Chapter-IV

SNR Analysis Using Kaiser Window
CHAPTER IV
SNR ANALYSIS USING KAISER WINDOW

4.1 INTRODUCTION

This chapter presents the effect of $\beta$ in Kaiser Window on the SNR of radar returns. It proposes an optimum value of $\beta$ used in the Kaiser Window with which data may be weighted for the MST Radar signal analysis. The SNR computation for the six sets of Radar data is carried on and presented in Figures 4.3-4.8 and also Figures 4.9-4.14. From the Figures 4.3-4.8, it is inferred that when $\beta=0$, which corresponds to the rectangular window, the average SNR in the negative side is minimum and increases with the increase in $\beta$ steadily. When $\beta$ is in between 4 and 6, the increase in SNR slowly reaches to a constant and further increase of $\beta$ no longer produces any appreciable change in average SNR value.

4.2 KAISER WINDOW

Another window designed in accordance with an optimality criterion, is the Kaiser-Bessel (or Prolate Spheroidal Wave Function) [84, 85]. The previous window was characterized by minimum main-lobe width for a given side-lobe level. An alternate, and related, optimality criterion is the problem of determining the wave shape on a finite support that maximizes the energy in a specified bandwidth. This wave-shape has been identified by Slepian, Landau, and Pollak as the Prolate Spheroid function (of order zero) which contains a selectable time-bandwidth product parameter. Kaiser has discovered a simple numerical approximation to this function in terms of the zero order modified Bessel function of the first kind (hence the designation Kaiser- Bessel). The Kaiser-Bessel window is defined in equation (4.1) where the parameter $\beta$ is the window's half time-bandwidth product. The series for the Bessel function converges quite rapidly due to the $k!$ in the denominator.
The transform of the Kaiser-Bessel window is (within very low level aliasing terms) the function shown in equation (4.2). We see that this function tends to \( \frac{\sin(x)}{x} \) when the spectral argument is evaluated beyond the time-bandwidth related main-lobe bandwidth.

It must be noted that the Kaiser window can also be approximated by samples of the main-lobe of its spectra since the window is self-replicating under the tune-limiting and band-limiting operations.

\[
w(n) = \frac{I_0 \left[ \beta \sqrt{1.0 - \left( \frac{n}{N/2} \right)^2} \right]}{I_0(\beta)}
\]

where

\[
I_0(x) = \sum_{k=0}^{\infty} \left[ \frac{(x/2)^k}{k!} \right]^3 \quad \text{for} \quad -N/2 \leq n \leq N/2
\]

\[
W(\omega) = \frac{N}{I_0(\beta)} \frac{\sinh \sqrt{\left( \beta^2 - (N\omega)^2 \right)}}{(\beta)^2 - (N\omega)^2^2}
\]

Where \( \beta \) is the shape parameter.
Figure: 4.1 Kaiser-Bessel (60dB, \( \beta=8.3 \)) window and its Fourier Transform

Figure: 4.2 Kaiser-Bessel (80dB, \( \beta=10.7 \)) window and its Fourier Transform.
Figures 4.1 and 4.2 represent the Kaiser-Bessel window for parameter selected to achieve 60 and 80 dB side-lobes. The changes in the main-lobe widths can be observed comparative to those of the earlier window.

Kaiser window has two parameters: The length of the sequence (N) and a shape parameter (β). As the length of the window is fixed to 512 data points in the case of MST radar data used, the shape parameter β can be varied. As the parameter β increases steadily, the side lobe level of the frequency response decreases [52, 76]. In this chapter we have investigated the effect of β on the SNR of MST radar data. The results are presented in Figure 4.3-4.8 and Figure 4.9-4.14.

4.3 KAISER-WINDOW APPLIED TO MST RADAR SIGNALS

As explained in chapter 1, the back-scattered signal from the atmospheric layers is very small in terms of power with which it was emitted. The received back-scattered signals otherwise called as Radar returns are associated with Gaussian noise. The noise dominates the signal as the distance between the Radar and the target increases and this leads to a decrease in Signal to Noise ratio. This makes the detection of the signal difficult. Doppler profile information is obtained from the power spectrum using Fast Fourier Transform. Frequency characteristics of the back-scattered signals of the Radar are analyzed with power spectrum, which specifies the spectral characteristics of a signal in frequency domain.

The specifications of the data are given in Table 3.1. The SNR analysis is performed on MST Radar data which corresponds to the lower stratosphere obtained from the NARL, Gadanki, India, on July 2008. The Radar was operated in Zenith X, Zenith Y, North, South, West and East with an angle of $10^\circ$ from the vertical, direction. The data obtained from the six directions are used to carry on the analysis. The complete implementation of the scheme using MATLAB, to study the effect of β on the SNR of the radar returns can be put as follows.

a) Compute the Kaiser window with β=0
b) Taper the radar data with the Kaiser window parameters specified.

c) Perform the Fourier analysis of the above tapered data.

d) Compute the SNR using the procedure as mentioned in Chapter 2.7.
e) Compute the Mean Value Below Zero SNR (MVBZ).
f) Compute the Mean Value Above Zero SNR (MVAZ).
g) Update the value of $\beta$ and repeat the steps (b)-(f).

4.4 RESULTS AND DISCUSSION

The SNR computation for the six sets of Radar data is carried on and presented in Figure 4.3-4.8 and Figure 4.9-4.14. From the Figures 4.3-4.8 it is inferred that when $\beta=0$, which corresponds to the rectangular window, the average SNR in the negative side (MVBZ) is minimum and increases with the increase in $\beta$ steadily. When $\beta$ is in between 4 and 6, (MVBZ) increases slowly and reaches to a constant. Further increase of $\beta$ produces no appreciable change in average SNR value (MVBZ).

On the other hand in all the six sets of data, the average positive (MVAZ) SNR has not suffered any major change when $\beta$ is increased. Moreover, slight decrease in average positive (MVAZ) SNR is observed when $\beta$ is increased beyond 6.

From the Figures 4.9-4.14 it is also seen that for the lowermost 50 bins, the average SNRs in the negative and positive sides (MVBZ and MVAZ) are not improved appreciably. Moreover there is slight and marginal decrease in both SNR's is observed. For the middle 50 bins and the uppermost 50 bins the increase in average negative (MVBZ) SNR values is almost 5dB - 6dB when $\beta$ is around 6. Further improvement is also seen when $\beta$ is increased beyond 6. This result is important since the back-scattered signal from the middle and uppermost bins is very weak and improvement in SNR is highly desirable in spectral estimation.

Noting the above observations, it is concluded that the Kaiser window can be used with $\beta$ greater than or equal to 6 to taper the data for spectral analysis in place of a rectangular window. The results also suggest that, the effect of side lobe reduction in the improvement of SNR of noisy data and demands for the design of optimal windows. Table 4.1 presents the maximum SNR improvement for various bins when MST Radar signals are processed using Kaiser window.
Figure 4.3: Average SNR EAST Beam

Figure 4.4: Average SNR WEST Beam
Figure 4.5: Average SNR NORTH Beam

Figure 4.6: Average SNR SOUTH Beam
Figure 4.7: Average SNR ZENITH-X Beam

Figure 4.8: Average SNR ZENITH-Y Beam
Figure 4.11(a): Average SNR Lower Bias - North Beam

Figure 4.11(b): Average SNR Middle Bias - North Beam

Figure 4.11(c): Average SNR Upper Bias - North Beam

Figure 4.12(a): Average SNR Lower Bias - South Beam

Figure 4.12(b): Average SNR Middle Bias - South Beam

Figure 4.12(c): Average SNR Upper Bias - South Beam
TABLE 4.1: Maximum SNR improvement in dB using Kaiser Window

<p>| Beam Direction | MVBZ          | MVAZ          |</p>
<table>
<thead>
<tr>
<th></th>
<th>Lower bins dB</th>
<th>Middle bins dB</th>
<th>Upper bins dB</th>
<th>Lower bins dB</th>
<th>Middle bins dB</th>
<th>Upper bins dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>0.5</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>West</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>0.2</td>
<td>0.1</td>
<td>2</td>
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<tr>
<td>North</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>South</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>0.1</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>Zenith-x</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Zenith-y</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>