Appendices

Appendix 1

List of operators in Cartesian and cylindrical polar coordinates:
The scalar function ‘ψ’ and the vector function ‘A’ depend on the Cartesian and cylindrical polar coordinates \((x, y, z)\), \((r, \phi, z)\). If \(\hat{x}, \hat{y}, \hat{z}\) and \(\hat{r}, \hat{\phi}, \hat{z}\) are the unit vectors in Cartesian and cylindrical polar coordinates then the operators in Cartesian and cylindrical coordinates are defined as

**Cartesian coordinates**

Gradient: \(\nabla \psi(x, y, z) = \nabla_i \psi + \frac{\partial \psi}{\partial z} = \hat{x} \frac{\partial \psi}{\partial x} + \hat{y} \frac{\partial \psi}{\partial y} + \hat{z} \frac{\partial \psi}{\partial z}\)

Divergence: \(\nabla \cdot A(x, y, z) = \nabla_i \cdot A_i + \frac{\partial A_x}{\partial z} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\)

Curl: \(\nabla \times A(x, y, z) = \left\{ \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ \partial / \partial x & \partial / \partial y & \partial / \partial z \\ A_x & A_y & A_z \end{array} \right\} \) and \(\nabla \times A_i = \hat{z} \left\{ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right\}\)

Scalar Laplacian operator \(\nabla^2\): \(\nabla^2 \psi(x, y, z) = \nabla_i^2 \psi + \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\)

Vector Laplacian operator \(\nabla^2\): \(\nabla^2 A(x, y, z) = \hat{x} (\nabla^2 A_x) + \hat{y} (\nabla^2 A_y) + \hat{z} (\nabla^2 A_z)\)

Where

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]
Cylindrical polar coordinates

Gradient: \( \nabla \psi (r, \phi, z) = \nabla_i \psi + \hat{z} \frac{\partial \psi}{\partial z} = \hat{r} \frac{\partial \psi}{\partial r} + \hat{\phi} \frac{\partial \psi}{\partial \phi} + \hat{z} \frac{\partial \psi}{\partial z} \)

Divergence: \( \nabla \cdot \mathbf{A} (r, \phi, z) = \nabla_i \cdot \mathbf{A_i} + \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r \sin \phi} \frac{\partial}{\partial \phi} (\sin \phi A_\phi) + \frac{\partial}{\partial z} A_z \)

Curl: \( \nabla \times \mathbf{A} (r, \phi, z) = \left( \begin{array}{c} i \hat{r} \hat{\phi} \hat{z} \\ \frac{\partial}{\partial r} \frac{\partial}{\partial \phi} \frac{\partial}{\partial z} \\ A_r, r A_\phi, A_z \end{array} \right) \) and \( \nabla \times \mathbf{A} = \frac{\hat{z}}{r} \left( \frac{\partial (r A_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi} \right) \)

Scalar Laplacian operator \( \nabla^2 \psi \):

\( \nabla^2 \psi (r, \phi, z) = \nabla_i^2 \psi + \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} \)

Vector Laplacian operator:

\( \nabla^2 \mathbf{A} (r, \phi, z) = \hat{r} \left\{ \nabla^2 A_r - \frac{2}{r^2} \frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2} \right\} + \hat{\phi} \left\{ \nabla^2 A_\phi + \frac{2}{r^2} \frac{\partial A_r}{\partial r} - \frac{A_\phi}{r^2} \right\} + \hat{z} (\nabla^2 A_z) \)

here \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \)

Relation between the coordinates and unit vectors:

\( x = r \cos \phi, \quad y = r \sin \phi; \quad r = \left( x^2 + y^2 \right)^{1/2}, \quad \phi = \tan^{-1} (y / x) \)

\( \hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi; \quad \hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi, \)

\( \hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi; \quad \hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi, \)

\( \hat{r} \times \hat{\phi} = \hat{z}; \quad \hat{z} \times \hat{r} = \hat{\phi} \)

From the above relations

\( \frac{\partial \hat{r}}{\partial r} = 0; \quad \frac{\partial \hat{r}}{\partial \phi} = \hat{\phi}; \quad \frac{\partial \hat{\phi}}{\partial r} = 0; \quad \frac{\partial \hat{\phi}}{\partial \phi} = -\hat{r} \)
Appendix 2

Formation of LP modes:

The LP\(_{11}^e\) mode is formed due to beating between the HE\(_{21}^e\) and TM\(_{01}\) vector modes. The electric field of the two vector modes are

\[
e_{\text{HE}_{21}^e} = F_1(R) \{ \hat{x} \cos \varphi - \hat{y} \sin \varphi \} \exp(i\beta_1z)
\]

\[
e_{\text{TM}_{01}} = F_1(R) \{ \hat{x} \cos \varphi + \hat{y} \sin \varphi \} \exp(i\beta_2z)
\]

where \(\hat{x}, \hat{y}\) are the unit vectors, \(F_1(R)\) is the radial distribution of the field and \(\beta_1, \beta_2\) are the vector propagation constants of the HE\(_{21}^e\) and TM\(_{01}\) modes respectively. The vector and scalar propagation constants of the fiber modes are related by

\[
\beta_1 = \tilde{\beta} + \delta\beta_1, \quad \beta_2 = \tilde{\beta} + \delta\beta_2,
\]

where \(\tilde{\beta}\) is the scalar propagation constant and \(\delta\beta_i\) is the polarization correction to the \(i^{th}\) mode.

The electric field of the vector LP\(_{11}^e\) mode is sum of the two vector modes and is written as

\[
\text{LP}_{11}^e = (\{ \hat{x} \cos \varphi - \hat{y} \sin \varphi \} \exp(i\delta\beta_1z) + \{ \hat{x} \cos \varphi + \hat{y} \sin \varphi \} \exp(i\delta\beta_2z)) F_1(R) \exp(i\tilde{\beta}z)
\]

\[
\text{LP}_{11}^e = \{ \hat{x} \cos \varphi (e^{i\delta\beta_1z} + e^{i\delta\beta_2z}) + \hat{y} \sin \varphi (e^{i\delta\beta_1z} - e^{i\delta\beta_2z}) \} F_1(R) \exp(i\tilde{\beta}z)
\]

\[
\text{LP}_{11}^x = \{ \hat{x} \cos \varphi e^{-i(\delta\beta_1 - \delta\beta_2)x^2} + e^{i(\delta\beta_2 - \delta\beta_1)x^2} \} \times \\
\{ \hat{y} \sin \varphi e^{i(\delta\beta_1 - \delta\beta_2)x^2} - e^{-i(\delta\beta_2 - \delta\beta_1)x^2} \} \} \times \\
\exp(i\tilde{\beta}z)
\]

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\[ \text{LP}_{11}^\text{ex} = \{ \hat{x}\cos(\phi)\cos(\delta \beta_{21} z) + i\hat{y}\sin(\phi)\sin(\delta \beta_{21} z) \} F_1(R)\exp(i\beta' z) \]

where \( \delta \beta_{21} = (\delta \beta_2 - \delta \beta_1)/2 \) is half the correction between the polarization correction terms of \( \text{HE}_{21}^e \) and \( \text{TM}_{01} \) vector modes. \( \beta' = \tilde{\beta} + (\delta \beta_1 + \delta \beta_2)/2 \) is the vector propagation constant of \( \text{LP}_{11}^\text{ex} \) mode.

Similarly the other LP modes are written in terms of the vector propagation constants:

\[ \text{LP}_{11}^{\text{oy}} = \text{HE}_{21}^e - \text{TM}_{01} = \left( \begin{array}{c} -i\cos \phi \sin \delta \beta_{21} z \\ -\sin \phi \cos \delta \beta_{21} z \end{array} \right) F_1(R)\exp(i\beta' z) \]

\[ \text{LP}_{11}^{\text{ox}} = \text{HE}_{21}^o + \text{TE}_{01} = \left( \begin{array}{c} \sin \phi \cos \delta \beta_{41} z \\ -i\cos \phi \sin \delta \beta_{41} z \end{array} \right) F_1(R)\exp(i\beta' z) \]

\[ \text{LP}_{11}^{\text{oy}} = \text{HE}_{21}^{\text{od}} - \text{TE}_{01} = \left( \begin{array}{c} -i\sin \phi \sin \delta \beta_{41} z \\ \cos \phi \cos \delta \beta_{41} z \end{array} \right) F_1(R)\exp(i\beta' z) \]

**Appendix 3**

The electric field due to coherent superposition of orthogonal circularly polarized Gaussian beam and vortex beam is written as

\[
E_i = (\hat{x} + i\sigma_1 \hat{y})a \exp(-R^2) + (\hat{x} + i\sigma_2 \hat{y})R\exp(-R^2 + i\sigma_2 \varphi)
\]  

(A3.1)

The beam passing through the rotating analyzer gives the different projection of the field. The field projections in the new coordinate for \((\hat{x} + i\sigma_1 \hat{y})\) is
\[
\begin{pmatrix}
\hat{X} \\
\hat{Y}
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
1 \\
i \sigma_i
\end{pmatrix}
\] (A3.2)

\((\cos \theta + i \sigma_i \sin \theta) \hat{X} + (-\sin \theta + i \sigma_i \cos \theta) \hat{Y}\) since \(\sigma_i = \pm 1\), writing \(\sigma_i\) in the numerator or in the denominator will not effect the equation, then the above equation becomes

\((\cos \theta + i \sigma_i \sin \theta) \hat{X} + i \sigma_i (\cos \theta + i \sigma_i \sin \theta) \hat{Y}\)  (A3.3)

\((\hat{X} + i \sigma_i \hat{Y}) \exp(i \sigma_i \theta)\)  (A3.4)

Similarly for \((\hat{X} + i \sigma_i \hat{Y})\), it is, \((\hat{X} + i \sigma_2 \hat{Y}) \exp(i \sigma_2 \theta)\)  (A3.5)

The projected field in the new coordinates is (eqns. (A3.4 and A3.5))

\[E_0 = (\hat{X} + i \sigma_1 \hat{Y}) a \exp(-R^2 + i \sigma_1 \theta) + (\hat{X} + i \sigma_2 \hat{Y}) \Re \exp(-R^2 + i \sigma_2 \varphi + i \sigma_2 \theta)\]

At the singular point the real and imaginary components of the field is zero. By equating the real and imaginary components of the field to zero the above equation becomes

\[0 = a \exp(-R^2 + i \sigma_1 \theta) + \Re \exp(-R^2 + i \sigma_2 \varphi + i \sigma_2 \theta)\]

\[0 = a + \Re \exp(i (\sigma_2 \varphi + \sigma_2 \theta - \sigma_1 \theta))\]
List of publications related to thesis

Journals

Conferences

