Chapter 2

Generation and Characterization of Vector-vortex Beams

In this chapter, we first present the literature on different methods to generate cylindrical vector beams using active and passive optical components. In this context we present our experimental results on the generation of cylindrical vector-vortex beams using a two-mode optical fiber and switching between the different VV mode combinations. The VV beams are generated using a two-mode step-index optical fiber by coupling the Gaussian beam into the fiber as skew-offset beam. The vector nature of the output modes are analyzed by the rotating analyzer at the output end of the fiber and the vortex nature of the modes are identified by constructing a two-beam interferometer. The switching between the different VV beams are achieved (i) by changing the input launch angle to a diametrically opposite position in the fiber cross-section and (ii) by changing the input quarter wave plate to launch right and left circularly polarized light for both input launch angles. In this case the lateral and angular shifts of the four output beams are analyzed using the weighted average method. Further, the spatial polarization across the beam cross section of VV beams are manipulated by a combination of two HWPs at the fiber output end to get the pure cylindrical VV beams. We also qualitatively address the role of different experimental parameters such as the fiber length and numerical aperture (NA) on the generation of vector-vortex beams.
2.1 Introduction

In recent years the generation of azimuthally polarized TE$_{01}$ and radially polarized TM$_{01}$ or their combination received a lot of interest due to their spatial polarization properties. These beams are called cylindrical vector (CV) beams due to their cylindrical symmetry in the electric or the magnetic fields in the beam cross section. The linear combination of TE and TM modes are known as generalized vector beams. There are many active and passive methods to generate these CV beams detailed in Refs [19, 20]. In the active methods, the CV beams are generated inside a laser cavity using intracavity devices such as axial birefringent / dichroic components to provide the necessary mode discrimination. For example, by using a calcite crystal as intracavity axial birefringent material CV modes were generated by Pohl [57], and this method was extended further in [58, 59], and to improve the efficiency of generation [60, 61]. The other methods used to generate CV beams including using a conical axicon [62] Brewster angle reflector [63, 64], polarization sensitive cavity mirrors [65], and intra cavity interferometric method [66].

In the passive method, the CV beams are generated outside the cavity. In general, these methods convert the commonly known spatially homogeneous polarization into the spatially inhomogenous CV polarization. The CV modes are generated using birefringent or dichroic radial polarizers [67, 68], liquid crystal SLM [69], combining HG$_{01}$ and HG$_{10}$ modes through an interferometric method [70], space varying sub-wavelength gratings [20], several segmented $\lambda/2$ or $\lambda/4$ plate for a linear or circular polarized beams [71, 72]. Interestingly, optical fibers are used both as active and passive element to generate CV beams [31, 33, 73].
In all the methods discussed above, apart from the sub-wavelength grating, the generated CV beams don’t carry helical phase structure and have only the polarization singularities. The polarization singularities of these beams were also studied under the high focusing lens [74-76].

2.2 Generation of cylindrical vector beams using optical fiber

As discussed in Sec 1.2.2, the two-mode optical fiber supports six waveguide modes [23] of which the four 0th order guided modes (HE$_{21}^{0}$, TM$_{01}$, HE$_{21}^{0}$, and TE$_{01}$) are singular or annular modes. These modes are reminiscent of the CV beams due to their spatial polarization structure. The radial (TM$_{01}$) and azimuthal (TE$_{01}$) modes are cylindrically symmetric in polarization and the HE$_{21}^{x}$, HE$_{21}^{0}$ modes are hybrid in polarization. The power distributions of the fundamental and annular modes across the cross section of the fiber are different. For the fundamental mode optical power concentrated on the axis whereas for the annular modes the optical power is distributed in a doughnut shape around the axis. By illuminating the fiber with a Gaussian beam with an off-axis and tilt (Fig.2.1(c, d)) or the on-axis coupling of externally generated LG$_{01}$ beam into the fiber, the four annular modes are excited in the fiber with negligible fundamental mode.

By tuning the input Gaussian beam polarization and the input launch angle or by selecting the appropriate phase and polarization of the input LG$_{01}$ beam we can selectively excite the desired annular mode or the combination of annular modes out of four possible guided modes [31-33]. This is a simple and inexpensive method to generate CV beams and also the beams generated using this method are highly stable making it suitable for several applications.
Grosjean et al. [31, 32] generated these fiber modes using an all fiber device in which light from a single mode fiber is coupled into the two mode fiber through index matching liquid. By misaligning the two fibers, annular modes are excited. In this experiment it was observed that when the two fibers are shifted in the direction of the incident beam the TM$_{01}$ beam is generated whereas for a perpendicular shift TE$_{01}$ beam is generated and for other shifts HE$_{21}$ beam is obtained. Later on Volpe et al., [33] increased the purity and efficiency of the mode generation by coupling the externally generated Laguerre-Gaussian (LG) beam into the two-mode fiber through on axis coupling. When the input LG mode has opposite signs of circular polarization ($\sigma = \pm 1$) and helical phase structure ($l = \mp 1$), TE$_{01}$ and TM$_{01}$ modes with same efficiency are excited with a constant phase of $\pi/2$ in the fiber whereas for the same sign of circular polarization ($\sigma = \pm 1$) and helical phase structure ($l = \pm 1$), HE$^c_{21}$ and HE$^o_{21}$ modes are excited in the fiber with same efficiency and with constant phase of $\pi/2$. At appropriate lengths of the fiber the input LG beam is converted into generalized CV beams.

Figure 2.1: (a) Fiber cross section with coordinates and incident angle, $\theta_i$, (b) on-axis launching of the Gaussian beam (c) tilted ($\theta_t$) illumination of Gaussian beam (d) offset ($r_d$) illumination
In our method, we excite the different combinations of the guided modes in a two-mode step-index circular core optical fiber using a Gaussian beam from the He-Ne laser as skew-offset beam. By keeping the input end of the fiber at the focal plane of the microscopic objective lens and giving tilt and/or offset to the fiber or to the input beam (Fig. 2.1(c, d)) we selectively excite 0th order vector mode combinations in the fiber. Further, an optimum condition for the desire mode excitation is achieved by varying the input beam polarization and the fiber length. In this method, the coupling of the input Gaussian beam as off/set skew beam always excites vortex modes along with the vector modes. Hence, the output modes of the fiber have both vector and vortex nature. This method has significant advantage as far as generation of polarization singularities such as two C-points and a single C-point and switching between the different topological charges of the C-points in the output beam are concerned. The presence of two partial vortices in the output beam forms two C-points and the coherent superposition of the fundamental mode with the vortex modes form isolated C-point which are discussed in Chapters 3 and 4.

2.3 Experimental details
A schematic of the experimental setup used for controlled generation of cylindrical VV beams and switching between the different VV beams using linearly and circularly polarized Gaussian beam is shown in Fig. 2.2 (a). Partially polarized Gaussian (TEM$_{00}$) beam from the He-Ne laser ($\lambda = 632.8$ nm) pass through a Glan-Thompson polarizer (P) to obtain linearly polarized light. The linearly polarized light then passes through a half-wave plate (HWP) or a quarter wave plate (QWP) mounted on a rotation stage to enable adjustment of the polarization of the beam launched into the fiber. The beam
after passing through the polarization components is focused using a 0.25 NA 10x microscope objective lens (L₁) onto the cleaved end of the fiber, positioned using a three-axis ultra positioner stage. The V number of the circular core step-index optical fiber, calculated using the available fiber parameters is 3.805, implying that the fiber will support two LP modes, the fundamental LP₀₁ mode and the first higher-order LP₁₁ mode [23]. The TMF is kept horizontal with reduced twist, and bend along its length. By adjusting the position of the focused input beam with respect to the fiber axis we launch skew beam into the fiber. The angular tilt of the input beam to excite the vector-vortex modes are measured as follows: for linearly polarized input beam, starting from the maximum output intensity measured for on-axis Gaussian beam illumination the fiber input end is carefully adjusted for off-axis illumination using the x- and y-axis controls to get the desired LG₀₁ output beam. By measuring the half-width at half-maximum (HWHM) of the output Gaussian and LG beams directly using a CCD camera, connected through an IEEE 1394 card to the computer, positioned at 80 mm from the fiber tip we determine an angular tilt in the input beam of 30 μrad from the on-axis position to achieve the desired LG₀₁ output beam. The TMF output intensity is then collimated using a lens (L₂) and imaged using the CCD for data acquisition and analysis. A rotating analyzer (A) at the fiber output end is used to characterize the polarization content of the output beam for a fixed input beam polarization. A two-beam interferometer is constructed in parallel using two beam splitters and two mirrors (BS₁, BS₂ and M₁, M₂) to verify the presence of transverse and longitudinal OV in the output beam [29, 30].
Chapter 2

2.4 Results and discussion

2.4.1 Generation of vector-vortex beams

For the input HWP oriented at 0°, the vertical linearly polarized Gaussian beam is coupled into the fiber, by placing its cleaved end at the focal point of the objective lens (L1) and adjusting the fiber ‘x’ and ‘y’ positions to a particular angle such that skew beam is coupled into the fiber. The fiber output is a HG10 beam as shown in Fig.2.3 (a). Now by fixing the fiber input launch angle, the input HWP is rotated in steps of 5° from 0° to 90° and the resulting intensity patterns for every 10° are shown in Fig.2.3(a)–(j). Counterclockwise (CCW) rotation of the input beam polarization results in CCW rotating and a dramatically changing output beam pattern. As a function of the plane of
polarization of the input beam the output HG$_{10}$ beam (at $\theta = 0^\circ$) transforms into an LG$_{01}$ beam for $\theta = 35^\circ$ and a tilted HG beam for $\theta = 60^\circ$ before returning back to the HG$_{10}$ beam for $\theta = 90^\circ$. The behavior of the output intensity pattern repeats itself for larger rotation angles of the HWP. The behavior of the output beams for different input polarization with same input launch angle is due to the polarization selectivity of the guided vector modes [22, 23].

![Figure 2.3: Intensity pattern at the fiber output as a function of input HWP angle 'θ'.](image)

The polarization content of the HG and LG output beams are then analyzed by rotating the analyzer kept at the output end of the fiber. The HG$_{10}$ and LG$_{01}$ modes for every 45° of analyzer rotation are as shown in Fig.2.4. From the output beam behavior, it is clear that the output HG$_{10}$ mode is LP$_{11}$ mode and it is formed due to the beating between the HE$_{21}^+$ and TM$_{01}$ guided vector modes. For the LG$_{01}$ mode, the two-lobe pattern after the analyzer rotates in the same sense (CCW) as the analyzer. Further, for the analyzer at 0° the null line between the two lobes makes 45° in the clockwise direction (Fig.2.4). This behavior indicates that the fiber output is due to the linear combination of radially and azimuthally polarized TM$_{01}$ and TE$_{01}$ modes excited within the fiber with same phase.
Figure 2.4: Fiber output as function of analyzer rotation for HG\textsubscript{10} and LG\textsubscript{01} output beams and their corresponding edge and fork interference patterns.

The corresponding interference patterns for the two output beams shows the pure edge and screw dislocations as shown in Fig.2.4. The interference forklet pattern is due to the combination of TE and TM modes (partial vortices) to form the helical phase structure and hence the forklet pattern. The complete polarization information and the presence of partial vortices in the beam cannot be obtained from the analyzer rotation alone and we need special techniques such as Stokes parameter method and state of polarization map at each and every point of the beam, which we discuss in the Chapter 3. At the other input polarizations the output beams are due to the simultaneous excitation of different vector-vortex modes and they possess polarization singularities across the beam cross section because of the different propagation constants of the excited modes.

Further, we plot the measured intensity variation as a function of analyzer rotation (from 0° to 360°), for three output beams from the fiber as shown in Fig.2.5. The intensity variation of the HG\textsubscript{10} mode shows sinusoidal oscillation with maximum contrast for analyzer rotation as shown in Fig.2.5 (triangles) which confirms further that the output mode is linearly (x) polarized even (LP\textsubscript{11}ex) mode. For an in phase TE\textsubscript{01} and TM\textsubscript{01} mode combination the small
intensity variation of the two lobes as a function of the analyzer angle (Fig. 2.5) is possibly due to the variation in the state of elliptical polarization across the beam cross section. The resulting (linearized) polarization vectors for this mode are shown in Fig. 2.5 inset.

![Figure 2.5: Average beam intensity as a function of analyzer rotation angle for the different output beams. HG_{10} = 0° (open triangles), LG_{01} = 35° (filled squares), and tilted HG_{01} = 65° (open circles). The symbols are fitted to a sinusoidal curve. The beam patterns and the corresponding field vectors are given in the insets.](image)

When the HWP is oriented at 65°, the resulting tilted HG beam after passing through the rotating analyzer shows little rotation from its position along with the intensity variation. This behavior is due to the presence of the vector singularities (or disclinations) across the beam cross section. In this case we plotted only the intensity variations in the intervals 45°–135° and 225°–315°, of the analyzer orientation (Fig. 2.5, open circles) where the beam shift from its position is very less.

Now, changing the input launch condition by adjusting the fiber ‘x’ and ‘y’ positions for input HWP at 0° the output is a HG_{01} beam. The different output modes (HG_{10} and HG_{01}) for the same input polarization are possible due
to the change in the input cone angle of the skew-offset beam with respect to fiber axis [22, 23, 31]. Without changing the input launch angle, HWP is rotated upto 90° in steps of 10°. The output modes are as shown in Fig.2.6.

![Figure 2.6: Intensity pattern at the fiber output as a function of HWP rotation. (a) – (j) θ = 0° – 90° in steps of 10°.](image)

In this case, for the input HWP at 22.5°, the output beam passing through a counter clockwise rotating analyzer results in the two-lobe pattern rotating in opposite direction to the analyzer rotation as shown in Fig.2.7. This implies that the output beam is a combination of even and odd HE_{21} modes. The intensity variation in the two lobe pattern for different analyzer orientation is due to the simultaneous presence of the other fiber modes. In this case also the presence of the vortex character in the output beam shows up as up forklet.

![Figure 2.7: For a counter clockwise rotation of analyzer the two lobe pattern rotates in a clockwise direction and the corresponding interference fork pattern shows the helical phase structure.](image)

From the above two cases, it is clear that by changing the input launch conditions for the same input polarization two different mode combinations can be excited in the fiber. The rotation of the HWP for the two different launch
angles results in TE and TM modes excited for the HWP orientation at 35° with HG\(_{10}\) mode as the starting mode whereas even and odd HE\(_{21}\) mode combination is excited for the HWP at 22.5° with HG\(_{01}\) as the starting mode. In both the cases the presence of partial vortices in the output beam shows up as down and up forklet patterns.

### 2.4.2 Switching between different vector-vortex beams

The experimental method used to switch between the different vector-vortex beams is shown in Fig.2.2 (a, b). Starting from adjusting the fiber input end to achieve maximum output intensity corresponding to on-axis illumination of circularly polarized input Gaussian beam, the fiber position is carefully adjusted for an off-axis and tilted illumination using the x- and y-axes controls to selectively launch skew beam into the fiber and hence to get the desired ‘doughnut’ output beam for a fixed input beam polarization. In this case we measured the tilt angle using the back-reflected beam from the fiber input end, imaged using the CCD camera positioned at 137.3 cm. The measured angular tilt in the input beam are -27 \(\mu\)rad and 34.3 \(\mu\)rad respectively for doughnut output beams with respect to the Gaussian output beam, for the two input beam positions of I\(_1\) and I\(_2\) (Fig.2.2(b)).

Switching between the different VV beams is achieved using the input QWP: the QWP oriented at 45° with respect to the polarizer direction results in right-circular polarized (RCP) light which is first launched into the TMF at -27 \(\mu\)rad angle corresponding to I\(_1\) shown in Fig.2.2 (b). The output doughnut beam (row 1 of Fig.2.8) upon passing through a vertically oriented analyzer results in a two-lobe pattern with its null line making 45° in the clockwise (CW) direction. Rotating the analyzer axis in the counter-clockwise (CCW) direction
results in the two-lobe pattern rotating in the same sense (row 1 of Fig.2.8) indicating that the output doughnut beam is due to the linear combination of radially and azimuthally polarized \( \text{TM}_{01} \) and \( \text{TE}_{01} \) modes (\( \text{TM}_{01} - \text{TE}_{01} \)) excited with same phase in the TMF [23, 33]. A small variation in the lobe intensity as a function of the analyzer rotation is possibly due to the contribution from the residual fundamental \( \text{HE}_{11} \) mode excited simultaneously in the fiber. Interference of this beam with the reference beam shows a downward forklet (Fig.2.9 (e)) indicating the presence of vortex in the beam. The corresponding electric field pattern is shown in Fig.2.9 (a).

Without disturbing the fiber input launch condition the QWP is rotated to 135° and the resulting left circularly polarized (LCP) light excites different waveguide modes in the fiber as evidenced by the behavior of the output doughnut beam with respect to the rotating analyzer (row 2 of Fig.2.8). For the doughnut beam excited by the LCP Gaussian input light, the vertical analyzer axis orientation results in two lobes with vertical null line and a CCW rotation of the analyzer axis results in the two-lobe \( \text{HG}_{10} \) pattern rotating in the opposite, CW sense. This behavior of the output \( \text{LG}_{01} \) beam from the TMF is an odd \( \text{HE}_{21} \) hybrid eigen mode of the waveguide excited in the fiber (Fig.2.8 (b)). Now, interference of this vector beam with the reference beam, confirms the presence of on-axis screw dislocation via the appearance of a single downward forklet as shown in Fig.2.9 (f). From the two outputs (for QWP at 45° and 135°), it is clear that just by rotating the QWP between 45° and 135° the resulting right and left circular polarized input skew ray launched into the TMF results in characteristics due to the different modes excited in the fiber. Both the beams have the same helical structure (down forklet) due to the same skew angle for
the inputs but the different vector nature is due to the polarization selectivity of the fiber modes.

In addition we also explored the effect of changing the skew ray launch angle as discussed above with respect to the fiber axis and its effect on the characteristics of the generated beams. By adjusting the x and y-axis controls of the fiber input end we move the launch angle to 34.3 µrad with respect to the fiber axis corresponding to the skew ray I₂ shown by dotted line in Fig.2.2 (b). This launch condition results in a completely different behavior of the vector-vortex beams generated for RCP and LCP input beam. For the RCP input Gaussian beam the output doughnut beam, after passing through vertically oriented analyzer results in two-lobe pattern with a horizontal null-line (row 3 of Fig.2.8). Subsequent CCW rotation of the analyzer axis results in a CW rotation of the two-lobe pattern, characteristic of the even hybrid mode (HE_{21}^e) excited in the TMF. It is obvious that there is a \pi/2 phase difference between the two-lobe HG_{10} and HG_{01} beams corresponding to the I₁ and I₂ skew rays launched into the TMF for vertical orientation of the analyzer axis. In addition, the two-lobe intensity pattern rotates in CW for a CCW rotation of the analyzer, implying that the beam generated corresponds to even HE_{21} waveguide mode, as shown in Fig.2.9 (c). The interference of this beam with the reference beam results in an upward forklet (Fig.2.9 (g)) due to the skew ray I₂ launched from the opposite side of the fiber axis. Similar to the change in the forklet direction – from upwards to downwards or vice-versa – reported for a change in the direction of the reference beam [77] we report here the change in the direction of the input Gaussian beam with respect to the fiber axis for generating the LG_{01} beam results in the flipping of the forklet direction (from downwards to upwards) due to the excitation of the vector-vortex beams with
opposite topological charges. The reversal of the vortex charge sign for the two input beams $I_1$ and $I_2$ is simply due to an image inversion in geometrical optics, similar to the results of Molina-Terriza et al., [78].

<table>
<thead>
<tr>
<th>QWP</th>
<th>A</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>45°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>135°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_2$</td>
<td>45°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>135°</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2.8:** VV beams generated for two different QWP orientations (45° and 135°), and two input beam positions ($I_1$ and $I_2$) and the corresponding behaviour as a function of the analyzer axis orientation.

**Figure 2.9:** Row 1: Electric field distribution of the VV beams for the two different QWP orientations and two input beam positions. Row 2: the corresponding two-beam interference patterns.

Next, by keeping the input beam launch condition the same ($I_2$), rotating the QWP to 135° the behavior of the resulting LG$_{01}$ beam after passing through the rotating analyzer shown in row 4 of Fig.2.8 corresponds to the linear
combination of TM\textsubscript{01} and TE\textsubscript{01} waveguide modes (TM\textsubscript{01} + TE\textsubscript{01}) but with $\pi/2$ phase difference compared to that shown in row 1 of Fig.2.8. The resulting generalized vector-vortex beam rotation sense for CCW rotation of the analyzer axis is opposite to that reported earlier. The electric field pattern corresponding to the generalized VV beam is shown in Fig.2.9 (d). As before, the two-beam interference of this LG\textsubscript{01} VV beam with the reference beam results in a single upward directed forklet (Fig.2.9 (h)). Similar to what was mentioned earlier, changing the QWP orientation between 45° and 135° (corresponding to RCP and LCP input Gaussian light) it is possible to switch between the different VV beams respectively generated using a TMF. The sense of rotation of the vector-vortex beams for a fixed orientation of the QWP however is opposite for the two input launch conditions I\textsubscript{1} and I\textsubscript{2} with respect to the fiber axis indicating clearly that the reversal of the topological charge is simply due to the image inversion [77].

2.4.3 Weighted average analysis
We further analyze the changes observed in the output beam pattern for the four beams generated as a function of the change in the input beam polarization (from RCP to LCP) and as a function of the launch angle with reference to the fiber axis. Custom Matlab program was written to calculate the weighted average (centre of gravity) and the tilt angle of the beam. The weighted average of the doughnut beams were calculated by using

$$X_c = \sum x_i \frac{I(i,j)}{\sum I(i,j)} \quad ; \quad Y_c = \sum y_i \frac{I(i,j)}{\sum I(i,j)}$$

where $x_i$ and $y_i$ are the pixel numbers and $I(i,j)$ are the intensities of the corresponding pixel number. The X- and Y-axes are then drawn through the
centroïd of the doughnut beam as shown in Fig.2.10. We then find the average high intensity and average low intensity (null of the doughnut beam) pixels within the doughnut beam and then draw a straight line connecting the high and low intensity pixels. This line passing through the centroïd horizontal axes, with respect to which we calculate the slope. The slope is calculated using the formula \( \phi = \arctan \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \), where \((x_1, y_1)\) and \((x_2, y_2)\) are the pixel numbers of the low and high intensities of the doughnut beams respectively.

**Figure 2.10:** Weighted average analysis of the four output OVV beams

The analysis figures shown in Fig.2.10 (a – d) correspond to the beams shown in Fig.2.8. It is interesting to see that the doughnut beams (Fig.2.10 (a) and (b)) corresponding to the same input angle of -27 µrad have same slope of 82° but have different weighted average beam centers of (435, 374) and (461, 376) respectively for the LCP and RCP polarization of the input beam, corresponding to a change in the weighted centre of (26, 2). This implies that
there is a transverse shift in the output beam position with respect to the change in the input beam polarization but no angular rotation of the beam. However, when we changed the input beam angle to 34.3 µrad on the opposite side of the fiber axis, the output doughnut beams (Fig. 2.10 (c) and (d)) have the same weighted average centers of (433, 405) but now they have different slopes with respect to the horizontal axis. The slope changes from 49° to 67° for a change in the input beam polarization (from LCP to RCP), a difference of 18°. This corresponds to the fact that in this case, there is no transverse shift in the beam but the output beam rotates angularly while changing the polarization of the input beam.

2.4.4 Conversion of generalized vector-vortex beams to cylindrical vector-vortex beams

The conversion of generalized vector-vortex beams into the cylindrical VV beam is achieved using a combination of two half wave plates [79]. In principle a single HWP rotates the initial polarization state by 2θ, which for an inhomogeneous polarized input beam will give different amount of rotation to different parts of the beam. But the combination of two HWP results in a pure polarization rotator. This feature of using two HWP can be understood by using the corresponding Jones matrices. The total effect of the two wave plates is

\[
T = R(-\theta_2) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} R(\theta_2) R(-\theta_1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} R(\theta_1)
\]

where \( R(\theta_1) \) and \( R(\theta_2) \) are the rotation matrices for the two HWPs. After simplification, we get
From this it is clear that the combination of two HWPs works as a pure polarization rotator and irrespective of the initial polarization state of the input beam or its inhomogeneity in polarization, the polarization state of the output beam is rotated by the relative angle between the two HWP \( 2(\theta_2 - \theta_1) \) i.e., all the electric vectors are rotated by the same amount. The polarization state of the two generalized VV beams shown in the Figs.2.4 and 2.7, are converted into pure CVV beams using the two HWP combination. The output beams after passing through the two HWP combination is shown in Figs.2.11 and 2.12, for different relative angles. For the in phase combination of TE\(_{01}\) and TM\(_{01}\) beams, the relative angle between two HWP at +45° gives the pure azimuthally polarized TE\(_{01}\) beam and for a -45° it is converted into radially polarized TM\(_{01}\) beam. The corresponding two lobe patterns after the analyzer are as shown in Fig.2.11 (b). For the combination of even and odd HE\(_{21}\) modes the relative angle between the two HWP at +45° gives radially polarized TM\(_{01}\) beam and for -45° azimuthally polarized TE\(_{01}\) beam. The corresponding behaviors for the analyzer rotation are shown in Fig.2.11 (b).
2.5 Effect of other parameters on vector-vortex beam generation

In addition to the input beam polarization and the launching angle the generation of vector-vortex beams also depends on the fiber length and numerical aperture (NA) of the lens. Based on our experimental observations we qualitatively address these effects.
Chapter 2

2.5.1 Length of the fiber

The vector fiber modes are length dependent due to the polarization correction term to the scalar propagation constants which plays a crucial role in the generation optical VV beams. The ‘β’ values of the vector modes calculated using eqn. (1.28), are used to select the appropriate length of the fiber for the desired mode generation. The on axis launch of externally generated HG or the LG beam into the fiber, in which the input and output beams with same orientation and polarization is possible only at a particular length of the fiber where the beat length interms of the fiber eigen modes is equal to the fiber length. In these experiments the desired modes are obtained by cutting the fiber to appropriate lengths. In coupling the Gaussian beam into the fiber as off-axis skew beam, to within the first few millimeters of the fiber length the input Gaussian beam is converted into the fiber modes. After this length, the modes propagate in the fiber the same way as the coupling of the externally generated HG or the LG beams. The length dependence of the fiber modes in this case can be studied by fixing one input condition and cutting the fiber output end to get back to the starting mode or by tuning the input wavelength of the Gaussian beam (which effects the ‘β’ values of the modes) for a fixed fiber length and input conditions. Using second method we studied the propagation of modes in the fiber interms of the evolution of polarization singularities in Chapter 3.

2.5.2 Numerical aperture of the lens

The NA of the input lens effects the cone of light coupled into the fiber. For an on axis launching (Fig. 2.1 (b)) the NA of the lens which is close to the fiber NA will couple maximum light into the guided modes. In our experiments the effect of NA on the mode excitation is carried out for three different NAs. Since the
NA of the fiber used in this thesis is 0.20, the three different NAs used here are 0.10, 0.25, and 0.40 above and below the fiber NA. The diameter of the focused spot for the three different NAs are calculated using formula $w = \frac{4\lambda f}{\pi D}$, where $\lambda$ is the wavelength (632.8 nm), ‘f’ focal length of the lens (25.4mm, 16.5mm and 9.0mm) and ‘D’ the beam diameter incident on the lens (2.5mm). The measured diameters of the focused spot for the three different NAs are: 8.1\(\mu\)m, 5.3\(\mu\)m and 2.8\(\mu\)m respectively with reference to the core diameter of the fiber 3.8 \(\mu\)m.

The effect of NA on the output beam is understood by adjusting input conditions for the same output beam for three different NAs by adjusting the x and y positions of the fiber for the same input polarization. For the input HWP at 0° the output beams for the three different NAs are as shown in Fig.2.13 (b). The intensity difference between the two lobes is more for the input NAs of 0.10 and 0.25 whereas for 0.45 the variation is small. This is probably due to the different spot sizes due to the lens: bigger spot sizes exciting more fundamental mode than the smaller spot sizes along with the vector modes and the beating between these modes results to the intensity variation between the two lobes [23]. By changing the input polarization for the same input launch conditions, to HWP at 67.5° the output beams for all the three NAs are shown in Fig.2.13 (c).
Figure 2.13: (a) Different NA of the objective lens used to couple light into the TMF (b) Starting modes for different NAs. (c) Output mode for the input HWP at 67.5° (d) Output beams for different NAs analyzer orientation.

For the output beams passing through the clockwise rotating analyzer, the two lobe pattern rotates in the same way for all the three NAs shown in Fig.2.13 (d). From the analyzer rotation the behavior of the output beam is a combination of out of phase TE\textsubscript{01} and TM\textsubscript{01} modes. The intensity distribution across the beam cross section is different for the three different NAs and is also reflected in the analyzer data as the intensity variation in the two lobes. The intensity variation in the two lobes after the analyzer is more for the 0.10 and small for the 0.45 NA. This behavior is strongly connected with our starting mode which has different intensities in the two lobes. The lens with higher NA than the fiber has the advantage in order to suppressing the effect of fundamental mode in the output beam.
2.6 Summary

In this chapter, we presented our results on the vector-vortex beams generated using short length of two-mode step-index fiber by coupling Gaussian beam as off-set skew beam. The positive and negative cone angle of the input beam with respect to the fiber axis excite different mode combinations and opposite helical charges. For the output beam passing through the rotating analyzer the polarization content of the output beams are shown as linear vectors are as approximation to elliptically polarized output as will be shown using Stokes polarization measurements. Analyzer rotation alone can not give the complete information about the polarization of electric vectors across the beam cross section for which we need some other techniques to measure the state of polarization across the beam at every point. In the next chapter we present Stokes parameter techniques to measure the presence of partial vortices and the state of polarization at every point in the beam to better understand the beams generated from the TMF under different conditions.