CHAPTER-IV

PROFIT ANALYSIS OF A COLD-STANDBY SYSTEM RELIABILITY MODEL WITH PRIORITY TO PREVENTIVE MAINTENANCE OVER REPAIR

4.1 Introduction

The technique of redundancy in cold standby mode has been adopted in most of the complex industrial systems for providing reliable services to the customers. However, most of the operating systems are vulnerable to damage caused by wear out and other unforeseen reasons. Therefore, preventive maintenance of such systems becomes necessary after a pre-specified operating time in order to maintain their efficiency. Malik [2013] introduced the concept of preventive maintenance while analyzing a computer system with cold standby redundancy. Further, system availability may be improved by giving priority in repair disciplines of one unit over the other. In view of these facts, profit analysis of a cold standby system has been made in the previous chapter by considering the concept of priority to repair over preventive maintenance.

While considering these practical situations in mind, the motive of the present chapter is to analyze a cold standby system of two identical units by giving priority to preventive maintenance over repair. Each unit has two modes- operative and complete failure. There is a single server who visits the system immediately for conducting maintenance and repair. Server conducts preventive maintenance of the unit after a maximum operation time ‘t’. However, repair of the unit is done at its complete failure. Priority is given to preventive maintenance of one unit over repair of the other unit. The random variables associated with failure time, completion of maximum operation time, preventive maintenance and repair times are statistically independent. The failure time of the unit and the time by which unit undergoes for preventive maintenance and repair times are taken as arbitrary. Several measures of system effectiveness such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to maintenance and repair, expected number of maintenances and repairs of the unit and profit function are obtained using semi-Markov process and
regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit function for particular values of various parameters and costs. The profit of the present model is also compared with that of the models discussed in chapter 2nd and 3rd.

4.2 Notations

- \( E_0 \) : Set of regenerative states.
- \( O/Cs \) : The unit is operative/cold standby.
- \( \alpha_0 \) : The rate by which unit undergoes for preventive maintenance.
- \( \lambda \) : Constant failure rate of the unit.
- \( f(t)/F(t) \) : pdf/cdf of preventive maintenance time.
- \( g(t)/G(t) \) : pdf/cdf of repair time of a failed unit.
- \( P_m/WP_m \) : The unit is under preventive maintenance/waiting for preventive maintenance.
- \( PM/FUR \) : The unit is under preventive maintenance/under repair continuously from previous state.
- \( Fu/Fw_r \) : The failed unit is under repair/waiting for repair.
- \( m_{ij} \) : The unconditional mean time taken by the system to transit from any regenerative state \( S_i \) when it (time) is counted from epoch of entrance in to that state \( S_j \). Mathematically, it can be written as

\[
m_{ij} = \int_0^\infty t \, d[Q_{ij}(t)] = -q_{ij}^*(0).
\]
\( \mu_i \) : The mean Sojourn time in state \( S_i \) which is given by

\[
\mu_i = E(t) = \int_0^\infty P(T > t) dt = \sum_{j} m_{ij} ,
\]

where \( T \) denotes the time to system failure.

\( W_i(t) \) : Probability that the server is busy in the state \( S_i \) up to time \( t \) without making any transition to any other regenerative state or returning to the same state via one or more regenerative states.

\( \otimes/\oplus \) : Symbol for Laplace Stieltjes convolution/Laplace convolution.

\( ~/\ast \) : Symbol for Laplace Stieltjes transform/ Laplace transform.

\'(desh) : Used to represent alternative result.

### 4.3 System Description and Assumptions

1. The system has of two identical units which may fail directly from normal mode.
2. Initially one unit is operative and the other is kept as spare in cold standby.
3. There is a single server who visits immediately to the system.
4. The preventive maintenance of the unit is carried out after a maximum operation time.
5. Repair of the unit is done at its failure.
6. The server cannot leave the system while performing jobs.
7. The unit works as a new after repair and preventive maintenance.
8. The switch over is instantaneous and perfect.
9. Priority is given to preventive maintenance over repair.
10. The failure time of the unit and the rate by which unit undergoes for preventive maintenance follow negative exponential distributions.
11. The distributions for preventive maintenance and repair times of the units are taken as arbitrary with different probability density functions.
12. All random variables are statistically independent.

4.4 Transition Probabilities and Mean Sojourn Times

The differential transition probabilities are:

\[ dQ_{01}(t) = \alpha_0 e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{02}(t) = \lambda e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{10}(t) = f(t) e^{-\alpha_0 t} \, dt \]
\[ dQ_{13}(t) = \overline{F}(t) \alpha_0 e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{16}(t) = \overline{F}(t) \lambda e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{20}(t) = g(t) e^{-\alpha_0 t} \, dt \]
\[ dQ_{24}(t) = \overline{G}(t) \alpha_0 e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{25}(t) = \overline{G}(t) \lambda e^{-(\alpha_0 + \lambda)t} \, dt \]
\[ dQ_{31}(t) = f(t)dt \]
\[ dQ_{42}(t) = f(t)dt \]
\[ dQ_{52}(t) = g(t)dt \]
\[ dQ_{62}(t) = g(t)dt \]  

(4.1)

Simple probabilistic considerations yield the following expressions for non-zero elements

\[ p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t) \, dt \]  

(4.2)
\[ p_{01} = \frac{\alpha_0}{\alpha_0 + \lambda}, \quad p_{02} = \frac{\lambda}{\alpha_0 + \lambda}, \quad p_{10} = f^*(\alpha_0 + \lambda), \quad p_{13} = p_{113} = \frac{\alpha_0}{\alpha_0 + \lambda}[1 - f^*(\alpha_0 + \lambda)] \]

\[ p_{16} = p_{126} = \frac{\lambda}{\alpha_0 + \lambda}[1 - f^*(\alpha_0 + \lambda)], \quad p_{20} = g^*(\alpha_0 + \lambda), \quad p_{23} = p_{223} = \frac{\lambda}{\alpha_0 + \lambda}[1 - g^*(\alpha_0 + \lambda)] \]

\[ p_{24} = p_{214} = \frac{\alpha_0}{\alpha_0 + \lambda}[1 - g^*(\alpha_0 + \lambda)], \quad p_{34} = p_{42} = p_{52} = p_{62} = 1 \quad (4.3) \]

It can be easily verified that

\[ p_{01} + p_{02} = p_{10} + p_{13} + p_{16} = p_{20} + p_{24} + p_{25} = 1 \quad (4.4) \]

The mean sojourn time \( \mu_i \) in the regenerative state \( S_i \) are defined as the time of stay in that state before transition to any other state. If \( T \) denotes the sojourn time in the regenerative state \( S_i \), then

\[ \mu_i = E(T) = P_i(T > t)dt \quad (4.5) \]

Thus,

\[ \mu_0 = \frac{1}{\alpha_0 + \lambda} \]

\[ \mu_1 = \frac{1}{\alpha_0 + \lambda}[1 - f^*(\alpha_0 + \lambda)] \]

\[ \mu_2 = \frac{1}{\alpha_0 + \lambda}[1 - g^*(\alpha_0 + \lambda)] \]

\[ \mu'_1 = \frac{(1 - f^*(\lambda + \alpha_0))(1 - \alpha_0 f^* (0))}{(\lambda + \alpha_0)} \]

and

\[ \mu'_2 = \frac{(1 - g^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0) g^* (0))}{(\lambda + \alpha_0)} \quad (4.6) \]

The unconditional mean time taken by the system to transit for any regenerative state \( S_i \) when it (time) is counted from the epoch of entrance into the state \( S_i \) is mathematically, states as
\[ m_y = \int_0^\infty t q_y(t) \, dt = -q_y''(0) \quad (4.7) \]

\[ \mu_0 = m_{01} + m_{02} \]

\[ \mu_i = m_{i0} + m_{i3} + m_{i6} \]

\[ \mu'_i = m_{i0} + m_{i1,3} + m_{i2,6} \]

\[ \mu''_i = m_{20} + m_{22,5} + m_{21,4} \quad (4.8) \]

The possible state transition diagram is shown in figure in 4.1.

[State Transition Diagram]

Figure 4.1
4.5 Reliability and Mean Time to System Failure (MTSF)

Let \( \phi_i(t) \) be the c.d.f. of first passage time from the regenerative state \( S_i \) to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for \( \phi_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \times \phi_1(t) + Q_{02}(t) \times \phi_2(t) \\
\phi_1(t) &= Q_{10}(t) \times \phi_0(t) + Q_{13}(t) + Q_{16}(t) \\
\phi_2(t) &= Q_{20}(t) \times \phi_0(t) + Q_{25}(t) + Q_{24}(t)
\end{align*}
\]

(4.9)

Taking Laplace–Stieltjes transform of above relations (4.9) and solving for \( \tilde{\phi}_i(s) \).

We have

\[
R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s}
\]

(4.10)

The reliability of the system model can be obtained by taking Laplace inverse transform of (4.10).

The mean time to system failure (MTSF) is given by

\[
MTSF = \lim_{s \to 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{\mu_0 + \mu_1 p_{01} + \mu_2 p_{02}}{1 - p_{01} p_{10} - p_{02} p_{20}}
\]

\[
= \frac{(\alpha_0 + \lambda) + \alpha_0 [1 - f^*(\alpha_0 + \lambda)] + \lambda [1 - g^*(\alpha_0 + \lambda)]}{(\alpha_0 + \lambda)^2 - [\alpha_0 f^*(\alpha_0 + \lambda) + \lambda g^*(\alpha_0 + \lambda)]}
\]

(4.11)

4.6 Availability Analysis

Let \( A_i(t) \) be the probability that the system is in up-state at instant \( t \) given that the system entered regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( A_i(t) \) are given as

\[
A_0(t) = M_0(t) + q_{01}(t) \oplus A_1(t) + q_{02}(t) \oplus A_2(t)
\]
\[ A_1(t) = M_1(t) + q_{10}(t) \oplus A_0(t) + q_{11,3}(t) \oplus A_1(t) + q_{12,6}(t) \oplus A_2(t) \]

\[ A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + q_{22,5}(t) \oplus A_2(t) + q_{24}(t) \oplus A_4(t) \]

\[ A_4(t) = q_{42}(t) \oplus A_2(t) \quad (4.12) \]

Here, \( M_i(t) \) is the probability that the system is up initially in state \( S_i \in E \) is up at time \( t \) without visiting to any other regenerative state, we have

\[ M_0(t) = e^{-(\alpha_0 + \lambda)t} \]

\[ M_1(t) = e^{-(\alpha_0 + \lambda)t} \overline{F(t)} \]

\[ M_2(t) = e^{-(\alpha_0 + \lambda)t} \overline{G(t)} \quad (4.13) \]

Taking L.T. of above relations (4.12) and solving for \( A_0^*(s) \), we get

\[ A_0^*(s) = \frac{M_0^*(1 - q_{11,3}^* - q_{22,5}^* - q_{24}^* q_{42}^*)}{(1 - q_{10}^* - q_{02}^*)(1 - q_{22,5} - q_{24}^* q_{42}^*)} + M_1^* \{ q_{01}^*(1 - q_{22,5}^*) - q_{24}^* q_{42}^* \} + M_2^* \{ q_{01}^* q_{12,6}^* + q_{02}^* (1 - q_{11,3}^*) \}
\]

\[ + (1 - q_{10}^* - q_{11,3}^* - q_{12,6}^*) q_{01}^* \{ 1 - q_{22,5}^* - q_{24}^* q_{42}^* \} + (1 - q_{20}^* - q_{24}^* - q_{22,5}^*) q_{01}^* q_{12,6}^* + q_{02}^* (1 - q_{11,3}^*) \]

\[ + q_{42}^* q_{01}^* q_{12,6}^* q_{24}^* + q_{02}^* q_{24}^* (1 - q_{11,3}^*) \]  

(4.14)

The steady state availability is given by

\[ A_0(\infty) = \lim_{s \to 0} s A_0^*(s) = \frac{N}{D} \]

where

\[ N = [g^*(\lambda + \alpha_0)(\lambda + \alpha_0 g^*(\lambda + \alpha_0)) + \alpha_0 g^*(\lambda + \alpha_0)(1 - f^*(\lambda + \alpha_0)) + \lambda(1 - g^*(\lambda + \alpha_0))]/(\lambda + \alpha_0)^2 \]
\[ D = [\alpha(\lambda + \alpha_0)g^*(\lambda + \alpha_0)(\lambda + \alpha_0) + \alpha \alpha_0(\lambda + \alpha_0)g^*(\lambda + \alpha_0)(1 - f^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0)g^*(0)) + \lambda \alpha(\lambda + \alpha_0)(1 - g^*(\lambda + \alpha_0))(\lambda + \alpha_0)] / \alpha(\lambda + \alpha_0)^3 \]

(4.15)

4.7 (a) Busy Period Analysis of Server Due to Preventive Maintenance

Let \( B_{t}^p(t) \) be the probability that the server is busy in preventive maintenance of the unit at an instant \( 't' \) given that system entered state \( S_i \) at \( t=0 \). The recursive relations for \( B_{t}^p(t) \) are as follows:

\[ B_{0}^p(t) = q_{01}(t) \oplus B_{1}^p(t) + q_{02}(t) \oplus B_{2}^p(t) \]

\[ B_{1}^p(t) = W_{1}(t) + q_{10}(t) \oplus B_{0}^p(t) + q_{11,5}(t) \oplus B_{1}^p(t) + q_{11,6}(t) \oplus B_{2}^p(t) \]

\[ B_{2}^p(t) = q_{20}(t) \oplus B_{0}^p(t) + q_{22,5}(t) \oplus B_{2}^p(t) + q_{24}(t) \oplus B_{4}^p(t) \]

\[ B_{4}^p(t) = W_{4}(t) + q_{42}(t) \oplus B_{2}^p(t) \]

(4.16)

where \( W_{1}(t) \) and \( W_{4}(t) \) be the probability that the server is busy in state \( S_1 \) and \( S_4 \) due to preventive maintenance up to time \( 't' \) without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

\[ W_{1}(t) = \{e^{-(\alpha_0 + \lambda)t} + (\alpha_0 e^{-(\alpha_0 + \lambda)t} \oplus 1) + (\lambda e^{-(\alpha_0 + \lambda)t} \oplus 1)\}F(t) \]

\[ W_{4}(t) = \overline{F(t)} \]

Taking Laplace transform of above relations (4.16) and solving for \( B_{0}^p(s) \). We get
The time for which server is busy due to preventive maintenance is given by

\[
B_0^p(s) = \lim_{s \rightarrow 0} s B_0^p(s) = \frac{N_1^p}{D}
\]

(4.17)

where \( N_1^p = W_1^* \left( p_{01}(1 - p_{22.5} - p_{24}p_{42}) + p_{24}W_4(P_{01}P_{12.6} + P_{02}(1 - P_{11.3})) \right) \)

\[
= \frac{\lambda \alpha_0 g^*(\lambda + \alpha_0)(1 - f^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0)g^*(0)) + \lambda \alpha_0(1 - g^*(\lambda + \alpha_0))}{\alpha(\lambda + \alpha_0)^2}
\]

and \( D \) has already mentioned in relation (4.15).

4.7 (b) Busy Period Analysis of the Server Due to Repair

Let \( B_i^R(t) \) be the probability that the server is busy in repair of the unit at an instant ‘t’
given that system entered state \( S_i \) at \( t=0 \). The recursive relations for \( B_i^R(t) \) are as follows:

\[
B_0^R(t) = q_{01}(t) \oplus B_1^R(t) + q_{02}(t) \oplus B_2^R(t)
\]

\[
B_1^R(t) = q_{10}(t) \oplus B_0^R(t) + q_{11.3}(t) \oplus B_1^R(t) + q_{12.6}(t) \oplus B_2^R(t)
\]

\[
B_2^R(t) = W_2(t) + q_{20}(t) \oplus B_0^R(t) + q_{22.5}(t) \oplus B_2^R(t) + q_{24}(t) \oplus B_4^R(t)
\]

\[
B_4^R(t) = q_{42}(t) \oplus B_2^R(t)
\]

(4.18)
where $W_2(t)$ is the probability that the server is busy in state $S_2$ respectively, due to repair up to time ‘t’ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

$$W_2(t) = e^{-\alpha(t)} e^{-\lambda t} G(t)$$

Taking Laplace transform of above relations (4.18) and solving for $B_0^R(s)$. We obtained

$$B_0^R(s) = \frac{W_2^*\{q_{01}^* q_{12.6}^* + q_{02}^* (1-q_{11.3}^*)\}}{(1-q_{01}^* - q_{02}^* \{ (1-q_{22.5}^* - q_{24}^* q_{42}^* )
\n+ (1-q_{10}^* - q_{11.3}^* - q_{12.6}^*) q_{01}^* (1-q_{22.5}^* - q_{24}^* q_{42}^* )
\n+ (1-q_{20}^* - q_{24}^* - q_{22.5}^* ) q_{01}^* q_{12.6}^* + q_{02}^* (1-q_{11.3}^*)
\n+ q_{42}^* q_{01}^* q_{12.6}^* q_{42}^* + q_{02}^* q_{24}^* (1-q_{11.3}^*)\}}$$

The time for which server is busy due to repair is given by

$$B_0^R = \lim_{s \to 0} s B_0^R(s) = \frac{N_2^R}{D}, \quad (4.19)$$

where $N_2^R = W_2^*\{ p_{01} p_{12.6} + p_{02} (1-p_{11.3})\}$

$$= \frac{\lambda (1-g^*(\lambda + \alpha_0)(1-(\lambda + \alpha_0)g^*(0))}{(\lambda + \alpha_0)^3}$$

and $D$ has already mentioned in relation (4.15).

4.8 (a) Expected Number of Preventive Maintenances of the Unit

Let $R_i^P(t)$ be the expected number of preventive maintenance of unit by the server in (0,t] given that the system entered the regenerative state $S_i$ at t=0. The recursive relations for $R_i^P(t)$ is given as
\[ R^p_0(t) = Q_{01}(t) \otimes R^p_1(t) + Q_{02}(t) \otimes R^p_2(t) \]

\[ R^p_i(t) = Q_{10}(t) \otimes [1 + R^p_0(t)] + Q_{12}(t) \otimes [1 + R^p_i(t)] + Q_{16}(t) \otimes [1 + R^p_2(t)] \]

\[ R^p_2(t) = Q_{20}(t) \otimes R^p_0(t) + Q_{22.5}(t) \otimes R^p_2(t) + Q_{24}(t) \otimes R^p_4(t) \]

\[ \tilde{R}^p_0(s) = \frac{g_0^*(1 - q^*_{22.5} - q^*_{24}q^*_{42}) + q^*_{24}q^*_{42} \{q^*_{02}(1 - q^*_{11.3}) + q^*_{01}q^*_{12.6}\}}{(1 - q^*_{01} - q^*_{02}) \{(1 - q^*_{22.5} - q^*_{24}q^*_{42})\}} \]

\[ + (1 - q^*_{10} - q^*_{11.3} - q^*_{12.6})q^*_{01}\{(1 - q^*_{22.5}) - q^*_{24}q^*_{42}\} \]

\[ + (1 - q^*_{20} - q^*_{24} - q^*_{22.5})\{q^*_{01}q^*_{12.6} + q^*_{02}(1 - q^*_{11.3})\} \]

\[ + q^*_{42}\{q^*_{01}q^*_{12.6}q^*_{24} + q^*_{02}q^*_{24}(1 - q^*_{11.3})\} \]

The expected number of preventive maintenances per unit time are respectively of given by

\[ R^p_0(\infty) = \lim_{s \to 0} s\tilde{R}^p_0(s) = \frac{N^p_3}{D} \quad (4.21) \]

where \( N^p_3 = p_{01}(1 - p_{22.5} - P_{24}P_{42}) + p_{24}P_{42}(p_{01}P_{12.6} + p_{02}(1 - p_{11.3})) \)

\[ = \frac{\alpha_0(\lambda + \alpha_0)^2(\lambda + \alpha_0)}{\lambda + \alpha_0} \quad \text{and} \quad D \text{ has already defined in relation (4.15).} \]

4.8(b) Expected Number of Repairs

Let \( R^R_i(t) \) be the expected number of repairs of unit by the server in \((0, t]\) given that the system entered the regenerative state \( S_i \) at \( t = 0 \).

The recursive relations for \( R^R_i(t) \) is given as
\[ R^R_0(t) = Q_{01}(t) \otimes R^R_1(t) + Q_{02}(t) \otimes R^R_2(t) \]
\[ R^R_1(t) = Q_{10}(t) \otimes R^R_0(t) + Q_{11.3}(t) \otimes R^R_1(t) + Q_{12.6}(t) \otimes R^R_2(t) \]
\[ R^R_2(t) = Q_{20}(t) \otimes [1 + R^R_0(t)] + Q_{22.5}(t) \otimes [1 + R^R_1(t)] + Q_{24}(t) \otimes [1 + R^R_2(t)] \]
\[ R^R_4(t) = Q_{42}(t) \otimes R^R_5(t) \]  
(4.22)

Taking L.S.T of relations (4.22) and, solving for \( \tilde{R}^R_0(s) \). We get

\[
\tilde{R}^R_0(s) = \frac{(q_{20} + q_{22.5} + q_{24})[q_{02}(1-q_{11.3}) + q_{01}q_{12.6}]}{(1-q_{01} - q_{02})[(1-q_{22.5} - q_{24}q_{42})]}
\]
+ \( (1 - q_{10} - q_{11.3} - q_{12.6})q_{01}[(1 - q_{22.5}) - q_{24}q_{42}] \)
+ \( (1 - q_{20} - q_{24} - q_{22.5})q_{01}q_{12.6} + q_{02}(1 - q_{11.3}) \)
+ \( q_{42}[q_{01}q_{12.6}q_{24} + q_{02}q_{24}(1 - q_{11.3})] \)

The expected numbers of repairs per unit time are respectively given by

\[ R^R_0(\infty) = \lim_{s \to 0} s\tilde{R}^R_0(s) = \frac{N^R_4}{D} \]  
(4.23)

where \( N^R_4 = (p_{20} + p_{22.5} + p_{24})[p_{01}p_{12.6} + p_{02}(1 - p_{11.3})] \)

\[ = \frac{\lambda}{(\lambda + \alpha_0)} \]

and \( D \) has already defined in relation (4.15).

4.9 Profit Analysis

The profit incurred to the system model in steady state can be obtained as

\[ P = K_0A_0 - K_1B^P_0 - K_2B^R_0 - K_3R^R_0 - K_4R^P_0 - K_5 \]  
(4.24)

\( K_0 = \) Revenue per unit up-time of the system

\( K_1 = \) Cost per unit time for which server is busy due preventive maintenance
\(K_2=\) Cost per unit time for which server is busy due to repair

\(K_3=\) Cost per unit time repair

\(K_4=\) Cost per unit time preventive maintenances done by the server

\(K_5=\) Total installation cost of the system

### 4.10 Particular Case

Let us take \(g(t) = \theta e^{-\theta t}\) and \(f(t) = \alpha e^{-\alpha t}\), then the following results are obtained:

\[
\text{MTSF} = \frac{(\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0) + \alpha_0(\theta + \lambda + \alpha_0) + \lambda(\alpha + \lambda + \alpha_0)}{\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0) - \alpha\alpha_0(\theta + \lambda + \alpha_0) - \lambda(\alpha + \lambda + \alpha_0)}
\]  \tag{4.25}

Availability

\[
A_0 = \frac{\alpha(\lambda + \theta)(\alpha + \lambda + \alpha_0)}{\alpha \theta^2(\alpha + \lambda) + \alpha_0\theta^2(\alpha + \lambda + \alpha_0) + \lambda\alpha(\theta + \lambda)(\alpha + \lambda + \alpha_0) + \lambda\alpha_0\theta(\alpha_0 + \lambda)(\alpha + \lambda)}
\]  \tag{4.26}

Busy period due to preventive maintenance

\[
B_0^p = \frac{\alpha_0(\lambda + \theta)(\alpha + \lambda + \alpha_0)}{\alpha \theta^2(\alpha + \lambda) + \alpha_0\theta^2(\alpha + \lambda + \alpha_0) + \lambda\alpha(\theta + \lambda)(\alpha + \lambda + \alpha_0) + \lambda\alpha_0\theta(\alpha_0 + \lambda)(\alpha + \lambda)}
\]  \tag{4.27}

Busy period due to Repair

\[
B_0^r = \frac{\alpha\lambda(\theta + \lambda)(\alpha + \lambda + \alpha_0)}{\alpha \theta^2(\alpha + \lambda) + \alpha_0\theta^2(\alpha + \lambda + \alpha_0) + \alpha\lambda(\theta + \lambda)(\alpha + \lambda + \alpha_0) + \lambda\alpha_0\theta(\alpha_0 + \lambda)(\alpha + \lambda)}
\]
Expected Number of visits by the server for conducting preventive maintenance

\[ R^p_0 = \frac{\alpha \alpha_0 \theta (\lambda + \theta + \alpha_0)(\lambda + \theta)}{\alpha \theta^2 (\alpha + \lambda) + \alpha_0 \theta^2 (\alpha + \lambda + \alpha_0) + \alpha \lambda (\theta + \lambda)(\alpha + \lambda + \alpha_0) + \lambda \alpha_0 \theta (\alpha_0 + \lambda)(\alpha + \lambda)} \]

(4.29)

Expected Number of visits by the server for doing repair

\[ R^r_0 = \frac{\lambda \alpha \theta (\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0)}{\alpha \theta^2 (\alpha + \lambda) + \alpha_0 \theta^2 (\alpha + \lambda + \alpha_0) + \alpha \lambda (\theta + \lambda)(\alpha + \lambda + \alpha_0) + \lambda \alpha_0 \theta (\alpha_0 + \lambda)(\alpha + \lambda)} \]

(4.30)

4.11 Conclusion

The graphs for mean time to system failure, availability and profit function have been drawn with respect to preventive maintenance rate giving particular values to the parameters and costs as shown respectively in figures 4.2, 4.3 and 4.4. It is observed that the values of these reliability measures go on increasing with the increase of preventive maintenance and repair rates. However, their values decline with the increase of maximum constant rate of operation (\(\alpha_0\)) and failure rate. The profit of the system has been evaluated after reducing the installation cost of the system which was generally ignored by the researchers.

Finally, it is concluded that the performance of a cold stand by system can be improved by conducting preventive maintenance after a pre-specific period of operation rather than increasing the repair rate of the system at its failure.

4.12 Comparative Study

The profit of the present model has been compared with that of the models discussed in previous chapters. It is observed that the present model is less profitable as compared to these
models. However, profit difference of this model with that of the models discussed in chapters 2nd and 3rd go on decreasing with the increase of preventive maintenance rate (α). Thus, study reveals that priority to preventive maintenance of one unit over repair of the other unit is not economically beneficial. The results are shown numerically and graphically in tables 4.5, 4.6 and figures 4.5, 4.6 respectively.

### 4.13 Graphical Presentation of Reliability Measures

Figure 4.2: Graph between MTTF and Preventive Maintenance rate (α)

![Graph between MTTF and Preventive Maintenance rate (α)](image)

Figure 4.3: Graph between Availability and Preventive Maintenance rate (α)
Figure 4.4: Graph Between Profit and Preventive Maintenance rate ($\alpha$)
Figure 4.5: Graph of Profit Difference (Model 2.1-Model 4.1) with respect to Preventive Maintenance rate ($\alpha$)

Figure 4.6: Graph of Profit Difference (Model 3.1-Model 4.1) with respect to Preventive Maintenance rate ($\alpha$)
4.14 Numerical Presentation of Reliability Measures

1. MTSF (Table 4.1)

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate (α) ↓</th>
<th>(α_0=5, \lambda=.01)</th>
<th>(α_0=5, \lambda=.01)</th>
<th>(α_0=7, \lambda=.01)</th>
<th>(α_0=5, \lambda=.02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(θ=2.5, a=.7, b=.3)</td>
<td>0.598139</td>
<td>0.598205</td>
<td>0.386965</td>
<td>0.59629</td>
</tr>
<tr>
<td>(θ=3, a=.7, b=.3)</td>
<td>0.637837</td>
<td>0.637917</td>
<td>0.407268</td>
<td>0.635688</td>
</tr>
<tr>
<td>(θ=2.5, a=.7, b=.3)</td>
<td>0.677513</td>
<td>0.677608</td>
<td>0.427564</td>
<td>0.675045</td>
</tr>
<tr>
<td>(θ=2.5, a=.7, b=.3)</td>
<td>0.717169</td>
<td>0.71728</td>
<td>0.447855</td>
<td>0.714361</td>
</tr>
<tr>
<td>(θ=3, a=.7, b=.3)</td>
<td>0.756803</td>
<td>0.756933</td>
<td>0.468139</td>
<td>0.753634</td>
</tr>
<tr>
<td>(θ=2.5, a=.7, b=.3)</td>
<td>0.796417</td>
<td>0.796565</td>
<td>0.488418</td>
<td>0.792867</td>
</tr>
<tr>
<td>(θ=2.5, a=.7, b=.3)</td>
<td>0.836009</td>
<td>0.836178</td>
<td>0.50869</td>
<td>0.832057</td>
</tr>
</tbody>
</table>
### 2. Availability (Table 4.2)

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate (α) \ θ, a, b</th>
<th>( \alpha_0=5, \lambda=.01 )</th>
<th>( \alpha_0=5, \lambda=.01 )</th>
<th>( \alpha_0=7, \lambda=.01 )</th>
<th>( \alpha_0=5, \lambda=.02 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ( \theta=2.5 ), ( a=0.7, b=0.3 )</td>
<td>0.526337</td>
<td>0.599714</td>
<td>0.383252</td>
<td>0.526344</td>
</tr>
<tr>
<td>6 ( \theta=3 ), ( a=0.7, b=0.3 )</td>
<td>0.559481</td>
<td>0.632455</td>
<td>0.410768</td>
<td>0.559624</td>
</tr>
<tr>
<td>7 ( a=0.7, b=0.3 )</td>
<td>0.587671</td>
<td>0.659664</td>
<td>0.435125</td>
<td>0.587912</td>
</tr>
<tr>
<td>8 ( a=0.7, b=0.3 )</td>
<td>0.612096</td>
<td>0.682772</td>
<td>0.457043</td>
<td>0.612408</td>
</tr>
<tr>
<td>9 ( a=0.7, b=0.3 )</td>
<td>0.633551</td>
<td>0.702721</td>
<td>0.47699</td>
<td>0.633915</td>
</tr>
<tr>
<td>10 ( a=0.7, b=0.3 )</td>
<td>0.652598</td>
<td>0.720164</td>
<td>0.495296</td>
<td>0.653</td>
</tr>
<tr>
<td>11 ( a=0.7, b=0.3 )</td>
<td>0.669654</td>
<td>0.735574</td>
<td>0.512203</td>
<td>0.670085</td>
</tr>
<tr>
<td>12 ( a=0.7, b=0.3 )</td>
<td>0.685038</td>
<td>0.749305</td>
<td>0.527897</td>
<td>0.685488</td>
</tr>
<tr>
<td>13 ( a=0.7, b=0.3 )</td>
<td>0.698996</td>
<td>0.761631</td>
<td>0.542527</td>
<td>0.699462</td>
</tr>
<tr>
<td>14 ( a=0.7, b=0.3 )</td>
<td>0.711729</td>
<td>0.772764</td>
<td>0.556211</td>
<td>0.712204</td>
</tr>
</tbody>
</table>

### 3. Profit (Table 4.3)
### Preventive Maintenance Rate ($\alpha$) \downarrow

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate ($\alpha$)</th>
<th>$\alpha_0 = 5$, $\lambda = .01$</th>
<th>$\alpha_0 = 5$, $\lambda = .01$</th>
<th>$\alpha_0 = 7$, $\lambda = .01$</th>
<th>$\alpha_0 = 5$, $\lambda = .02$</th>
</tr>
</thead>
<tbody>
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<td>$\theta = 3,$</td>
<td>$\theta = 2.5,$</td>
<td>$\theta = 2.5,$</td>
<td></td>
</tr>
<tr>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>720.3071</td>
<td>975.2925</td>
<td>-492.439</td>
<td>716.9044</td>
</tr>
<tr>
<td>6</td>
<td>1055.413</td>
<td>1325.466</td>
<td>-159.879</td>
<td>1051.469</td>
</tr>
<tr>
<td>7</td>
<td>1333.14</td>
<td>1612.275</td>
<td>116.9706</td>
<td>1328.825</td>
</tr>
<tr>
<td>8</td>
<td>1568.98</td>
<td>1853.103</td>
<td>354.2738</td>
<td>1564.38</td>
</tr>
<tr>
<td>9</td>
<td>1772.858</td>
<td>2059.113</td>
<td>561.9856</td>
<td>1768.022</td>
</tr>
<tr>
<td>10</td>
<td>1951.539</td>
<td>2237.902</td>
<td>746.6485</td>
<td>1946.495</td>
</tr>
<tr>
<td>11</td>
<td>2109.852</td>
<td>2394.879</td>
<td>912.7901</td>
<td>2104.618</td>
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<tr>
<td>12</td>
<td>2251.372</td>
<td>2534.031</td>
<td>1063.677</td>
<td>2245.961</td>
</tr>
<tr>
<td>13</td>
<td>2378.826</td>
<td>2658.381</td>
<td>1201.75</td>
<td>2373.248</td>
</tr>
<tr>
<td>14</td>
<td>2494.342</td>
<td>2770.276</td>
<td>1328.885</td>
<td>2488.606</td>
</tr>
</tbody>
</table>

4. **Profit Difference Model 2.1 and Model 4.1** (Table 4.4)

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate ($\alpha$)</th>
<th>$\alpha_0 = 5$, $\lambda = .01$</th>
<th>$\alpha_0 = 5$, $\lambda = .01$</th>
<th>$\alpha_0 = 7$, $\lambda = .01$</th>
<th>$\alpha_0 = 5$, $\lambda = .02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 2.5,$</td>
<td>$\theta = 3,$</td>
<td>$\theta = 2.5,$</td>
<td>$\theta = 2.5,$</td>
<td></td>
</tr>
<tr>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td>$a = .7$, $b = .3$</td>
<td></td>
</tr>
<tr>
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<td>1272.612</td>
<td>1018.491</td>
<td>1590.687</td>
<td>1268.952</td>
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<tr>
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<td>903.6897</td>
<td>1485.501</td>
<td>1168.506</td>
</tr>
<tr>
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<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>1079.85</td>
<td>801.9189</td>
<td>1398.701</td>
<td>1075.164</td>
</tr>
<tr>
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<td>991.0501</td>
<td>708.2616</td>
<td>1320.501</td>
<td>985.9441</td>
</tr>
<tr>
<td>9</td>
<td>905.5666</td>
<td>620.7572</td>
<td>1246.522</td>
<td>900.1215</td>
</tr>
<tr>
<td>10</td>
<td>823.3815</td>
<td>538.557</td>
<td>1174.868</td>
<td>817.6704</td>
</tr>
<tr>
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<td>744.6306</td>
<td>461.2205</td>
<td>1104.798</td>
<td>738.7157</td>
</tr>
<tr>
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<td>669.4219</td>
<td>388.4466</td>
<td>1036.096</td>
<td>663.3542</td>
</tr>
<tr>
<td>13</td>
<td>597.7907</td>
<td>319.9756</td>
<td>968.7693</td>
<td>591.6115</td>
</tr>
<tr>
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<td>529.7017</td>
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<td>523.4443</td>
</tr>
</tbody>
</table>

5. **Profit Difference Model 3.1 and Model 4.1** (Table 4.4)

<p>| Preventive Maintenance Rate (α) | ( \alpha_0=5, , \lambda=.01 ) &amp; ( \alpha_0=5, , \lambda=.01 ) &amp; ( \alpha_0=7, , \lambda=.01 ) &amp; ( \alpha_0=5, , \lambda=.02 ) |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                                 | ( \theta=2.5, ) &amp; ( a=.7, , b=.3 ) | ( \theta=3, ) &amp; ( a=.7, , b=.3 ) | ( \theta=2.5, ) &amp; ( a=.7, , b=.3 ) | ( \theta=2.5, ) &amp; ( a=.7, , b=.3 ) |
| 5                              | 1437.818                        | 1183.909                        | 1858.422                        | 1432.691                        |
| 6                              | 1335.584                        | 1066.829                        | 1741.237                        | 1329.446                        |
| 7                              | 1238.014                        | 960.3625                        | 1641.728                        | 1230.943                        |
| 8                              | 1143.314                        | 860.8276                        | 1550.946                        | 1135.437                        |
| 9                              | 1051.458                        | 766.9691                        | 1464.926                        | 1042.906                        |
| 10                             | 962.8275                        | 678.3376                        | 1381.959                        | 953.716                         |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>877.777</td>
<td>594.7129</td>
<td>1301.364</td>
<td>868.2061</td>
</tr>
<tr>
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<td>515.9108</td>
<td>1222.92</td>
<td>786.5838</td>
</tr>
<tr>
<td>13</td>
<td>719.1797</td>
<td>441.7274</td>
<td>1146.599</td>
<td>708.9251</td>
</tr>
<tr>
<td>14</td>
<td>645.7082</td>
<td>371.9326</td>
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<td>635.2026</td>
</tr>
</tbody>
</table>