Chapter—6

INVENTORY MODELS WITH VARIABLE DEMAND

INTRODUCTION

We have seen that classical EOQ model [55, 75] does not have involvement of stock level q or time t on demand rate. So many authors have produced in past, several inventory models, taking the dependence of stock level or time on demand rate. Mandal and Phaujdar [72, 73], Gupta and Vrat [50] considered the stock dependent demand rate in their EOQ models. Silver and Meal [90] developed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. The classical no-shortage inventory problem for a linear trend in demand over a finite time-horizon was analytically solved by Donaldson [23]. Silver [95] derived a heuristic for the special case of a positive, linear trend in demand and applied it to the problem of Donaldson to cut short his complicated computational procedure. Ritchie [85] obtained an exact solution, having the simplicity of the EOQ formula, for Donaldson problem for linear, increasing demand. Deb and Chaudhari [30] studied the inventory replenishment policy for items having a deterministic demand pattern with a linear trend and shortages. They developed a heuristic to determine the decision rule and sizes of replenishments over a finite time horizon to keep total costs minimum. Further this work was extended by Murdeshwar [71].

In all the above inventory models, demand is either function of stock level or time. But in some cases demand may depend on both stock-level as well as time. For example, products showing a seasonal trend.

The present chapter deals with those inventory models which take care of demand rate on stock level and time both. Replenishment is

[Note: A paper on the work of this chapter has been communicated in Opsearch.]
instantaneous and shortages are not permitted.

ANALYSIS

In the first section profit maximization technique is used to estimate total average profit per unit time, while in the second section cost minimization technique is applied to determine minimum average cost per unit time of the system respectively.

SECTION – 1

PROFIT MAXIMIZATION CRITERIA

The EOQ model has been derived for the following functionals of demand rate and all the assumptions are same as in the inventory model of third chapter.

(i) \( r(q) = t e^{\alpha q} \)

(ii) \( r(q) = \frac{\alpha + \beta q}{\alpha + \beta t} \), where \( \alpha, \beta \) are constants and \( q \) is the stock level at any time \( t \).

Case (i): Let \( r(q) = t e^{\alpha q} \), then

\[
\frac{dq}{dt} = -t e^{\alpha q} \tag{6.1}
\]

Now considering \( q(0) = S \) and \( q(T) = 0 \), the length \( T \) of each cycle is calculated using equation (6.1) as,

\[
\int_{0}^{T} t \, dt = -\int_{0}^{S} \frac{dq}{e^{\alpha q}}
\]

or

\[
\frac{T^2}{2} = \int_{0}^{S} \frac{dq}{e^{\alpha q}} = \frac{1}{\alpha} \left[ e^{-\alpha q} \right]_{0}^{S}
\]

or

\[
\frac{T^2}{2} = \frac{1}{\alpha} (1 - e^{-\alpha S})
\]

or

\[
T = \frac{\sqrt{2}}{\sqrt{\alpha}} \sqrt{1 - e^{-\alpha S}} = F(S) \quad \text{Say}, \tag{6.2}
\]
Differentiating expression (6.2) with respect to $S$, we get,

$$F'(S) = \frac{\sqrt{\alpha}}{\sqrt{2}} \frac{e^{-\alpha S}}{\sqrt{1 - e^{-\alpha S}}}$$  \hspace{1cm} (6.3)

Also from (6.1), on integrating we have,

$$- \frac{e^{-\alpha q}}{\alpha} = - \frac{t^2}{2} + B, \text{ B is constant of integration} \hspace{1cm} (6.4)$$

Applying $q(0) = S$

$$B = - \frac{e^{-\alpha S}}{2}$$

So equation (6.4) reduces to

$$e^{\alpha q} = \frac{\alpha}{2} t^2 + e^{-\alpha S}$$

or

$$q = \frac{1}{\alpha} \log \left( \frac{\alpha}{2} t^2 + e^{-\alpha S} \right) \hspace{1cm} (6.5)$$

Hence total amount of inventory during the cycle $(0, T)$ is

$$G_r(S) = \int_0^T q \, dt$$

$$= - \frac{1}{\alpha} \int_0^T \log \left( \frac{\alpha t^2}{2} + e^{-\alpha S} \right) \, dt$$

$$= - \frac{1}{\alpha} \left[ t \log \left( \frac{\alpha t^2}{2} + e^{-\alpha S} \right) \right]_0^T + \frac{1}{\alpha} \int_0^T \frac{\alpha t^2}{\left( \frac{\alpha t^2}{2} + e^{-\alpha S} \right)} \, dt$$

$$= - \frac{T}{\alpha} \log \left( \frac{\alpha T^2}{2} + e^{-\alpha S} \right) + \frac{2}{\alpha} \int_0^T dt - \frac{2}{\alpha} e^{-\alpha S} \int_0^T \frac{dt}{\left( \frac{\alpha t^2}{2} + e^{-\alpha S} \right)}$$

$$= - \frac{T}{\alpha} \log \left( \frac{\alpha T^2}{2} + e^{-\alpha S} \right) + \frac{2T}{\alpha} - \frac{4}{\alpha^2} e^{-\alpha S} \int_0^T \frac{dt}{t^2 + \left( \frac{\sqrt{2}}{\sqrt{\alpha}} e^{-\alpha S} \right)^2}$$
\[
= -\frac{T}{\alpha} \log \left( \frac{\alpha T^2}{2} + e^{-\alpha S} \right) + \frac{2T}{\alpha} - \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \sqrt{e^{-\alpha S}} \tan^{-1} \left( \frac{T\sqrt{\alpha}}{\sqrt{2} \sqrt{e^{-\alpha S}}} \right)
\]

(6.6)

Substituting the value of \( T \) from equation (6.2) in above equation (6.6), this reduces to,

\[
G(S) = \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \sqrt{1 - e^{-\alpha S}} - \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \sqrt{e^{-\alpha S}} \tan^{-1} \left( \frac{\sqrt{1 - e^{-\alpha S}}}{\sqrt{e^{-\alpha S}}} \right)
\]

\[
= \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \sqrt{e^{\alpha S} - 1} - \tan^{-1} \left( \sqrt{e^{\alpha S} - 1} \right)
\]

(6.7)

Also, differentiating equation (6.7) with respect to \( S \), we get,

\[
G'(S) = \frac{\sqrt{2}}{\alpha \sqrt{e^{\alpha S}}} \tan^{-1} \left( \sqrt{e^{\alpha S} - 1} \right)
\]

(6.8)

Finally for the total profit to be maximum, optimal condition (3.4) of chapter (3.3), using values of \( F(S) \), \( F'(S) \), \( G(S) \) and \( G'(S) \) from equations (6.2), (6.3), (6.7) and (6.8) respectively, implies,

\[
\frac{\sqrt{2}}{\alpha} \frac{\sqrt{e^{\alpha S} - 1}}{\sqrt{e^{\alpha S}}} \left[ (p - C) - \frac{\sqrt{2}}{\alpha \sqrt{\alpha}} \frac{C_1}{\sqrt{e^{\alpha S}}} \tan^{-1} \left( \sqrt{e^{\alpha S} - 1} \right) \right]
\]

\[
= \frac{\sqrt{2}}{\alpha \sqrt{e^{\alpha S} \sqrt{e^{\alpha S} - 1}}} \left[ (p-C)S - A - \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \frac{C_1}{\sqrt{e^{\alpha S}}} \left( \sqrt{e^{\alpha S} - 1} - \tan^{-1} \sqrt{e^{\alpha S} - 1} \right) \right]
\]

or

\[
2(e^{\alpha S} - 1)(p - C) - \frac{2\sqrt{2}}{\alpha \sqrt{e^{\alpha S}}} \frac{(e^{\alpha S} - 1)}{\sqrt{e^{\alpha S}}} C_1 \tan^{-1} \sqrt{e^{\alpha S} - 1}
\]

\[
= \alpha (p - C)S - \alpha A - \frac{2\sqrt{2}}{\alpha \sqrt{\alpha}} \frac{C_1}{\sqrt{e^{\alpha S}}} \left( \sqrt{e^{\alpha S} - 1} - \tan^{-1} \sqrt{e^{\alpha S} - 1} \right)
\]

or

\[
\alpha A + (p-C)(2e^{\alpha S} - \alpha S - 2) + \frac{2\sqrt{2}}{\alpha \sqrt{e^{\alpha S}}} \frac{C_1}{\sqrt{e^{\alpha S}}} \left[ \sqrt{e^{\alpha S} - 1} - e^{\alpha S} \tan^{-1} \sqrt{e^{\alpha S} - 1} \right] = 0
\]

(6.9)

And the condition for total average cost to be minimum is given by taking \( p = 0 \) in above expression,
\[ \alpha A - C(2 e^{\alpha S} - \alpha S - 2) + \frac{2\sqrt{2}}{\sqrt{\alpha}} \frac{C_1}{\sqrt{e^{\alpha S} - 1 - e^{\alpha S} \tan^{-1} \sqrt{e^{\alpha S} - 1}}} = 0 \]

(6.10)

The transcendental equations (6.9) and (6.10) can be solved by using some suitable numerical method to get optimal value \( S \) of ordered quantity. The total profit (cost) will be maximum (minimum) if

\[ Z''(S) < 0, \ ( > 0) \], for that value of \( S \) obtained from equation (6.9) and (6.10) respectively, where \( Z(S) \) is as given by equation (3.1) of chapter (3.3).

**Case (ii)**: Let \( r(q) = \frac{\alpha + \beta q}{\gamma + \beta t} \), then

\[ \frac{dq}{dt} = - \left( \frac{\alpha + \beta q}{\gamma + \beta t} \right) \]

(6.11)

or

\[ \frac{\gamma + \beta t}{\gamma} \int_0^T dt = - \int_0^S \frac{dq}{\alpha + \beta q} \], as \( q(0) = S \) and \( q(T) = 0 \)

or

\[ \frac{1}{\beta} \left[ \log (\gamma + \beta t) \right]_0^T = \frac{1}{\beta} \left[ \log (\alpha + \beta q) \right]_0^S \]

or

\[ \log \left( \frac{\gamma + \beta T}{\gamma} \right) = \log \left( \frac{\alpha + \beta S}{\alpha} \right) \]

or

\[ T = F(S) = \frac{\log S}{\alpha} \]

(6.12)

Differentiating above equation (6.12) with respect to \( S \), we get,

\[ F'(S) = \frac{\beta}{\alpha} \]

(6.13)

Also from equation (6.11), we have,

\[ \int \frac{dq}{\alpha + \beta q} = - \int \frac{dt}{\gamma + \beta t} \]

(6.14)
or \[
\frac{1}{\beta} \log (\alpha + \beta q) = -\frac{1}{\beta} \log (\gamma + \beta t) + \frac{1}{\beta} \log B
\]

where \( \frac{1}{\beta} \log B \) is a constant of integration.

or \[
\log (\alpha + \beta q) (\gamma + \beta t) = \log B
\]

or \[
(\alpha + \beta q) (\gamma + \beta t) = B
\]

Since, initially at \( t = 0, q = S \), we get,

\[B = \gamma (\alpha + \beta S)\]

Putting this value of \( B \), in equation (6.14), this implies,

\[(\alpha + \beta q) (\gamma + \beta t) = \gamma (\alpha + \beta S)\]

or \[
\beta q = \frac{\gamma (\alpha + \beta S)}{\gamma + \beta t} - \alpha
\]

or

\[
\beta q = \frac{\beta (\gamma S - \alpha t)}{\gamma + \beta t}
\]

or

\[
q = \left( \frac{\gamma S - \alpha t}{\gamma + \beta t} \right) (6.15)
\]

Hence the total amount of inventory during the cycle \((0, T)\) in the system is given by,

\[
G(S) = \int_0^T q \, dt
\]

\[
= \int_0^T \left( \frac{\gamma S - \alpha t}{\gamma + \beta t} \right) \, dt
\]

\[
= \int_0^T \frac{\gamma S}{\gamma + \beta t} \, dt - \alpha \int_0^T \frac{t}{\gamma + \beta t} \, dt
\]

\[
= \frac{\gamma S}{\beta} \log \left( \frac{\gamma + \beta T}{\gamma} \right) - \alpha \int_0^T \left( 1 - \frac{\gamma}{\gamma + \beta t} \right) \, dt
\]
\[
\frac{\nu S}{\beta} \log \left( \frac{\gamma + \beta T}{\gamma} \right) - \frac{\alpha}{\beta} T + \frac{\alpha \nu}{\beta^2} \log \left( \frac{\gamma + \beta T}{\gamma} \right) = \frac{\nu}{\beta} \left( S + \frac{\alpha}{\beta} \right) \log \left( 1 + \frac{\beta}{\alpha} S \right) - \frac{\gamma}{\beta} S
\]

(6.16)

Expression (6.16), on putting value of \(T\) from (6.12) reduces to,

\[
G(S) = \frac{\nu}{\beta} \left( S + \frac{\alpha}{\beta} \right) \log \left( 1 + \frac{\beta}{\alpha} S \right) - \frac{\gamma}{\beta} S
\]

(6.17)

Also, differentiating equation (6.17) with respect to \(S\), we get

\[
G'(S) = \frac{\nu}{\beta} \log \left( 1 + \frac{\beta S}{\alpha} \right)
\]

(6.18)

Substituting values of \(F(S), F'(S), G(S)\) and \(G'(S)\) from equations (6.12), (6.13), (6.17) and (6.18) respectively in condition (3.4) of chapter (3.3) for total profit to be maximum, this gives,

\[
\frac{\nu}{\alpha} S \left[ (p - C) - \frac{\nu}{\beta} C_1 \log \left( 1 + \frac{\beta}{\alpha} S \right) \right] = \frac{\nu}{\alpha} \left[ (p - C)S - A - \frac{\nu}{\beta} C_1 \left( S + \frac{\alpha}{\beta} \right) \log \left( 1 + \frac{\beta}{\alpha} S \right) + C_1 \frac{\nu S}{\beta} \right]
\]

or

\[
C_1 \frac{\nu}{\beta^2} \log \left( 1 + \frac{\beta}{\alpha} S \right) + A - C_1 \frac{\nu S}{\beta} = 0
\]

or

\[
A + C_1 \frac{\nu}{\beta^2} \log \left( 1 + \frac{\beta}{\alpha} S \right) - \beta S = 0
\]

(6.19)

Equation (6.19) is a transcendental equation and its solution can be obtained by Newton-Raphson method. The solution \(S^*\) of equation (6.19) which satisfy \(Z''(S^*) < 0\), gives the optimum value of \(S\), where \(Z(S)\) is given by equation (3.1) of chapter (3.3).

Expression (6.19) can also be written as
\[
A + \frac{\alpha \gamma}{\beta^2} C_1 \left[ \frac{\beta}{\alpha} - \frac{\beta^2 S^2}{2 \alpha^2} - \frac{\beta^3 S^3}{3 \alpha^3} \ldots \right] - \frac{\gamma}{\beta} C_1 S = 0
\]

or
\[
A + \alpha \gamma C_1 \left[ -\frac{1}{2} S^2 - \frac{\beta S^3}{3 \alpha^3} \ldots \right] = 0 \tag{6.20}
\]

In particular if \( \beta = 0 \), equation (6.20) reduces to,
\[
A = \frac{\gamma}{2 \alpha} C_1 S^2
\]

or
\[
S = \sqrt{\frac{2A \alpha}{C_1 \gamma}} \tag{6.21}
\]

This value of \( S \) is the same as obtained in classical EOQ model [55] and does not depend on cost price \( C \) and selling price \( p \) of unit item.

SECTION 2

COST MINIMIZATION CRITERIA

In this section, some more bi-variable demand rates have been considered. Cost minimization criteria is applied to derive the total average cost per unit time of the system. Functional forms of demand rate are,

(iii) \( r(q) = \alpha + \beta \gamma q^\gamma + \delta t^\delta \)

(iv) \( r(q) = \alpha + \beta e^q + \gamma e^l \)

(v) \( r(q) = \alpha + \beta q^2, \alpha, \beta, \gamma, \delta \) and \( \lambda \) are some constants.

Case I: Let demand rate is \( r(q) = \alpha + \beta \gamma q^\gamma + \delta t^\delta \) \tag{6.22}

Since the total average cost per unit time

\[
K = rC + \frac{A}{q} r + \frac{CC_1}{2} q
\]

Supplying value of \( r \) from equation (6.22), we have,
\[ K(q, t) = C(\alpha + \beta q^\gamma + \delta t^\lambda) + \frac{A}{q}(\alpha + \beta q^\gamma + \delta t^\lambda) + \frac{CC_1}{2}q \]

\[ = \alpha C + \beta Cq^\gamma + \delta Ct^\lambda + \frac{\alpha A}{q} + \beta Aq^\gamma - 1 + \frac{\delta At^\lambda}{q} + \frac{CC_1q}{2} \quad (6.23) \]

Differentiating partially w.r.t. \( q \), we get

\[ \frac{\delta K}{\delta q} = \beta r Cq^{\gamma - 1} - \frac{\alpha A}{q^2} + A \beta (\gamma - 1) q^{\gamma - 2} - \frac{A \delta t^\lambda}{q^2} + \frac{CC_1}{2} \quad (6.24) \]

Differentiating once again w.r.t. \( q \) partially, we get

\[ \frac{\delta^2 K}{\delta q^2} = C \beta r (\gamma - 1) q^{\gamma - 2} + \frac{2A \alpha}{q^3} + A \beta (\gamma - 1)(\gamma - 2) q^{\gamma - 3} + \frac{2A \delta t^\lambda}{q^3} \quad (6.25) \]

Differentiating equation (6.23) partially w.r.t. \( t \), this gives

\[ \frac{\delta K}{\delta t} = C \lambda \delta t^\lambda - 1 + \frac{A \lambda \delta t^\lambda - 1}{q} \quad (6.26) \]

Further differentiating equation (6.26) w.r.t. \( t \), this gives

\[ \frac{\delta^2 K}{\delta t^2} = C \lambda (\lambda - 1) \delta t^\lambda - 2 + A \frac{\delta}{q} \lambda (\lambda - 1) t^\lambda - 2 \quad (6.27) \]

Differentiating equation (6.24) partially w.r.t. \( t \), we get

\[ \frac{\delta^2 K}{\delta q \delta t} = -\frac{A \lambda \delta t^\lambda - 1}{q^2} \quad (6.28) \]

For \( K(q, t) \) to be minimum, the necessary condition,

\[ \left( \frac{\delta^2 K}{\delta q^2} \right) \left( \frac{\delta^2 K}{\delta t^2} \right) - \left( \frac{\delta^2 K}{\delta q \delta t} \right) > 0 \]

or

\[ \left[ C \beta r (\gamma - 1) q^{\gamma - 2} + \frac{2A \alpha}{q^3} + A \beta (\gamma - 1)(\gamma - 2) q^{\gamma - 3} + \frac{2A \delta t^\lambda}{q^3} \right] \times \]

\[ \left[ C \lambda (\lambda - 1) + \frac{A \delta t^\lambda (\lambda - 1)}{q} \right] > \left( \frac{A \delta t^\lambda}{q^2} \right)^2 t^\lambda \quad (6.29) \]
or \[
\begin{align*}
C^2 \beta \delta \lambda (\lambda - 1) (\gamma - 1) q^\gamma - 2 + 2AC \frac{\alpha \beta \delta \lambda}{q^3} (\lambda - 1) + AC \beta \delta \lambda (\lambda - 1) & \\
(\gamma - 1)(\gamma - 2) q^\gamma - 3 + 2AC \frac{\delta \lambda}{q^3} (\lambda - 1) t^\gamma + A\beta \delta \lambda (\lambda - 1) & \\
(\gamma - 1)(\gamma - 1) q^\gamma - 3 + 2AC \frac{\delta \lambda}{q^3} (\lambda - 1) t^\gamma + A\beta \delta \lambda (\lambda - 1) & \\
+ 2AC^2 \frac{\lambda}{q^4} q^\gamma - 3 + 2AC \frac{\lambda}{q^4} q^\gamma - 3 + A\beta \delta \lambda (\lambda - 1)(\gamma - 1)(\gamma - 2) q^\gamma - 4
\end{align*}
\]

\[+ 2A \frac{\lambda}{q^4} \lambda (\lambda - 1) t^\gamma \frac{A^2 \lambda^2}{q^4} t^\gamma \quad (6.30)
\]

or \[
\begin{align*}
\beta \delta \lambda (\lambda - 1)(\gamma - 1) q^\gamma - 4 \{\gamma C^2 q^2 + (\gamma - 2)ACq + AC\gamma q + A^2 (\gamma - 2)\} & \\
+ 2AC \frac{\delta \lambda (\lambda - 1)}{q^4} (Cq + A) + A\frac{\delta \lambda}{q^4} & \\
\{2C(\lambda - 1)q + 2A(\lambda - 1) - A\lambda\} t^\gamma & \geq 0
\end{align*}
\]

\[+ 2AC(\lambda - 1)(Cq + A) q^\gamma - 4 \{\gamma C^2 q^2 + (\gamma - 1)CqA + A^2 (\gamma - 2)\} + 2AC(\lambda - 1)(Cq + A)
\]

\[t^\gamma q^\gamma - 4 \{2C(\lambda - 1)q + A(\lambda - 2)\} \geq 0 \quad (6.31)
\]

Total average cost \(K\) will be minimum if \(\frac{\partial^2 K}{\partial t^2} > 0\) at values \(q = q^*\) and \(t = t^*\) given by optimality condition (6.32) provided \(\alpha, \beta, \delta, \gamma > 0\) and \(\gamma \geq 2\).

**Case II.** Let \(r = \alpha + \beta e^q + \gamma e^t\), so that total average cost per unit time

\[K(q, t) = C(\alpha + \beta e^q + \gamma e^t) + \frac{A}{q} (\alpha + \beta e^q + \gamma e^t) + \frac{CCq}{2} \quad (6.33)
\]

Differentiating equation (6.33) partially w.r.t. \(q\), we get

\[\frac{\partial K}{\partial q} = \beta Ce^q + \frac{A}{q^2} (q\beta e^q - \alpha - \beta e^q - \gamma e^t) + \frac{CCq}{2} \quad (6.34)
\]

Differentiating once again with respect to \(q\), partially, we have,
\[ \frac{\partial^2 K}{\partial q^2} = \beta Ce^q + \frac{A}{q^4} \left[ \beta q^3 e^q - 2q(\beta q e^q - \alpha - \beta e^q - \gamma e^t) \right] \]

\[ = \beta Ce^q + \frac{A}{q^4} \left( \beta q^3 e^q - 2\beta q^2 e^q + 2\beta q e^q + 2\gamma q e^t + 2\alpha q \right) \]  \hspace{1cm} (6.35)

Differentiating once equation (6.33) w.r.t. \( t \) partially,

\[ \frac{\partial K}{\partial t} = \gamma Ce^t + \frac{A\gamma}{q} e^t = \gamma \left( C + \frac{A}{q} \right) e^t \]  \hspace{1cm} (6.36)

and

\[ \frac{\partial^2 K}{\partial t^2} = \gamma \left( C + \frac{A}{q} \right) e^t \]  \hspace{1cm} (6.37)

Also

\[ \frac{\partial^2 K}{\partial q \partial t} = -\frac{A\gamma}{q^2} e^t \]  \hspace{1cm} (6.38)

Now total cost to be minimum, necessary condition is,

\[ \left[ \beta Ce^q + \frac{A}{q^4} \left( \beta q^3 e^q - 2\beta q^2 e^q + 2\beta q e^q + 2\gamma q e^t + 2\alpha q \right) \right] \times \]

\[ \left[ \gamma \left( C + \frac{A}{q} \right) e^t \right] > \frac{A^2\gamma^2}{q^4} e^{2t} \]

or

\[ \left[ \left( Cq^3 + Aq^2 - 2Aq + 2A \right) \beta e^q + 2A(2\gamma e^t + \alpha) \right] \left( Cq + A \right) > A^2\gamma e^t \]  \hspace{1cm} (6.39)

Total cost will be minimum if \( \frac{d^2 K}{dt^2} > 0 \) at values \( q = q^* \) and \( t = t^* \) obtained from equation (6.39).

**Case III.** Let demand rate be of the form, \( r = \alpha + \beta q^2 t^2, \beta \neq 0 \)

Total cost can be given by

\[ K(q, t) = C(\alpha + \beta q^2 t^2) + \frac{A}{q} (\alpha + \beta q^2 t^2) + \frac{CC_1q}{2} \]  \hspace{1cm} (6.40)

Differentiating equation (6.40) partially w.r.t. \( q \), successively twice,

\[ \frac{\partial K}{\partial q} = 2\beta Cqt^2 - \frac{A\alpha}{q^2} + A\beta t^2 + \frac{CC_1}{2} \]  \hspace{1cm} (6.41)
and \[
\frac{\partial^2 K}{\partial q^2} = 2\beta C t^2 + \frac{2A\alpha}{q^3}
\] (6.42)

Differentiating equation (6.40) w.r.t. \( t \) partially twice, we get,

\[
\frac{\partial K}{\partial t} = 2\beta C q^2 t + 2A\beta qt
\] (6.43)

\[
\frac{\partial^2 K}{\partial t^2} = 2\beta C q^2 + 2A\beta q
\] (6.44)

also \[
\frac{\partial^2 K}{\partial q \partial t} = 4\beta C qt + 2A\beta t
\] (6.45)

Now for total cost to be minimum, the necessary condition is,

\[
\left(\beta C t^2 + \frac{A\alpha}{q^3}\right) (Cq + A)q - \beta t^2 (2Cq + A)^2 > 0
\]

or \[
\frac{AC\alpha}{q} + \frac{A^2\alpha}{q^2} - 3\beta C^2 q^2 t^2 - A\beta^2 C t^2 q - \beta A^2 t^2 > 0
\]

or \[
3\beta C^2 t^2 q^4 + A\beta C t^2 q^3 + A^2 \beta t^2 q^2 - AC\alpha q - A^2 \alpha < 0
\] (6.46)

Also \( K(q, t) \) will be minimum if \( \frac{\partial^2 K}{\partial t^2} > 0 \) at values \( q^* \) and \( t^* \), obtained from equation (6.46).

**CONCLUDING REMARKS**

In the EOQ model considered in this chapter, demand rate depends on stock-level and time both—a more realistic situation. In classical EOQ model in fixed demand with no shortages allowed, results do not depend on unit cost and selling price of the item, while it may depend on item for variable demand.

Some more functional forms of demand rate can be chosen suitably in order to extend the above EOQ model.