CHAPTER-II

PROFIT ANALYSIS OF A COLD-STANDBY SYSTEM RELIABILITY MODEL WITH PREVENTIVE MAINTENANCE AND REPAIR

2.1 Introduction

A lot of research work on reliability modeling of standby systems has been carried out by the researchers under a common assumption that systems can perform adequately for a long time without requiring any maintenance. But this assumption seems to be unrealistic when a system has to operate in varying environment conditions. Also, system deteriorates due to continued operation and ageing. In such a situation, preventive maintenance of the system can be conducted after a maximum operation time in order to slow the deteriorate process as well as to restore the system in a younger age or state. Osaki and Asakura [1970], Kapur and Kapoor [1974] obtained time to first system down for standby systems under repair and preventive maintenance considering arbitrary distributions. However, Kapur and Kapoor [1974] also evaluated transition probabilities for the system models with different standby modes.

The reliability, availability and profit function which are measured in terms of system state probabilities have been identified as a major stumbling block in achieving a high or required level of system performance. On the other hand, the technique of cold standby redundancy has been considered as an effective strategy in order to improve the performance and to achieve high reliability of the system. Therefore, extensive research has been carried out by the researchers including Gopalan and Naidu [1983], Singh [1989] and Kadyan et al. [2010] on reliability modeling of a system with cold standby redundancy under different repair policies. Also, Sureria et al. [2012] carried out cost benefit analysis of a computer system with cold standby redundancy. Recently, Malik [2013] discussed reliability model of a computer system with cold standby redundancy under preventive maintenance and repair.

Hence in the present chapter some important reliability measures such as mean time to system failure (MTSF), availability and profit incurred to the system model have been obtained for a two-unit cold standby system with preventive maintenance and repair. The units are identical in nature subject to constant failure from normal mode. The operative unit undergoes
for preventive maintenance after a pre-specific time ‘t’ up to which no failure occurs. However, repair of the unit is done at its failure. The unit works as new after repair and preventive maintenance. The switch devices work properly. The failure time of the unit and the time by which unit undergoes for preventive maintenance and repair times are assumed as arbitrary with different probability density functions. All random variables are statistically independent. The expressions for some more reliability measures such as mean sojourn times, busy period of the server, expected number of preventive maintenance and repair have been derived in steady state using semi-Markov process and regenerative point technique. The graphical behavior of MTSF, availability and profit function have also been observed with respect to preventive maintenance rate for particular values of other parameters and costs. The application of the present study can be visualized in a water supply system of two identical electric pumps- one unit is initially working and the other is kept spare in cold standby.

2.2 Notations

\( E_0 \) : Set of regenerative states.

\( O/Cs \) : The unit is operative/cold standby.

\( \alpha_0 \) : The rate by which unit undergoes for preventive maintenance.

\( \lambda \) : Constant failure rate of the unit.

\( f(t)/F(t) \) : pdf/cdf of preventive maintenance time of the unit.

\( g(t)/G(t) \) : pdf/cdf of repair time of the unit.

\( P_{m}/WP_{m} \) : The unit is under preventive maintenance/waiting for preventive maintenance.

\( PM/FUR \) : The unit is under preventive maintenance/under repair continuously from previous state.

\( Fu_{i}/Fw_{i} \) : The failed unit is under repair/waiting for repair.
The unconditional mean time taken by the system to transit from any regenerative state $S_i$ when it (time) is counted from epoch of entrance in to that state $S_j$. Mathematically, it can be written as

$$m_{ij} = \int_0^\infty d[Q_{ij}(t)] = -q_{ij}'(0).$$

The mean Sojourn time in state $S_i$ which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t)dt = \sum_j m_{ij},$$

where $T$ denotes the time to system failure.

Probability that the server is busy in the state $S_i$ up to time $t'$ without making any transition to any other regenerative state or returning to the same state via one or more regenerative states.

Symbol for Laplace Stieltjes convolution/Laplace convolution.

Symbol for Laplace Stieltjes transform/ Laplace transform.

Used to represent alternative result.

2.3 System Description and Assumptions

1. The system has of two identical units which may fail directly from normal mode.
2. Initially one unit is operative and the other is kept as spare in cold standby.
3. There is a single server who visits immediately to the system.
4. The preventive maintenance of the unit is carried out after a maximum operation time.
5. Repair of the unit is done at its failure.
6. The server cannot leave the system while performing jobs.
7. The unit works as a new after repair and preventive maintenance.
8. The switch over is instantaneous and perfect.
9. The failure time of the unit and the rate by which unit undergoes for preventive maintenance follow negative exponential distributions.
10. The distributions for preventive maintenance and repair times of the units are taken as arbitrary with different probability density functions.
11. All random variable are statistically independent.

2.4 Transition Probabilities and Mean Sojourn Times

The differential transition probabilities are:

\[ dQ_{01}(t) = \alpha_0 e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{02}(t) = \lambda e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{10}(t) = f(t) e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{13}(t) = \bar{F}(t) \alpha_0 e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{15}(t) = \bar{F}(t) \lambda e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{20}(t) = g(t) e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{24}(t) = \bar{G}(t) \alpha_0 e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{25}(t) = \bar{G}(t) \lambda e^{-\left(\alpha_0 + \lambda\right) t} \, dt \]
\[ dQ_{31}(t) = f(t) dt \]
\[ dQ_{41}(t) = g(t) dt \]
\[ dQ_{52}(t) = g(t) dt \]
\[ dQ_{02}(t) = g(t) dt \]  

(2.1)
Simple probabilistic considerations yield the following expressions for non-zero elements

\[ p_{ij} = Q_i(\infty) = \int q_{ij}(t) \, dt \]  

(2.2)

\[ p_{01} = \frac{\alpha_0}{\alpha_0 + \lambda}, \quad p_{02} = \frac{\lambda}{\alpha_0 + \lambda}, \quad p_{10} = f^*(\alpha_0 + \lambda), \quad p_{13} = p_{113} = \frac{\alpha_0}{\alpha_0 + \lambda} [1 - f^*(\alpha_0 + \lambda)] \]

\[ p_{16} = p_{126} = \frac{\lambda}{\alpha_0 + \lambda} [1 - f^*(\alpha_0 + \lambda)], \quad p_{20} = g^*(\alpha_0 + \lambda), \quad p_{25} = p_{225} = \frac{\lambda}{\alpha_0 + \lambda} [1 - g^*(\alpha_0 + \lambda)] \]

\[ p_{24} = p_{214} = \frac{\alpha_0}{\alpha_0 + \lambda} [1 - g^*(\alpha_0 + \lambda)], \quad p_{31} = p_{41} = p_{52} = p_{62} = 1 \]  

(2.3)

It can be easily verified that

\[ p_{01} + p_{02} = p_{10} + p_{13} + p_{16} = p_{20} + p_{24} + p_{25} = 1 \]  

(2.4)

The mean sojourn time \((\mu_i)\) in the regenerative state \(S_i\) are defined as the time of stay in that state before transition to any other state. If \(T\) denotes the sojourn time in the regenerative state \(S_i\) then

\[ \mu_i = E(T) = P_i(T > t)dt \]  

(2.5)

Thus,

\[ \mu_0 = \frac{1}{\alpha_0 + \lambda} \]

\[ \mu_1 = \frac{1}{\alpha_0 + \lambda} [1 - f^*(\alpha_0 + \lambda)] \]

\[ \mu_2 = \frac{1}{\alpha_0 + \lambda} [1 - g^*(\alpha_0 + \lambda)] \]

\[ \mu_1' = \frac{(1 - f^*(\lambda + \alpha_0))(1 - \alpha_0 f^*(0))}{(\lambda + \alpha_0)} \]

and

\[ \mu_2' = \frac{(1 - g^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0) g^*(0))}{(\lambda + \alpha_0)} \]  

(2.6)
The unconditional mean time taken by the system to transit for any regenerative state $S_j$ when it (time) is counted from the epoch of entrance into the state $S_i$ is mathematically, states as

$$m_{ij} = \int_0^\infty tq_{ij}(t)dt = -q''_{ij}(0)$$  \hspace{1cm} (2.7)

$$\mu_0 = m_{01} + m_{02}$$

$$\mu_t = m_{10} + m_{13} + m_{16}$$

$$\mu'_t = m_{10} + m_{113} + m_{126}$$

$$\mu''_t = m_{20} + m_{225} + m_{214}$$  \hspace{1cm} (2.8)

The possible state transition diagram is shown in figure in 2.1.
2.5 Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f. of first passage time from the regenerative state $S_i$ to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

\[
\phi_0(t) = Q_{01}(t) \otimes \phi_1(t) + Q_{02}(t) \otimes \phi_2(t)
\]
\[
\phi_1(t) = Q_{10}(t) \otimes \phi_0(t) + Q_{13}(t) + Q_{16}(t)
\]
\[
\phi_2(t) = Q_{20}(t) \otimes \phi_1(t) + Q_{23}(t) + Q_{24}(t)
\]

(2.9)

Taking Laplace–Stieltjes transform of above relations (2.9) and solving for $\tilde{\phi}_i(s)$.

We have

\[
R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s}
\]

(2.10)

The reliability of the system model can be obtained by taking Laplace inverse transform of (2.10).

The mean time to system failure (MTSF) is given by

\[
MTSF = \lim_{s \to 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{\mu_0 + \mu_1 p_{01} + \mu_2 p_{02}}{1 - p_{01} p_{10} - p_{02} p_{20}}
\]

\[
= \frac{(\alpha_0 + \lambda) + \alpha_0 \{1 - f^*(\alpha_0 + \lambda)\} + \lambda \{1 - g^*(\alpha_0 + \lambda)\}}{(\alpha_0 + \lambda)^2 - [\alpha_0 f^*(\alpha_0 + \lambda) + \lambda g^*(\alpha_0 + \lambda)]}
\]

(2.11)

2.6 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state $S_i$ at t=0. The recursive relations for $A_i(t)$ are given as
$$A_0(t) = M_0(t) + q_{01}(t) \oplus A_1(t) + q_{02}(t) \oplus A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) \oplus A_0(t) + q_{113}(t) \oplus A_1(t) + q_{126}(t) \oplus A_2(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + q_{214}(t) \oplus A_1(t) + q_{225}(t) \oplus A_2(t)$$

(2.12)

Here, $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time $t$ without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\alpha_0 + \lambda)t}$$

$$M_1(t) = e^{-(\alpha_0 + \lambda)t} \overline{F(t)}$$

$$M_2(t) = e^{-(\alpha_0 + \lambda)t} \overline{G(t)}$$

(2.13)

Taking L.T. of above relations (2.12) and solving for $A^*_0(s)$, we get

$$A^*_0(s) = \frac{M_0^*(s)\{1-q_{113}^*(s)(1-q_{225}^*(s) - q_{214}^*(s)q_{126}^*(s))\}}{(1-q_{01}^*(s)-q_{02}^*(s))\{(1-q_{113}^*(s)(1-q_{225}^*(s))-q_{214}^*(s)q_{126}^*(s))\}}$$

$$+ M_1^*(s)\{q_{01}^*(s)(1-q_{225}^*(s)) + q_{02}^*(s)q_{214}^*(s)\}$$

$$+ M_2^*(s)\{q_{02}^*(s)q_{01}^*(s) + q_{02}^*(s)(1-q_{113}^*(s))\}$$

(2.14)

The steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA^*_0(s) = \frac{N}{\lambda}$$

where

$$N = [(\lambda + \alpha_0)(\lambda + \alpha_0)f^*(\lambda + \alpha_0))(\alpha_0 + \lambda g^*(\lambda + \alpha_0)) - \lambda \alpha_0(1-g^*(\lambda + \alpha_0))$$

$$\{1 - f^*(\lambda + \alpha_0)\} + (1 - f^*(\lambda + \alpha_0))\{\alpha_0(\lambda + \alpha_0)(\alpha_0 + \lambda g^*(\lambda + \alpha_0))$$

$$+ \lambda \alpha_0(1-g^*(\lambda + \alpha_0)) + (1-g^*(\lambda + \alpha_0))\{\lambda \alpha_0(1-f^*(\lambda + \alpha_0))$$

$$+ \lambda(\lambda + \alpha_0f^*(\lambda + \alpha_0))\}/(\lambda + \alpha_0)^3$$
\[ D = \left[ (\lambda + \alpha_o)(\lambda + \alpha_o f^*(\lambda + \alpha_o))(\alpha_o + \lambda g^*(\lambda + \alpha_o)) - \lambda \alpha_o(1 - g^*(\lambda + \alpha_o)) \right] \\
(1 - f^*(\lambda + \alpha_o)) + (1 - f^*(\lambda + \alpha_o))(1 - (\lambda + \alpha_o)f^*(0)) \{ \alpha_o(\lambda + \alpha_o) \}
\]

\[ \alpha_o + \lambda g^*(\lambda + \alpha_o) + \lambda \alpha_o(1 - g^*(\lambda + \alpha_o)) + (1 - g^*(\lambda + \alpha_o))(1 - (\lambda + \alpha_o)g^*(0)) \]

\[ \{ \lambda \alpha_o(1 - f^*(\lambda + \alpha_o)) + \lambda(\lambda + \alpha_o f^*(\lambda + \alpha_o)) \} / (\lambda + \alpha_o)^3 \]  

(2.15)

2.7 (a) Busy Period Analysis for Server Due to Preventive Maintenance

Let \( B^p_i(t) \) be the probability that the server is busy in preventive maintenance of the unit at an instant ‘t’ given that system entered state \( S_i \) at \( t=0 \). The recursive relations for \( B^p_i(t) \) are as follows:

\[ B^p_0(t) = q_{01}(t) \oplus B^p_1(t) + q_{02}(t) \oplus B^p_2(t) \]

\[ B^p_1(t) = W_1(t) + q_{10}(t) \oplus B^p_0(t) + q_{113}(t) \oplus B^p_1(t) + q_{126}(t) \oplus B^p_2(t) \]

\[ B^p_2(t) = q_{20}(t) \oplus B^p_0(t) + q_{214}(t) \oplus B^p_1(t) + q_{225}(t) \oplus B^p_2(t) \]  

(2.16)

where \( W_1(t) \) be the probability that the server is busy in state \( S_1 \) due to preventive maintenance up to time ‘t’ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

\[ W_1(t) = \{ e^{-(\alpha_o + \lambda)t} + (\alpha_o e^{-(\alpha_o + \lambda)t} \oplus 1) + (\lambda e^{-(\alpha_o + \lambda)t} \oplus 1) \} F(t) \]

Taking Laplace transform of above relations (2.16) and solving for \( B^p_0(s) \).

\[ B^p_0(s) = \frac{W_1^*(s)\{ q_{01}^*(s)(1 - q_{225}^*(s)) + q_{02}^*(s)q_{214}^*(s) \}}{(1 - q_{01}^*(s) - q_{02}^*(s))\{ (1 - q_{113}^*(s)(1 - q_{225}^*(s)) - q_{214}^*(s)q_{126}^*(s)) \}
\]

\[ + (1 - q_{10}^*(s) - q_{113}^*(s) - q_{126}^*(s))\{ q_{01}^*(s)(1 - q_{225}^*(s)) + q_{02}^*(s)q_{214}^*(s) \}
\]

\[ + (1 - q_{20}^*(s) - q_{214}^*(s) - q_{225}^*(s))\{ q_{02}^*(s)(1 - q_{113}^*(s)) + q_{01}^*(s)q_{126}^*(s) \} \]

The time for which server is busy due to preventive maintenance is given by
\[ B_{0}^{p}(t) = \lim_{s \to 0} sB_{0}^{p}(t) = \frac{N_{1}^{p}}{D} \]  

(2.17)

where \( N_{1}^{p}(t) = W_1' \left( p_{01}(1 - p_{22.3}) + p_{02}p_{21.4} \right) \)

\[
= \{ (1 - f^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0) f^*(0)) \{ \alpha_0 (\lambda + \alpha_0) \} \times (\alpha_0 + \lambda^* g^*(\lambda + \alpha_0)) + \lambda \alpha_0 (1 - g^*(\lambda + \alpha_0)) \} f(\lambda + \alpha_0)^3
\]

and \( D \) has already mentioned in relation (2.15).

2.7 (b) Busy Period Analysis of the Server Due to Repair

Let \( B_{i}^{R}(t) \) be the probability that the server is busy in repair of the unit at an instant ‘t’ given that system entered state S\(_i\) at t=0. The recursive relations for \( B_{i}^{R}(t) \) are as follows:

\[
B_{0}^{R}(t) = q_{01}(t) \oplus B_{1}^{R}(t) + q_{02}(t) \oplus B_{2}^{R}(t)
\]

\[
B_{1}^{R}(t) = q_{10}(t) \oplus B_{0}^{R}(t) + q_{1.3}(t) \oplus B_{1}^{R}(t) + q_{1.6}(t) \oplus B_{2}^{R}(t)
\]

\[
B_{2}^{R}(t) = W_{2}(t) + q_{20}(t) \oplus B_{0}^{R}(t) + q_{2.4}(t) \oplus B_{1}^{R}(t) + q_{2.5}(t) \oplus B_{2}^{R}(t)
\]  

(2.18)

where \( W_{2}(t) \) be the probability that the server is busy in state S\(_2\) due to repair up to time ‘t’ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

\[
W_{2}(t) = \left[ e^{-(\alpha_0 + \lambda)t} + (\alpha_0 e^{-(\alpha_0 + \lambda)t} \oplus 1) + (\lambda e^{-(\alpha_0 + \lambda)t} \oplus 1) \right] \overline{G}(t)
\]

Taking Laplace transform of above relations (2.18) and solving for \( B_{0}^{R}(s) \).

\[
B_{0}^{R}(s) = \frac{W_{2}^{*}(s)\{q_{01}^{*}(s)q_{1.2.4}(s) + q_{02}^{*}(s)(1 - q_{1.1.3}(s))\}}{(1 - q_{01}^{*}(s) - q_{02}^{*}(s))\{(1 - q_{1.1.3}(s)(1 - q_{2.2.5}(s)) - q_{2.1.4}(s)q_{1.2.6}(s))\}}
\]

\[
+ (1 - q_{10}^{*}(s) - q_{1.3}(s) - q_{2.6}(s))\{q_{01}(s)(1 - q_{2.2.5}(s)) + q_{02}(s)q_{2.1.4}(s)\}
\]

\[
+ (1 - q_{20}^{*}(s) - q_{2.1.4}(s) - q_{2.2.5}(s))\{q_{02}(s)(1 - q_{1.1.3}(s)) + q_{01}(s)q_{1.2.6}(s)\}
\]
The time for which server is busy due to repair is given by

\[ B_0^R = \lim_{s \to 0} sB_0^*(s) = \frac{N_2^R}{D}, \]  

(2.19)

where \( N_2^R(t) = W_2^* \{ p_{01}p_{126} + p_{02}(1 - p_{113}) \} \)

\[ = [(1 - g^*(\lambda + \alpha_0))(1 - (\lambda + \alpha_0)g^*(0))] \]
\[ \times [(\lambda + \alpha_0)(1 - f^*(\lambda + \alpha_0)) + \lambda(\lambda + \alpha_0f^*(\lambda + \alpha_0))]/(\lambda + \alpha_0)^3 \]

and \( D \) has already mentioned in relation (2.15).

2.8 (a) Expected Number of Preventive Maintenances of the Unit

Let \( R_i^p(t) \) be the expected number of preventive maintenance of unit by the server in \((0,t]\) given that the system entered the regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( R_i^p(t) \) is given as

\[ R_0^p(t) = Q_{01}(t) \otimes R_0^p(t) + Q_{02}(t) \otimes R_2^p(t) \]
\[ R_1^p(t) = Q_{10}(t) \otimes [1 + R_0^p(t)] + Q_{113}(t) \otimes [1 + R_1^p(t)] + Q_{126}(t) \otimes [1 + R_2^p(t)] \]
\[ R_2^p(t) = Q_{20}(t) \otimes R_0^p(t) + Q_{214}(t) \otimes R_1^p(t) + Q_{225}(t) \otimes R_2^p(t) \]  

(2.20)

Taking L.S.T of relations (2.20) and, solving for \( \mathcal{R}_0^p(t) \).

\[ \mathcal{R}_0^p(s) = \frac{\{(q_{01}^*(s) + q_{113}^*(s) + q_{126}^*(s))\{q_{01}^*(s)(1 - q_{225}^*(s)) + q_{02}^*(s)q_{214}^*(s)\}}{(1 - q_{01}^*(s) - q_{02}^*(s))\{(1 - q_{113}^*(s)(1 - q_{225}^*(s)) - q_{214}^*(s)q_{126}^*(s))\}} \]
\[ + (1 - q_{01}^*(s) - q_{113}^*(s) - q_{126}^*(s))\{q_{01}^*(s)(1 - q_{225}^*(s)) + q_{02}^*(s)q_{214}^*(s)\} \]
\[ + (1 - q_{20}^*(s) - q_{214}^*(s) - q_{225}^*(s))\{q_{02}^*(s)(1 - q_{113}^*(s)) + q_{01}^*(s)q_{126}^*(s)\} \]

The expected number of preventive maintenances per unit time are respectively of given by
\[ R^R_i(\infty) = \lim_{s \to 0} s \tilde{R}^R_i(s) = \frac{N^R_i}{D} \tag{2.21} \]

where \( N^R_i = (p_{10} + p_{11,3} + p_{12,6})\{p_{01}(1 - p_{22,5}) + p_{02}p_{21,4}\} \)

\[ = \alpha_0(\lambda + \alpha_0)(\alpha_0 + \lambda g^*(\lambda + \alpha_0)) + \alpha_0\lambda(1 - g^*(\lambda + \alpha_0)) \]

and \( D \) has already defined in relation (2.15).

\[ \] 2.8(b) Expected Number of Repairs of the Units

Let \( R^R_i(t) \) be the expected number of repairs of unit by the server in \((0,t]\) given that the system entered the regenerative state \( S_i \) at \( t=0 \).

The recursive relations for \( R^R_i(t) \) is given as

\[ R^R_0(t) = Q_{01}(t) \otimes R^R_0(t) + Q_{02}(t) \otimes R^R_2(t) \]

\[ R^R_1(t) = Q_{10}(t) \otimes R^R_0(t) + Q_{11,3}(t) \otimes R^R_1(t) + Q_{12,6}(t) \otimes R^R_2(t) \]

\[ R^R_2(t) = Q_{20}(t) \otimes [1 + R^R_0(t)] + Q_{21,4}(t) \otimes [1 + R^R_1(t)] + Q_{22,5}(t) \otimes [1 + R^R_2(t)] \tag{2.22} \]

Taking L.S.T of relations (2.22) and, solving for \( \tilde{R}^R_0(s) \).

\[ \tilde{R}^R_0(s) = \frac{\{(q_{20}^*(s) + q_{21,4}^*(s) + q_{22,5}^*(s))\{q_{01}(s)q_{12,6}^*(s) + q_{02}(s)(1 - q_{11,3}^*(s))\}}{(1-q_{01}(s)-q_{02}(s))(1-q_{11,3}^*(s)(1-q_{22,5}(s)) - q_{21,4}(s)q_{12,6}(s))} \]

\[ + (1-q_{01}(s)-q_{02}(s)-q_{11,3}(s)-q_{12,6}(s))\{q_{01}(s)(1-q_{22,5}(s)) + q_{02}(s)q_{21,4}(s)\} \]

\[ + (1-q_{20}(s)-q_{21,4}(s)-q_{22,5}(s))\{q_{02}(s)(1-q_{11,3}(s)) + q_{01}(s)q_{12,6}(s)\} \]

The expected numbers of repairs per unit time are given by

\[ R^R_i(\infty) = \lim_{s \to 0} s \tilde{R}^R_i(s) = \frac{N^R_i}{D} \tag{2.23} \]
where \( N_R^p \) = \( p_{01}p_{12.6} + p_{02}(1 - p_{11.3}) \)

\[
= \alpha_0(\lambda + \alpha_0)(\alpha_0 + \lambda f^*(\lambda + \alpha_0)) + \lambda \alpha_0(1 - f^*(\lambda + \alpha_0))
\]

and \( D \) has already defined in relation (2.15).

### 2.9 Profit Analysis

The profit incurred to the system model in steady state can be obtained as

\[
P = K_0A_0 - K_1B_0^p - K_2B_0^r - K_3R_0^r - K_4R_0^p - K_5
\]

\( K_0 \)= Revenue per unit up-time of the system

\( K_1 \)= Cost per unit time for which server is busy due preventive maintenance

\( K_2 \)= Cost per unit time for which server is busy due to repair

\( K_3 \)= Cost per unit time repair

\( K_4 \)= Cost per unit time preventive maintenances done by the server

\( K_5 \)= Total installation cost of the system

### 2.10 Particular Case

Let us take \( g(t) = \theta e^{-\theta t} \) and \( f(t) = \alpha e^{-\alpha t} \), then the following results are obtained:

\[
\text{MTSF} = \frac{\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0) + \alpha_0(\theta + \lambda + \alpha_0) + \lambda(\alpha + \lambda + \alpha_0)}{(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0) - \alpha \alpha_0(\theta + \lambda + \alpha_0) - \lambda \theta(\alpha + \lambda + \alpha_0)}
\]

\[
\text{Availability} \quad A_0 = \frac{\alpha \theta \lambda(\theta + \alpha + \lambda + 2 \alpha_0) + (\alpha + \alpha_0)(\theta + \alpha_0)}{\alpha \theta(\lambda \theta + \alpha \alpha_0 + \alpha \theta + \theta(\lambda + \alpha + \alpha_0)(\alpha_0(\theta + \alpha_0) + \lambda \alpha]} \frac{\alpha(\theta + \alpha_0 + \lambda)[\lambda \alpha_0 + \lambda(\alpha + \lambda)]}{
\]

\[
(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\theta + \lambda + \alpha_0) - \alpha \alpha_0(\theta + \lambda + \alpha_0) - \lambda \theta(\alpha + \lambda + \alpha_0)
\]
Busy period due to preventive maintenance

\[ B_0^p = \frac{\alpha_0 \theta (\alpha + \lambda + \alpha_0)(\lambda + \theta + \alpha_0)}{\alpha \theta (\lambda \theta + \alpha \alpha_0 + \alpha \theta + \theta (\lambda + \alpha + \alpha_0))(\alpha_0 (\theta + \alpha_0) + \lambda \alpha)} \]

\[ \alpha (\theta + \alpha_0 + \lambda)[\lambda \alpha_0 + \lambda (\alpha + \lambda)] \]  

(2.27)

Busy period due to Repair

\[ B_0^r = \frac{\alpha \lambda (\alpha + \lambda + \alpha_0)(\lambda + \theta + \alpha_0)}{\alpha \theta (\lambda \theta + \alpha \alpha_0 + \alpha \theta + \theta (\lambda + \alpha + \alpha_0))(\alpha_0 (\theta + \alpha_0) + \lambda \alpha)} \]

\[ \alpha (\theta + \alpha_0 + \lambda)[\lambda \alpha_0 + \lambda (\alpha + \lambda)] \]  

(2.28)

Expected Number of visits by the server for conducting preventive maintenance

\[ R_0^p = \frac{\alpha \alpha_0 \theta (\alpha + \lambda + \alpha_0)(\lambda + \theta + \alpha_0)}{\alpha \theta (\lambda \theta + \alpha \alpha_0 + \alpha \theta + \theta (\lambda + \alpha + \alpha_0))(\alpha_0 (\theta + \alpha_0) + \lambda \alpha)} \]

\[ \alpha (\theta + \alpha_0 + \lambda)[\lambda \alpha_0 + \lambda (\alpha + \lambda)] \]  

(2.29)

Expected Number of visits by the server for doing repair

\[ R_0^r = \frac{\alpha \theta \lambda (\alpha + \lambda + \alpha_0)(\lambda + \theta + \alpha_0)}{\alpha \theta (\lambda \theta + \alpha \alpha_0 + \alpha \theta + \theta (\lambda + \alpha + \alpha_0))(\alpha_0 (\theta + \alpha_0) + \lambda \alpha)} \]

\[ \alpha (\theta + \alpha_0 + \lambda)[\lambda \alpha_0 + \lambda (\alpha + \lambda)] \]  

(2.30)

2.11 Conclusion

The graphs for mean time to system failure, availability and profit function have been drawn with respect to preventive maintenance rate giving particular values to the parameters and costs as shown graphically and numerically in figures 2.2, 2.3 and 2.4 and table 2.1, 2.2 and 2.3 respectively. It is observed that the values of these reliability measures go on increasing with the increase of preventive maintenance and repair rates. However, their values decline
with the increase of maximum constant rate of operation ($\alpha_0$) and failure rate. It is interesting to note that profit of the system has been evaluated after reducing the installation cost of the system which was generally ignored by the researchers.

2.12 Graphical Representation of Reliability Measures

Figure 2.2 Graph between MTSF and Preventive Maintenance rate ($\alpha$)
Figure 2.3 Graph between Availability and Preventive Maintenance rate ($\alpha$)

Figure 2.4 Graph between Profit and Preventive Maintenance rate ($\alpha$)
### 2.13 Numerical Presentation of Reliability Measures

1. **MTSF (Table 2.1)**

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate (α) (\downarrow)</th>
<th>(\alpha_0=5, \lambda=.01)</th>
<th>(\alpha_0=5, \lambda=.01)</th>
<th>(\alpha_0=7, \lambda=.01)</th>
<th>(\alpha_0=5, \lambda=.02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.598139</td>
<td>0.598205</td>
<td>0.386965</td>
<td>0.59629</td>
</tr>
<tr>
<td>(\theta=3, a=.7, b=.3)</td>
<td>0.637837</td>
<td>0.637917</td>
<td>0.407268</td>
<td>0.635688</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.677513</td>
<td>0.677608</td>
<td>0.427564</td>
<td>0.675045</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.717169</td>
<td>0.71728</td>
<td>0.447855</td>
<td>0.714361</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.756803</td>
<td>0.756933</td>
<td>0.468139</td>
<td>0.753634</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.796417</td>
<td>0.796565</td>
<td>0.488418</td>
<td>0.792867</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.836009</td>
<td>0.836178</td>
<td>0.50869</td>
<td>0.832057</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.875581</td>
<td>0.875771</td>
<td>0.528956</td>
<td>0.871207</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.915131</td>
<td>0.915344</td>
<td>0.549216</td>
<td>0.910315</td>
</tr>
<tr>
<td>(\theta=2.5, a=.7, b=.3)</td>
<td>0.95466</td>
<td>0.954898</td>
<td>0.56947</td>
<td>0.949383</td>
</tr>
</tbody>
</table>
2. **Availability** (Table 2.2)

<table>
<thead>
<tr>
<th>Preventive Maintenance Rate ($\alpha$)</th>
<th>$\alpha_0=5$, $\lambda=.01$</th>
<th>$\alpha_0=5$, $\lambda=.01$</th>
<th>$\alpha_0=7$, $\lambda=.01$</th>
<th>$\alpha_0=5$, $\lambda=.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.665262</td>
<td>0.66552</td>
<td>0.54942</td>
<td>0.663861</td>
</tr>
<tr>
<td>$\theta=3$, $a=.7$, $b=.3$</td>
<td>0.723658</td>
<td>0.723963</td>
<td>0.612908</td>
<td>0.722046</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.768856</td>
<td>0.769201</td>
<td>0.665201</td>
<td>0.767075</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.80428</td>
<td>0.804657</td>
<td>0.70842</td>
<td>0.802365</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.832407</td>
<td>0.832811</td>
<td>0.744325</td>
<td>0.830384</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.855024</td>
<td>0.855451</td>
<td>0.774338</td>
<td>0.852914</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.873432</td>
<td>0.873877</td>
<td>0.799591</td>
<td>0.87125</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.888581</td>
<td>0.889042</td>
<td>0.820981</td>
<td>0.88634</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.901177</td>
<td>0.901651</td>
<td>0.839218</td>
<td>0.898887</td>
</tr>
<tr>
<td>$\theta=2.5$, $a=.7$, $b=.3$</td>
<td>0.91175</td>
<td>0.912235</td>
<td>0.854866</td>
<td>0.90942</td>
</tr>
</tbody>
</table>

3. **Profit** (Table 2.3)

<table>
<thead>
<tr>
<th>Preventive Maintenance</th>
<th>$\alpha_0=5$, $\lambda=.01$</th>
<th>$\alpha_0=5$, $\lambda=.01$</th>
<th>$\alpha_0=7$, $\lambda=.01$</th>
<th>$\alpha_0=5$, $\lambda=.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta=2.5$,</td>
<td>$\theta=3$,</td>
<td>$\theta=2.5$,</td>
<td>$\theta=2.5$,</td>
</tr>
<tr>
<td></td>
<td>$\theta=2.5$,</td>
<td>$\theta=3$,</td>
<td>$\theta=2.5$,</td>
<td>$\theta=2.5$,</td>
</tr>
<tr>
<td>Rate (α)</td>
<td>a=.7, b=.3</td>
<td>a=.7, b=.3</td>
<td>a=.7, b=.3</td>
<td>a=.7, b=.3</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>------------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>6</td>
<td>2228.108</td>
<td>2229.156</td>
<td>1325.622</td>
<td>2219.975</td>
</tr>
<tr>
<td>7</td>
<td>2412.99</td>
<td>2414.194</td>
<td>1515.672</td>
<td>2403.989</td>
</tr>
<tr>
<td>8</td>
<td>2560.03</td>
<td>2561.365</td>
<td>1674.775</td>
<td>2550.324</td>
</tr>
<tr>
<td>9</td>
<td>2678.425</td>
<td>2679.87</td>
<td>1808.508</td>
<td>2668.143</td>
</tr>
<tr>
<td>10</td>
<td>2774.921</td>
<td>2776.459</td>
<td>1921.517</td>
<td>2764.165</td>
</tr>
<tr>
<td>11</td>
<td>2854.483</td>
<td>2856.099</td>
<td>2017.588</td>
<td>2843.334</td>
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<tr>
<td>12</td>
<td>2920.794</td>
<td>2922.477</td>
<td>2099.773</td>
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<tr>
<td>13</td>
<td>2976.616</td>
<td>2978.357</td>
<td>2170.52</td>
<td>2964.86</td>
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<tr>
<td>14</td>
<td>3024.043</td>
<td>3025.833</td>
<td>2231.792</td>
<td>3012.05</td>
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</tbody>
</table>