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CHAPTER - 2

METHODOLOGY FOR MODEL DEVELOPMENT

2.1. Introduction

ANSYS™ is a general purpose finite element program. Many different types of problems may be analyzed using the codes, some examples would include: structural, thermal, magnetic field, electric field, coupled field and fluids analysis. ANSYS™ is also a self contained program in which the Pre-processor (model definition), Processor (solver), and Post-processor (output) are all integrated into a single code. All of these components may be accessed either through a graphical user interface or through execution commands.

2.2 Technical Approach for Model Development

The friction and efficiency prediction model consists of three main components: (i) a gear contact analysis model, (ii) a friction coefficient model, and (iii) a gear pair mechanical efficiency computation formulation.

The gear contact analysis model uses the gear design parameters, machine setups and cutter parameters, operating conditions and errors associated with assembly, mounting and manufacturing of the tooth profile to predict load and contact pressure distributions at every contact point at each incremental mesh position.

Predicted load and contact pressure distributions together with geometry, kinematic parameters, surface finish, and lubricant parameters are input to a friction coefficient model to determine friction coefficient $\mu(z,0, \phi_m)$ of every contact point $(z,0)$ on the gear tooth surfaces, where $z$ and $0$ are the gear surface parameters and $\phi$ is the gear rotation angle. $\mu(z,0, \phi_m)$ is then used by the mechanical efficiency computation to determine instantaneous mechanical efficiency $\eta(\phi_m)$ of the gear pair at the $m$-th incremental rotational positions $(m=1,2,3,\ldots,M)$ spread at an increment $\Delta\phi$ ($\phi_m =$...
mΔϕ) to cover an entire mesh cycle. The instantaneous mechanical efficiency values \( \eta(ϕ_m) \) are then averaged over a complete mesh cycle to obtain the average mechanical efficiency of the gear pair.

### 2.3 Gear Contact Analysis Model

For computation of the contact loads of parallel-axis gears, a load distribution model that was developed by Houser [73] which was initially proposed by Conry and Seireg [74] is used. This model is designed to compute elastic deformations at any point of the gear surface given the tooth compliance, applied torque, and the initial tooth separations under no load. It assumes that the elastic deformations are small, and hence, the macro-level gear geometry remains unchanged so that the tooth contact is contained in the near vicinity of the contact line, even after the deformations. The formulation of this model is described in Chapter- 4, entitled application of model to parallel axis gear pairs.

For computation of contact load or pressure distribution of cross axis gear, a commercially available finite element based gear analysis package Calyx developed by Vijayakar [75] is used. This gear analysis model combines finite element method and a surface integral approach [76]. The contact analysis model has a special setup for finite element grids inside the contact zone. A set of very fine contact grid is defined automatically on hypoid gear teeth to capture the entire contact zone. This model also allows plug-ins developed by users so that the mechanical efficacy formulation can be linked to the gear contact analysis model. The formulation of this model is described detail in Chapter- 5, entitled application of model to cross axis gear pairs.

### 2.4. Gear Friction Coefficient Model

When gears are in contact, load distribution along the contact lines on consecutive gear teeth can be obtained by the gear contact model is illustrated schematically in Figure 2.1(a). Here, the load distribution along contact lines is discretized [73]. As most of the relevant parameters along the contact lines vary, including the rolling and
sliding velocities, radii of curvature, contact pressure (or load per unit contact length), and surface roughness, the friction coefficient should vary along the contact lines as well. Since it is not practical to analyze every possible contact point for its coefficient of friction value, a discretization such as the one shown in Figure 2.1(b) is required [73]. A more refined load distribution (more discrete segments) would result in more accurate predictions at the expense of more computational time.

Figure: 2.1 (a) Gear contact model representing gear load distribution
(b) The equivalent cylindrical contact representing contact segment q
At every instant in mesh, there can be one or several teeth in contact depending on the total gear contact ratio. Each contact line consists of several segments, and each segment is represented by the contact characteristics of a point located in the middle of the segment. The gear contact at each one of these discrete segments can be equated to the contact of two equivalent cylinders with radii of curvature $r_1$ and $r_2$, normal load $w$, and surface velocities $u_1$ and $u_2$ as shown in Figure 2.1(b) [73]. This is equivalent to a cylinder with a sliding velocity $u_1 - u_2$ and effective radius of curvature $R = r_1 r_2 / (r_1 + r_2)$ in contact with a flat surface. The sliding velocity is required to be perpendicular to the contact segment and the two contacting surfaces are usually separated by a certain amount of lubricant film. As a result, a fluid pressure and viscous shear are generated to cause friction along the contacting surfaces. This friction traction can be estimated by using published empirical friction coefficient formulae at each contact segment or analyzed by using a line contact elastohydrodynamic lubrication model.

A large number of empirical formulae for coefficient of friction can be found in the literature. Most of these formulae were obtained by curve fitting for measured data collected from twin-disk type tests. They have the following general form:

$$\mu = f(v_k, v, V_s, V_r, R, W', P_{\text{max}}, S, \ldots)$$

(2.1)

Where, $v_k$ and $v$ are the respective kinematic and dynamic viscosities of the lubricant, both of which are measured at the oil inlet, and are functions of inlet oil temperature at ambient pressure. Parameters $V_s$, $V_r$ and $R$ denote relative surface sliding velocity, sum of the rolling velocities and the combined effective radius of curvature, respectively. The load parameters are the unit normal load $W'$ and the contact pressure $P_{\text{max}}$. $S$ is a surface finish parameter.

A representative set of commonly cited friction coefficient formulae are listed in Table 2.1 are used for prediction of coefficient of friction. Several other empirical formulae are also available in the literature [5-7]. By examination of Table 2.1 indicates that these formulae are quite different from each other in terms of the parameters that are the parameter ranges represented. For instance, formula by Misharin [18] and Drozdov and Gavrikov [21] do not include any surface roughness.
parameter, and hence, influence of varying surface roughness cannot be studied using these formulae. The remaining three formulae in Table 2.1 consider a surface parameter $S$ that is either the root mean square (RMS) or the centerline average (CLA) of the roughness profiles. Similarly, formulae by Drozdov and Gavrikov and Misharin also exclude the radius of curvature $R$. In ISO TC 60 [43] formula does not include $V_s$. Similarly, the formulae of O’Donoghue and Cameron [20] and Misharin are not a function of the normal load ($W'$ or $P$) and hence they cannot account for any load effects on coefficient of friction. Also, Drozdov and Gavrikov formula uses the maximum Hertzian pressure as the load parameter, while in the formula of Benedict and Kelley [19] considers the load per unit length as the load parameter. In addition, each formula listed in Table 2.1 is valid within certain ranges of key parameters. All these formulae will be used in this study with special attention given to their parameter sets and ranges of applications.

<table>
<thead>
<tr>
<th>Authors and Formulae</th>
<th>Application Ranges</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misharin [18]</td>
<td>$\frac{V_s}{V_r} \in [0.4, 1.3]$</td>
<td>$V_s, V_r$: m/s</td>
</tr>
<tr>
<td></td>
<td>$P \geq 2500$ kg/cm$^2$</td>
<td>$V_k$: centistokes</td>
</tr>
<tr>
<td></td>
<td>$\mu \in [0.02, 0.08]$</td>
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<tr>
<td>Benedict and Kelley [19]</td>
<td>$\log_{10} \left[ \frac{50}{50-S} \right] \leq 3$</td>
<td>S: $\mu$m, RMS</td>
</tr>
<tr>
<td></td>
<td>$V_s, V_r$: in/s</td>
<td>$W'$: lbf/in</td>
</tr>
<tr>
<td></td>
<td>$R$: in</td>
<td>$V_s, V_r$: in/s</td>
</tr>
<tr>
<td></td>
<td>$v$: centipoises</td>
<td></td>
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<tr>
<td>O’donoghue and Cameron [20]</td>
<td>$\mu=0.6 \left[ (S+22)/25 \right] \left[ \frac{V_s^{1/6} V_r^{1/6} R^{1/2}}{S} \right]^{-1}$</td>
<td>S: $\mu$m, CLA</td>
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<tr>
<td>Drozdov and Gavrikov [21]</td>
<td>$\nu_k \in [4, 500]$</td>
<td>$V_s, V_r$: m/s</td>
</tr>
<tr>
<td></td>
<td>$V_s \leq 15, V_r \in [3, 20]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_{\text{max}} \in [40000, 200000]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_k \in [4, 500]$</td>
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<tr>
<td></td>
<td>$V_s, V_r$: m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_{\text{max}} = \text{kg/cm}^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_k$: centistokes</td>
<td></td>
</tr>
<tr>
<td>ISO TC60 [43]</td>
<td>$\mu=0.12 \left[ W' S/R V_r \right]^{0.25}$</td>
<td>S: $\mu$m, RMS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$W'$: N/mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_r$: m/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R$: mm</td>
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<td>$S$: $\mu$m, RMS</td>
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<td>$W'$: N/mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$v$: centipoises</td>
</tr>
</tbody>
</table>

Table 2.1: Published empirical friction coefficient formulae
For calculation of friction coefficient using elastohydrodynamic lubrication model, a deterministic elastohydrodynamic lubrication model proposed by Cioc, et al [77] will be used. In this model, the distribution of the film pressure and thickness across an elastic lubricated contact can be obtained by solving the transient Reynolds equation simultaneously with a number of other equations including the film thickness equation, viscosity-pressure-temperature relationship, density-pressure-temperature relationship, the energy balance equation and the load equation.

The load distribution across the contact segment is given by [77]

\[
\bar{W} = \int_{X_{in}}^{X_{out}} [P(X, \bar{t}) + P_c(X, \bar{t})]dX
\]  
(2.2)

where \(\bar{W}\) is the dimensionless unit load applied, \(X_{in}\) and \(X_{out}\) are the inlet and outlet of computational domain for the contact zone, \(P_c\) is the asperity contact pressure.

The friction traction force per unit width of the contact is given by [77]

\[
F_t = F_r + F_s
\]  
(2.3)

where \(F_r\) is the rolling friction per unit width of the contact that comes from the viscous flow in the formation of the fluid film thickness and squeeze motion during pure rolling,

\[
F_r = - \int h(x) \frac{dp}{dx} dx
\]  
(2.4)

and \(F_s\) is the sliding friction per unit width of the contact that is caused by fluid shear and the asperity contact, respectively [78],

\[
F_s = \int v(x) \frac{U_2x - U_2x}{h(x)} dx + \int \mu_s p(x) dx
\]  
(2.5)
where \( v(x) \) is viscosity, \( U_1 - U_2 \) are the surface velocities of two contacting surfaces, \( h(x) \) fluid film thickness distributions across the lubricated contact zone, \( \mu_s \) is the coefficient of friction between the contacting asperities and \( p(x) \) is the pressure.

Finally, the coefficient of sliding friction of the entire section of contact is given by

\[
\mu_s = \frac{F_s'}{W'}
\]

(2.6)

where \( W' \) is the applied normal load per unit width of contact.

The approach outlined above to compute coefficient of friction is based on principle of physics. One disadvantage of this approach is that it requires significant computational effort since a large number of individual elastohydrodynamic lubrication analysis must be carried out for each discretized segment along the lines of contact at each rotational gear position considered.

### 2.5 Gear Efficiency Computation Model

Once the coefficient of friction \( \mu(z,0,\phi_m) \) at each contact point \( (z,0) \) at each rotational angle \( \phi_m \) (\( m = 1, 2, 3, \ldots, M \)) are known, the sliding friction force at each contact position can be calculated by

\[
F_f(z,0,\phi_m) = \mu(z,0,\phi_m) W(z,0,\phi_m)
\]

(2.7)

where \( W(z,0,\phi_m) \) is the normal load at the intersect of contact point. The rolling friction force \( F_r \) can be obtained by multiplying \( F_r \) in Equation 2.4 with the width of the contact. When the empirical friction formulae listed in Table 2.1 are used, \( F_r \) can be estimated from an isothermal elastohydrodynamic lubrication contact [55] as

\[
F_{ro} = 4.318(G\hat{U})^{0.658}Q^{-0.0126}R/\alpha
\]

(2.8)

where \( \hat{G} = \alpha E' \) is the dimensionless material parameter, \( \hat{U} = v_0 (u_1+u_2)/E'R \) is the dimensionless speed parameter, \( \hat{Q} = W'/(E'R) \) is the dimensionless load parameter,
R is the effective radius of curvature and $\alpha$ is the pressure viscosity coefficient. In account for the effect of temperature rise at high speed conditions, a thermal reduction factor $\varphi_T$ can be used to modify this isothermal formula such that

$$F_r = \varphi_T F_{ro}$$

(2.9)

where $\varphi_T$ is defined in reference [55] as

$$\varphi_T = \frac{(1-13.2(\frac{P_h}{E^*}) (L^*)^{0.42}}{1+0.213 (1+2.23SR^{0.83})(L^*)^{0.64}}$$

(2.10)

where

$$L^* = \frac{\partial \nu}{\partial t_o} \left( \frac{\nu_e}{K_f} \right)^2$$

(2.11)

Here, $\nu$ is the absolute viscosity in cPs, $t_o$ is the lubricant temperature at inlet in $^0C$, $K_f$ is the lubricant thermal conductivity in W/m $^0C$, SR is the slide-to-roll ratio that is defined as $SR = 2(u_1-u_2)/(u_1+u_2)$ and $\nu_e = (u_1+u_2)/2$.

The instantaneous efficiency of a gear pair is defined as the ratio of the instantaneous output power to the input power

$$\eta(\psi_m) = \frac{L_{out}(\psi_m)\omega_{out}}{L_{in}\omega_{in}}$$

(2.12)

where $L_{out}(\psi_m)$ and $L_{in}$ are the values of torque acting on the output and input gears and $\omega_{out}$ and $\omega_{in}$ are the output and input rotational speeds, respectively. When only frictional losses are considered, the instantaneous output power can be written as the difference between the input power and the frictional power losses and hence the mechanical efficiency can be expressed as

$$\eta(\psi_m) = 1 - \frac{1}{L_{in}\omega_{in}} \sum_{q=1}^{Q} [ |F_t(u_1 - u_2)| + |F_r(u_1 + u_2)| ]_q$$

(2.13)
where $\sum_{q=1}^{Q}[|F_r(u_1 - u_2)| + |F_r(u_1 + u_2)|]_q$ is the sum of the sliding frictional and rolling frictional power losses at each rotational angle $\phi_m$ and $Q$ is the total number of load segments at the same position.

### 2.6 Other Sources of Power Losses in a Gear Pairs

The power losses in a gearbox are comprised of two types of losses: load dependent (mechanical) and no-load dependent (spin) power losses. The mechanical losses consist of tooth friction and bearing losses and are comprised of rolling and sliding friction generated on gear and bearing surfaces due to torque transmitted through the system. The spin power losses consist of churning and windage losses and are associated with fluid gear interaction.

#### 1. Windage Losses

The windage losses are dependent on many parameters such as the width and diameter of the gear blank, speed, gear blank/rim geometry, the shape of the housing, type of the lubrication method (dip or jet lubrication), and the operating temperature and viscosity of the oil. Dudly [79] proposed the following formula to estimate the windage loss $P_w$ of a gear having an outside diameter $D_o$ and face width $B$ as

$$P_w = 10^{-17} N^3 D_o^5 B^{0.7}$$

(2.14)

where $P_w$ is the horsepower loss due to windage, $N$ is the speed in rpm, $D_o$ and $B$ is the gear outside diameter and face width respectively.

Townsend [80] proposed another formula to determine windage losses as

$$P_w = 10^{-20} \Phi \lambda N^{2.9}(0.16D_l^{3.9} B^{0.75} m^{1.15})$$

(2.15)

where $\Phi$ is an oil mixtype function that indicates the state or type of the atmosphere within the gear unit ($\Phi=1$ for an oil free atmosphere), $\lambda$ is a gearbox space function.
(λ = 1 for free space; λ = 0.6 to 0.7 for large enclosure and λ = 0.5 for a fitting enclosure),

$D_r$ is the root diameter and $m$ is the module, both in mm.

Another similar empirical formula for windage losses was proposed by Anderson and Loewenthal [38] as

$$P_w = 2.82\times10^{-7} \left(1+2.3\frac{B}{R_p}\right) N^{2.8} R_p^{4.6} (0.028v + 0.019)^{0.2}$$

(2.16)

where $B$ and $R_p$ are face width and pitch radius in meter, $N$ is the rotational speed in rpm, $v$ is the dynamic viscosity in centipoises. These formulae are valid for spur gears. For helical gears, Dawson [67] provided a correction factor that describes the relationship between helix angle and the power loss as percentage of equivalent spur gears.

For windage power loss on gear side Seetharamen [81] proposed a formula as

$$P_{slide} = \frac{15000}{N_{1000}} \left[\frac{N}{1000}\right]^3 \left[\frac{D}{2.54}\right]^5$$

(2.17)

where $N$ is the shaft speed in rpm.

For peripheral windage power loss Diab, et al [82] proposed a formula as

$$P_{perip} = \frac{1}{2} C_m \rho \omega^3 R^5$$

(2.18)

where $\rho$ is the fluid density in kg/m$^3$, $\omega$ is the rotational speed in rad/s and $R = D/2$ the gear outer radius in m.

The total windage power loss is then recovered by

$$P = P_{perip} + P_{slide}$$

(2.19)

Petry- Johnson, et al [83] proposed a formula for calculating windage power loss as
P_{windage} = 0.025 \pi \omega^{2.86} r^{4.72} \mu_D^{0.14} \quad (2.20)

where \( \omega \) is the rotational speed, \( r \) is the gear pitch radius, \( \mu_D \) is the dynamic viscosity.

2. Churning Losses

The churning losses result from gear blanks revolving in the oil sump and generating splash for lubricating the gear teeth, bearing, and seals. These losses are a function of speed and oil level, oil viscosity and temperature, tooth geometry and submerged depth of the gears [34].

Luck and Olver [69] were proposed several empirical formulae to estimate the oil churning losses. British Standard BS 14179 [84] provides a formula for oil churning losses for a smooth outside diameter, smooth side faces, and for the tooth surfaces as

\[
P_{ch} = \frac{7.37 F_g \nu_k N^3 D_o^{0.7} L}{A_g^{10^{26}}} \quad (2.21-a)
\]

\[
P_{ch} = \frac{1.47 F_g \nu_k N^3 D_o^{4.7}}{A_g^{10^{26}}} \quad (2.21-b)
\]

\[
P_{ch} = \frac{7.37 F_{g,1} \nu_k N^3 D_o^{4.7} B_i \left( \frac{R_f}{\tan \beta_n} \right)}{A_g^{10^{26}}} \quad (2.21-c)
\]

In Equation 2.21, \( F_g \) is the dip factor (\( F_g = 0 \) for gear is not dipped into oil, and \( F_g = 1 \) for gear is fully dipped into oil), \( \nu_k \) is the kinematic viscosity at operating temperature, \( N \) is the rotational speed, \( R_f \) is a roughness factor, \( \beta_n \) is the normal helix angle and \( A_g \) is the arrangement factor. In Equation 2.21 (c), \( i = 1,2 \) for pinion and gear respectively.

Also Changenet and Valex [85] were proposed a formula for determine churning losses at high speed as
\[ C_m = 3.4 \left( \frac{h}{R_p} \right)^{0.1} \left( \frac{V_0}{D_p} \right)^{-0.85} F_r^{-0.88} \left( \frac{b}{D_p} \right)^{0.85} \]  \quad (2.22)

where \( h \) is dynamic immersion height of gear, \( R_p \) is pitch radius, \( V_0 \) is oil volume, \( D_p \) is pitch diameter and \( b \) is face width.

3. Bearing Friction Losses

Rolling element bearing losses results from various sources [70]:

i) Rolling friction due to elastic hysteresis and deformation at raceway contacts,

ii) Sliding friction from unequal curvatures in contact areas, sliding contact of cage with rolling elements and guiding surfaces, sliding between the ends of the rollers and ring flanges and seal friction,

iii) Lubricant friction due to viscous shearing on rolling element, cage and raceway surfaces, and

iv) Churning and working of lubricant dispersed within the bearing cavity.

The most widely accepted bearing efficiency formulation given by Harris [86] is based on the original formulations of Palmgren. Bearing friction moment of a rolling element bearing without a seal can be written as

\[ M_b = M_p + M_L \]  \quad (2.23)

where \( M_p \) is the load dependent moment resulting from rolling and sliding friction in loaded rolling contacts

\[ M_p = 0.5 \mu_b W_b d_{bore} \]  \quad (2.24)

Here, \( M_p \) is calculated at the bearing bore radius of \( 0.5d_{bore} \), where \( d_{bore} \) is the bearing bore diameter in mm, \( \mu_b \) is the coefficient of friction for the bearing, and \( W_b \) is the
bearing load in N. The $M_L$ (in N-mm), accounts for viscous friction and related cage friction in rolling element pockets, and at cage guiding surfaces given as [86].

$$M_L = \begin{cases} 10^{-7} f_L (v_k N)^{2/3} D_m^3 & v_k N > 2000 \\ 1.60 \times 10^{-5} f_L D_m^3 & v_k N \leq 2000 \end{cases} \quad (2.25)$$

Where, $f_L$ is a factor depending on bearing type and method of lubrication, $N$ is the bearing speed in rpm and $D_m$ the mean diameter of bearing in mm.

Finally, the total bearing power losses can be obtained by multiplying the total frictional moment in equation (2.23) with the bearing angular velocity as $P_b = M_b \omega_b$. 