aₙ}, n ≥ 3 be the circular-arc family corresponding to a circular-arc graph G. For any three consecutive arcs Aᵢ, Aⱼ and Aₖ if Aᵢ does not dominate any arc other than Aⱼ and Aₖ and furthermore, domination of Aᵢ by Aⱼ is the only other domination that occurs in the family, then

Y_{ctd}(G) = n-2
Proof: Let G be the circular-arc graph, whose circular-arc family \( A = \{ A_1, A_2, A_3, \ldots, A_n \} \) satisfies the conditions mentioned in the theorem. By the hypothesis, the arc \( A_1 \) dominates the arcs \( A_2 \) and \( A_3 \); the arc \( A_2 \) dominates the arcs \( A_1 \) and \( A_3 \); the arc \( A_3 \) dominates the arcs \( A_1 \) and \( A_2 \); \(
\) \ldots \ldots \ldots \ldots \ldots \\)
the arc \( A_{n-2} \) dominates the arcs \( A_1 \) and \( A_{n-1} \); the arc \( A_{n-1} \) dominates the arcs \( A_2 \) and \( A_n \); and the arc \( A_n \) dominates the arcs \( A_1 \) and \( A_{n-1} \).

Let \( v_1, v_2, v_3, \ldots, v_n \) be the vertices corresponding to the arcs \( A_1, A_2, A_3, \ldots, A_n \) respectively.

Let \( S_i = \{ v_i, v_{i+1}, v_{i+2}, \ldots, v_n \} \) for \( i = 1, 2, 3, \ldots, n-1 \). Also, \( V - S_i = \{ v_1, v_{i-1}, \ldots, v_{i} \} \).

Then, \( V - S_i = \{ v_i, v_{i+1}, \ldots, v_n \} \) for \( i = 1, 2, 3, \ldots, n-1 \). Since the set of

arcs \( A_i, A_{i+1}, A_{i+2}, \ldots, A_n \) and the set of arcs \( A_{i-1}, A_{i-2}, A_{i-3}, \ldots, A_n \) are two sets of three consecutive arcs, in the set \( V - S_i \), the vertex \( v_i \) is adjacent to the vertex \( v_{i+1} \) and the vertex \( v_{i+1} \) is adjacent to the vertex \( v_{i+2} \) for \( i = 1, 2, \ldots, n-1 \). So the set \( S_i \) is a dominating set and also the induced subgraph \( \langle V - S_i \rangle \) consists of only two vertices \( v_i \) and \( v_{i+1} \) with an edge between them and it is a tree. Therefore, the set \( S_i \) is a ctd-set for each \( i = 1, 2, 3, \ldots, n-1 \). Also, \( V - S_i = \{ v_i, v_{i+1}, \ldots, v_n \} \). By hypothesis, \( v_i \) is adjacent to \( v_{i+1} \) and \( v_{i+1} \) is adjacent to \( v_{i+2} \), and it follows that the induced subgraph \( \langle V - S_i \rangle \) is a tree. Therefore, the set \( S_i \) is a ctd-set. Hence, the set \( S_i \) is a ctd-set for each \( i = 1, 2, 3, \ldots, n-1 \). Cardinality of \( S_i \) is \( n-2 \) for each \( i = 1, 2, \ldots, n \). Let \( S' = \{ S_i \mid i = 1, 2, \ldots, n \} \). It can be observed that, any set of cardinality \( n-2 \) other than in \( S' \) is not a ctd-set. Therefore, any ctd-set with cardinality less than \( n-2 \) is a subset of \( S_i \), where \( i \) is not a ctd-set. It follows that, \( \gamma_{\text{td}}(G) = n-2 \).

Now we shall show that any subset of \( S_i \) is not a ctd-set for any \( i \), where \( i = 1, 2, \ldots, n \).

First, let \( S_i \) = \( S_i \cup v_i \) for \( i = 1, 2, \ldots, n-1 \). Let \( j = i+1 \) and \( 1 \leq j \leq n-1 \).

Here three cases will arise.

Case (i): The vertex \( v_i \) may be a predecessor of \( v_{i+1} \). Then \( v_i \) is not adjacent to any vertex of \( S_i \), implies \( S_i \) is not a dominating set. It follows that \( S_i \) is not a ctd-set.

Case (ii): The vertex \( v_i \) may be successor of the vertex \( v_{i-1} \). Then \( v_i \) is not adjacent to any vertex of \( S_i \), implies \( S_i \) is not a dominating set. It follows that \( S_i \) is not a ctd-set.

Case (iii): The vertex \( v_i \) may be neither predecessor of \( v_{i+1} \), nor successor of the vertex \( v_{i-1} \). Then \( S_i \) is a dominating set. But the induced subgraph \( \langle V - S_i \rangle \) is not a connected graph in this case. The graph \( \langle V - S_i \rangle \) is not a tree. Implies the set \( S_i \) is not a ctd-set. It follows that, \( S_i \) is not a ctd-set in all the possible three cases. Secondly, let \( S_i = S_i \cup v_i \), where \( 2 \leq j \leq n-1 \).

Again three cases will arise.

Case (i): The vertex \( v_i \) may be \( v_{i+1} \). Then \( v_i \) is not adjacent to any vertex of \( S_i \), implies \( S_i \) is not a dominating set. It follows that \( S_i \) is not a ctd-set.

Case (ii): The vertex \( v_i \) may be \( v_{i-1} \). Then \( v_i \) is not adjacent to any vertex of \( S_i \), implies \( S_i \) is not a dominating set. It follows that \( S_i \) is not a ctd-set.

Case (iii): The vertex \( v_i \) may be different from \( v_{i+1} \) and \( v_{i-1} \). Then \( S_i \) is a dominating set. But the induced subgraph \( \langle V - S_i \rangle \) is not a connected graph in this case. The graph \( \langle V - S_i \rangle \) is not a tree. Implies the set \( S_i \) is not a ctd-set. It follows that, \( S_i \) is not a ctd-set in all the possible three cases. Hence, any subset of \( S_i \), for \( i = 1, 2, \ldots, n \) is not a ctd-set.

From (1) and (2), it follows that \( \gamma_{\text{td}}(G) = n-2 \).

Illustration: Let the circular-arc family 
\( A = \{ A_1, A_2, A_3, \ldots, A_n \} \), \( n \geq 3 \) corresponding to a circular-arc graph \( G \) be as in Fig. 1.

![Fig.1. Circular-arc family](image-url)
Proof: Let $A^* = \{A_1, A_2, A_3, \ldots, A_n\}$, $n \geq 2$ be the circular-arc family corresponding to a circular-arc graph $G$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices corresponding to the arcs $A_1, A_2, A_3, \ldots, A_n$, respectively.

First, let the circular-arc family $A$ analogous to a circular-arc graph $G$ satisfy the conditions mentioned in the theorem and let all the arcs in $A - \{A_1, A_2\}$ dominate $A_i$. Then $G$ is a $k_{t, n}$ graph. Then the partite sets of the graph are $\{v_i\}$ and $\{v_i, v_{i-1}, v_{i+1}, \ldots, v_j\}$ for some $i$, where $1 \leq i < j \leq n$.

Therefore the vertices $v_1, v_2, v_3, \ldots, v_{n-2}$ are pendant vertices. Every ctd-set consists of all pendant vertices. Therefore the vertices $v_1, v_2, v_3, \ldots, v_j, v_{j+1}, \ldots, v_n$ are members of every ctd-set. As a result, 

$$\gamma_{ctd}(G) \geq n - 1 \text{ ...........................................(i)}$$

Similarly, it can be proved that result (i) holds even if all the arcs in $A - \{A_1, A_2\}$ dominate $A_i$. But for any connected graph $G$ with $n \geq 2$, $\gamma_{ctd}(G) \leq n - 1 \text{ .........................................(ii)}$

From (i) and (ii) 

$$\gamma_{ctd}(G) = n - 1 \text{ ...........................................(iii)}$$

Conversely, let $\gamma_{ctd}(G) = n - 1$ 

Let $D = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$ for some $1 \leq i \leq n - 1$. Vertices $v_1, v_2, v_3, \ldots, v_j, v_{j+1}, \ldots, v_n$ is a ctd-set. As a result, 

$$\gamma_{ctd}(G) = n - 1 \text{ ...........................................(iii)}$$

Similarly, it can be proved that result (i) holds even if all the arcs in $A - \{A_1, A_2\}$ dominate $A_i$. But for any connected graph $G$ with $n \geq 2$, $\gamma_{ctd}(G) \leq n - 1 \text{ .........................................(ii)}$

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$$\gamma_{ctd}(G) = n - 1 \text{ ...........................................(iii)}$$
consists of two vertices \( v \) and \( v' \) with an edge between them. Implies sub graph \( <V - S> \) is a tree. It follows that \( S \) is a ctd-set, wherein the cardinality of \( S \) is \( n - 2 \). Hence

\[
\gamma_{cd}(G) \leq n - 2 \quad \text{(2)}
\]

From (1) and (2), it is clear that \( \gamma_{cd}(G) = n - 2 \) with minimal ctd-set as \( A - \{ A_i, A_j \} \).

Illustration: Let the circular-arc family 
\( A = \{ A_1, A_2, A_3, \ldots, A_n \}, n \geq 4 \) corresponding to a circular-arc graph \( G \) be as in Fig. 3.

![Circular-arc family](image_url)

The circular-arc family satisfies all the conditions mentioned in the theorem for \( n = 7, i = 1 \) and \( j = 4 \). The complementary tree domination number of the graph, \( \gamma_{cd}(G) = 5 \). The minimal ctd-set of the graph is \( \{ A_2, A_3, A_5, A_6, A_7 \} \).

REFERENCES


Total Bondage Number of an Interval Graph

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Abstract—Dominating sets play predominant role in the theory of graphs. Among the various applications of the theory of domination the most often discussed is a communication network. This network consists of communication links between a fixed set of sites. By constructing a family of minimum dominating sets, we compute the total bondage number. Suppose, communication network fails due to link failure. Then the problem is to find a fewest number of communication links such that the communication with all sites is possible. This leads to the introducing of the concept of total bondage number of graph. In this paper we consider the total bondage number \( b(G) \) for an interval family corresponding to an interval graph \( G \), which is defined as the minimum number of edges whose removal results in a new graph with larger total domination number.

Keywords: Interval graph, Dominating set, Domination number, Total dominating set, Total bondage number.

1. INTRODUCTION

It is well known that the topological structure of an interconnection network can be modeled by a connected graph whose vertices represent sites of the network and whose edges represent physical communication links. A minimum dominating set in the graph corresponding to an interval family \( J \), where each \( I_i \), is an interval on the real line \( I_i = [a_i, b_i] \) for \( i = 1, 2, \ldots, n \). Here \( a_i \) is called the left end point and \( b_i \) is the right end point of \( I_i \), without loss of generality we may assume that all end points of the intervals \( I \) are distinct numbers between 1 and \( 2n \). Two intervals \( i \) and \( j \) are said to intersect each other if they have non-empty intersection. A subset \( D \) of \( V \) is said to be a dominating set of \( G \) if every vertex in \( V \setminus D \) is adjacent to a vertex in \( D \). The domination number \( \gamma(G) \) of \( G \) is the minimum cardinality of dominating set \( \gamma(G) \). Also a minimum dominating set in the graph corresponds to a smallest set of sites selected in the network for some particular uses, such as placing transmitters. Such a set may not work when some communication links happen fault.

The bondage number \( b(G) \) of a non-empty graph \( G \) is the minimum cardinality among all sets of edges \( E_i \), for which \( \gamma(G - E_i) > \gamma(G) \). Thus, the bondage number of \( G \) is the smallest number of edges whose removal will render every minimum dominating set in \( G \) a non-dominating set in the resultant spanning sub graph \( G \). Since the domination number of every spanning sub graph of a non-empty graph \( G \) is at least as great as \( \gamma(G) \), the bondage number of a non-empty graph is well defined [3,4,6,7].

A subset \( S \) of \( V \) is called a total dominating set if every vertex in \( V \) is adjacent to some vertex in \( S \). The total domination number \( \gamma_t(G) \) of \( G \) is the minimum cardinality taken over all total dominating sets of \( G \) [1]. The total bondage number \( b_t(G) \) of a non-empty graph \( G \) is the minimum cardinality among all sets of edges \( E_i \), for which \( \gamma_t(G - E_i) > \gamma_t(G) \).

2. MAIN THEOREMS

THEOREM 2.1: Let \( G \) be an interval graph corresponding to the interval family \( I = \{i_1, i_2, \ldots, i_n\} \). Let \( i_1, i_2 \in I \) and suppose \( i_2 \) is contained in \( i_1 \) and there is no other interval that intersects \( i_2 \) other than \( i_1 \), then \( b_i(G) = 1 \).

PROOF: Let \( G \) be an interval graph corresponding to the interval family \( I = \{i_1, i_2, \ldots, i_n\} \). Let \( i_1, i_2 \in I \) and suppose \( i_1 \) is contained in \( i_2 \) and there is no other interval that intersects \( i_2 \). Then clearly \( i_2 \in TDS \), the total dominating set of the interval graph \( G \). Since there is no other interval in \( I \) other than \( i_1 \) that totally dominates \( i_2 \).

Consider the edge \((i_2, i_1)\) in the interval graph \( G \). If we remove this edge from \( G \), then \( i_2 \) becomes an isolated vertex in \( G - e \), as there is no other vertex that intersects \( i_2 \) other than \( i_1 \), as given in the hypothesis. Hence \( TDS = TDS \cup \{i_2\} \) becomes the total dominating set of \( G - e \) and since TDS is the minimum total dominating set of \( G \), it follows that \( TDS \) is also a minimum total dominating set of \( G - e \).
Therefore \[ |TDS| = \gamma(G-e) = |TDS| + 1 > |TDS| \] Thus \( b(G) = 1 \).

This statement is proved in the following illustration clearly.

2.2: ILLUSTRATION
Consider the following interval family \( I \).

\[
\text{Fig. 1: Interval family } I
\]

The corresponding interval graph \( G \) is as follows,

\[
\text{Fig. 2: Interval graph } G
\]

(Dominating set \( D = \{4, 8\} \))
Total dominating set \( TDS = \{4, 5, 8\} \) and \( \gamma_d(G) = 3 \).
Remove the edge \( e = (3, 4) \) from \( G \), then the corresponding interval graph \( G-e \) is as follows,

\[
\text{Fig. 3: Interval graph } G-e
\]

Total dominating set of \( G-e \) = \( TDS = \{3, 4, 5, 8\} \) and \( \gamma_d(G-e) = 4 \).
Therefore \( \gamma_d(G-e) > \gamma_d(G) \) and hence \( b(G-e) = 1 \).
Hence the proof of the theorem.

THEOREM 2.3: Let \( G \) be an interval graph corresponding to an interval family \( I = \{i_1, i_2, \ldots, i_n\} \). Let the total dominating set \( TDS \) of \( G \) consists of four vertices only, say \( p, q, r \) and \( s \). Suppose \( p \) and \( q \) vertices totally dominates the vertex set \( S_1 = \{1, 2, \ldots, i_i\} \) and \( r \) and \( s \) vertices totally dominates the vertex set \( S_2 = \{i+1, \ldots, n\} \). Suppose there is no vertex in \( S_1 \) other than the vertices \( p \) and \( q \) that totally dominates \( S_1 \) and no vertex in \( S_2 \) other than the vertices \( r \) and \( s \) that dominates \( S_2 \). Then \( b(G) = 1 \).

PROOF: Let \( G \) be an interval graph corresponding to an interval family \( I = \{i_1, i_2, \ldots, i_n\} \) and the total dominating set \( TDS \) of \( G \) consists of four vertices only, say \( p, q, r \) and \( s \).
Suppose \( p \) and \( q \) vertices totally dominates the vertex set \( S_1 = \{1, 2, \ldots, i_i\} \) and \( r \) and \( s \) vertices totally dominates the vertex set \( S_2 = \{i+1, \ldots, n\} \) and also consider that there is no vertex in \( S_1 \) other than the vertices \( p \) and \( q \) that totally dominates \( S_1 \) and no vertex in \( S_2 \) other than the vertices \( r \) and \( s \) that dominates \( S_2 \).
Since p and q are the only two vertices that are totally dominates S, there is no vertex in S, = \{1,2,......i\} \{p, q\} that can dominate S,. Let (p, q) be the edge in the graph G. Then in the graph G-e, p and q vertices totally dominates every vertex in S except the two vertices p and q. Now consider a vertex x in S, which is adjacent to both p and q. Then clearly the set \{p, q\} totally dominates the set S, in G-e. If there is no vertex x which is adjacent to both p and q then the graph G becomes disconnected, as the vertex x is isolated, which is a contradiction.

Let us assume that any two vertices (y, z), y\#p and z\#q or y\#q and z\#p totally dominates the vertex set S, in G-e, this implies that y and z both are totally dominates the vertex set S, in G, a contradiction, because by the hypothesis p and q are the only two vertices which are totally dominates S, in G. Hence we can easily say that two vertices i.e., y and z cannot dominate S, in G-e.

Thus TDS = TDS \cup \{x\} becomes a dominating set of G-e. Since TDS is a minimum total dominating set in G, TDS, is also minimum in G-e, so that \gamma_t(G - e) > \gamma_t(G). Hence b_t(G)=1.

2.4: ILLUSTRATION

The corresponding interval graph G is as follows,

![Interval family I](image)

Fig. 4: Interval family I

![Interval graph G](image)

Fig. 5: Interval graph G

S_1 = \{1,2,3,4,5\}
S_2 = \{6,7,8,9\}
P = 2, q = 4, r = 8, s = 9
Total dominating set TDS = \{2,4,8,9\} and \gamma_t(G) = 4.

Remove the edge e = (2,4) from G,

![Interval graph G-e](image)

Fig. 6: Interval graph G-e

Total dominating set of G-e = TDS = \{2,3,4,8,9\} and \gamma_t(G - e) = 5.

Therefore \gamma_t(G - e) > \gamma_t(G) and hence b_t(G)=1.

**THEOREM 2.5:** Let G be an interval graph corresponding to an interval family I = \{i_1, i_2, ...... i_k\}. Let the total dominating set TDS of G consists of four vertices only, say p, q, r and s. Suppose p and q vertices totally dominates the
vertex set \( S_1 = \{1, 2, \ldots, i\} \) and \( r \) and \( s \) vertices totally dominates the vertex set \( S_2 = \{i+1, \ldots, n\} \). Suppose there are two more vertices \( \{u, v\} \subseteq S_1 \) or \( S_2 \) respectively. Then \( b,(G) = 1 \).

**PROOF:** Let \( G \) be an interval graph corresponding to an interval family \( I = \{i_1, i_2, \ldots, i_n\} \). Let the total dominating set \( TDS \) of \( G \) consists of four vertices only say \( p, q, r \) and \( s \). Suppose there are two more vertices \( u, v \subseteq S_1 \) or \( S_2 \). The total dominating set \( TDS = \{p, q, r, s\} \) and the vertices \( p \) and \( q \) totally dominates the set \( S_1 \) and the two vertices \( r \) and \( s \) totally dominates the vertex set \( S_2 \).

Let \( u, v \subseteq S_1 \) such that \( u \) and \( v \) also totally dominates the vertex set \( S_1 \). Let \( e_1 = (u, q), e_2 = (v, q) \) and \( (p, v) \). Consider the graph \( G - \{e_1, e_2, e_3\} \). In this graph the vertices \( u \) and \( q \) and \( v \) and \( q \) and \( v \) are not adjacent. Hence \( p \) and \( q \) cannot totally dominate the set \( S_1 \) in \( G - \{e_1, e_2, e_3\} \). We require at least three vertices in \( S_1 \) in \( G - \{e_1, e_2, e_3\} \).

Therefore the total dominating set of \( G - e \) contains more than four vertices. Thus \( \gamma'_1(G - e) > \gamma'_1(G) \). Hence \( b_1(G) = 3 \).

Hence the proof of the theorem.

2.6: ILLUSTRATION

![Fig. 7: Interval family 1](image)

![Fig. 8: Interval graph G](image)

**Fig. 7:** Interval family 1

**Fig. 8:** Interval graph \( G \)

\[ S_1 = \{1, 2, 3, 4\}, S_2 = \{5, 6, 7, 8, 9, 10\}, p = 2, q = 4, r = 8, s = 9, u = 1, v = 3. \]

Total dominating set \( TDS = \{2, 4, 8, 9\} \) and \( \gamma'_1(G) = 4 \).

Remove the edges \( e_1 = (1, 4), e_2 = (3, 4), e_3 = (2, 3) \) from \( G \) then the corresponding interval graph \( G - e \) is as follows,

![Fig. 9: Interval graph G-e](image)

Total dominating set of \( G - e \) \( TDS = \{2, 3, 4, 8, 9\} \) and \( \gamma'_1(G - e) = 5 \).

Therefore \( \gamma'_1(G - e) > \gamma'_1(G) \) and hence \( b_1(G) = 1 \).

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