1. Introduction

1.1 Review of the study on electromagnetically induced transparency (EIT):

Electromagnetically Induced Transparency (EIT) [1] describes transparency (nonabsorption) of a weak optical (probe) field at a resonant transition in three level medium due to interference effect induced by a relatively strong (control) field applied at an adjacent transition. The foundations of EIT were laid by Kocharovskaya and Khanin [2] in 1988 and independently by S. Harris in 1991 [3]. Since then numerous experimental and theoretical publications devoted to the study of EIT have appeared and there is considerable activity in this field in recent times.

A physical mechanism underlying the EIT phenomenon is the destructive atomic interference between alternate laser-induced atomic transition amplitudes (or atomic coherences) for a given transition process. The condition behind the EIT is a two-photon (or Raman) resonance. Some other physical explanations for EIT are (i) coherent population trap (CPT) analogy [4] (ii) Dressed state analysis (The CPT analogy and the dressed state explanation for EIT can be related to each other [5] (iii) Multiple routes to excitation, (iv) other alternatives for explaining the EIT process have been proposed such as the use of Feynman diagrams to represent the interfering processes [6], use of a 3D vector model[7] or methods involving stochastic wave function diagrams[8].

Several interaction schemes of atomic levels and laser fields are suggested and experimentally demonstrated the EIT effect, such as a Λ- scheme [3,9] where two lower levels are coupled by two fields with a single upper level, a V- scheme [10], where one ground level is coupled with two higher levels, a ladder Ξ or cascade scheme [11], in which the upper level is coupled with the ground level through an intermediate one, and many others utilizing more than two fields coupling higher numbers of levels. Above proposed three level schemes can be realized in gases or vapors [3, 12], semiconductors [13] and in solid media. [14]

The possibility of controlling the transparency of a medium has a useful application in lasing without inversion [15] and the dispersion property of probe field is also equally interesting
as the steepness of the dispersion function [16] near resonance plays the key role for reduction of group velocity of light pulse (slow light) in the medium and is directly related to the line width of the EIT resonance [17]. EIT can be used as an enhancing non linear optical process including non linear frequency conversion [18]. In addition, EIT can be used in squeezed-light generation [19] and low-light-level photon switching [20]. A very narrow EIT resonances, which is an order of Hz is observed in cell with buffer gas atoms [21] and antirelaxation coating cells [22]. These narrow EIT resonances are used in sensitive magnetometer [23], precision spectroscopy [24] and frequency standards [25]. It is impossible to mention all of the hundreds of papers devoted to the study regarding the EIT and its possible applications. More information about EIT can be found in the review articles on EIT [26, 27] and references therein.

1.2 Motivation:

Narrow EIT resonances are being used as the basis for making very sensitive measurements, like sensitive magnetometers [23], precession spectroscopy [24], frequency standards [25], and also in metrology field. Hence the study and identification of various mechanisms that lead to broadening of EIT resonance has attracted considerable attention in recent years. Many broadening mechanisms contribute to the width of the EIT resonance such as, i) time of flight broadening (atoms with a non-zero velocity perpendicular to the direction of propagation of the pump and probe beams will traverse the pump and probe beams in a finite time). ii) Residual Doppler broadening owing to either wave vector mismatch of pump-probe and iii) finite angular separation between the pump-probe beams and power broadening by coupling field intensity [28] etc.. However, no or minimal residual Doppler broadening is observed in EIT experiments performed with the addition of a buffer gas in vapor cells [21, 29]. The measured EIT (or CPT) resonance line widths in these experiments are found to be much narrower, about two orders of magnitude lower than the expected residual Doppler widths. Experimentally very low group velocities and large group delays, via EIT in vapor cells containing buffer gas have also been observed [30]. The existing theoretical treatments of EIT and slow light however do not properly account for various broadening mechanisms mentioned above. It would be of interest and importance to develop a more general treatment of EIT and ultraslow light propagation and related nonlinear optical phenomena including various broadening mechanisms and effects of buffer gas.
Our aim here is to develop a general theory of electromagnetically induced transparency (EIT) and examine various associated phenomena such as slow group velocity, nonlinear generation etc. in inhomogeneously broadened multilevel systems of various types. Different schemes such as ladder system (Ξ), V-system (V), and lambda system (Λ) will be considered for the study of the EIT and other associated phenomena. In gaseous media (such as atomic vapors) comprising the abovementioned multilevel systems, Doppler broadening of various transitions - one photon, two-photon can occur. Collisional effects of a buffer gas (with vapor atoms) such as velocity changing collisions, dephasing collisions and collisional shift of the ground state and optical resonances will also be taken into account. It is expected that velocity changing collisions would cause significant narrowing of EIT line widths which could lead to many interesting and significant effects such as complete transparency and ultraslow light generation at relatively low coupling field intensities and vapor densities. Simultaneously, a four wave mixing (FWM) scheme in a double lambda system using EIT technique will also be studied.

1.3 **EIT: Broadening effect**

The term broadening is used to denote the finite spectral width of the resonance of atomic system to electromagnetic fields. Broadening can be classified as two categories 1) homogeneous broadening and 2) inhomogeneous broadening. The cause of homogeneous broadening in EIT is due to spontaneous transitions or nonradiative transitions, broadening due to the interaction with an electromagnetic field (power broadening), phase perturbing (elastic) collisions, laser line *widths etc.* Physically this would correspond to experiments carried out in either a ‘cold’ atomic sample (Bose Einstein condensate, optical trap) or in an atomic beam. An Inhomogeneous broadening occurring in EIT resonance is due to Doppler shift of atomic resonance (Doppler broadening) caused by thermal motion of atoms in a gas medium or impurity ions in a host crystal. For example, if an experiment is carried out in a gas cell, we must also include the effects of Doppler broadening as we would expect that Doppler broadened line shape is much broader than that of the homogeneous line shape. Particularly Doppler broadening affects both EIT resonance and its width in lambda (Λ) and ladder (Ξ) schemes, but EIT can still be observed in these schemes however, it requires a larger coupling field Rabi frequency as compared to cold atomic ensembles.
1.4 Formulation:

To study EIT effect it is necessary to understand how coherent fields interact with an atomic system. We followed a semi classical approximation (where electromagnetic fields are treated classically and atom is a quantum object) through which density matrix analysis is considered. This allows us to examine the population of atomic levels and coherence established between levels. The atomic susceptibility is directly related to coherence established between the levels, and thus absorption and dispersion properties are estimated.

a) Interaction Hamilton:

The Hamiltonian in the atom-field interaction can be defined as

$$H = H_0 + V$$

(1.1)

Where $H_0$ is the unperturbed Hamiltonian and $V$ is an interaction term comprised of the electric dipole interactions induced within the atom due to the applied electric field. Thus we can write $H$ as

$$H = H_0 - \vec{\mu}_{ij} \cdot \vec{E}$$

(1.2)

Where $\vec{E}$ is the electric field

$$\vec{E} = \sum_a \vec{e}_a \exp [i (\vec{k}_a \cdot \vec{r} - \omega_a t)]$$

(1.3)

and $\vec{\mu}_{ij}$ is the electric dipole matrix element.

$$\vec{\mu}_{ij} = \sum_{ij} |i\rangle\langle i| \mu_{ij} |j\rangle\langle j| \quad (i \neq j)$$

(1.4)

The Hamiltonian $V$ in the interaction representation is given by

$$V^\text{int} = e^{i \frac{t}{\hbar} H_0 t} (-\vec{\mu}_{ij} \cdot \vec{E}) e^{-i \frac{t}{\hbar} H_0^*}$$

(1.5)

b) Density matrix equations of motion:

The time evolution of the density matrix of the system in the interaction representation is
\[ \dot{\rho} = \left( \frac{i}{\hbar} \right) [\rho, V^{\text{int}}] \] (1.6)

The equation of motion of \( ij^{\text{th}} \) element of the density matrix is

\[ \dot{\rho}_{ij} = \left( \frac{i}{\hbar} \right) [(i|[\rho, V^{\text{int}}]|j)] \] (1.7)

\[ \dot{\rho}_{ij} = \left( \frac{i}{\hbar} \right) [(i|\rho V^{\text{int}}|j) - (i|V^{\text{int}} \rho|j)] \]

Inserting between \( \rho \) and \( V^{\text{int}} \), and \( V^{\text{int}} \) and \( \rho \), an orthonormal set of states \( |m\rangle \) that obey the completeness property \( \sum_{m}|m\rangle\langle m| = 1 \)

We get from Eq.(1.7)

\[ \dot{\rho}_{ij} = \left( \frac{i}{\hbar} \right) \sum_{m} \left\{ \rho_{im} V_{mj}^{\text{int}} - V_{im}^{\text{int}} \rho_{mj} \right\} \] (1.8a)

Where \( V_{mn}^{\text{int}} = \langle m|V^{\text{int}}|n\rangle \) and \( \rho_{ij} = \langle i|\rho|j\rangle \) (1.8b)

A generalized equation of motion, including the relaxation process \((\rho_{ij})_{\text{relax}}\) such as spontaneous decays (\( \gamma_{ij} i,j = 1-3 \)), radiative decay of half diagonal elements (\( \gamma_{ij} i \neq j \)), line width of probe (\( \gamma_{p} \)) and pump lasers (\( \gamma_{c} \), \( \cdots \)) are incorporated in right hand side of Eq(1.8a). described is as follows

\[ \dot{\rho}_{ij} = \left( \frac{i}{\hbar} \right) \sum_{m} \left\{ \rho_{im} V_{mj}^{\text{int}} - V_{im}^{\text{int}} \rho_{mj} \right\} + (\rho_{ij})_{\text{relax}} \] (1.9)

Doppler shift of atomic resonance due to thermal motion of atoms in the medium gives rise to inhomogeneous broadening. To incorporate atomic motion, the derivative \( \dot{\rho}_{ij} \) on the left-hand side of Eq.(1.9) can be replaced by the following equation as

\[ \dot{\rho}_{ij} \rightarrow \left\{ \frac{\partial}{\partial t} + \bar{v} \cdot \bar{\nabla} \right\} \rho_{ij} \] (1.10)

The response of the atom moving with velocity \( \bar{v} \) and relaxation effects with buffer gas collisions terms is illustrated by a density matrix equation can now be written as

\[ \left( \frac{\partial}{\partial t} + \bar{v} \cdot \bar{\nabla} \right) \rho_{ij} = - \left( \frac{i}{\hbar} \right) \sum_{m} \left\{ \rho_{im} V_{mj}^{\text{int}} - V_{im}^{\text{int}} \rho_{mj} \right\} + (\rho_{ij})_{\text{relax}} \] (1.11)
1.5 Buffer gas collisions effect

In vapors cells atoms are not confined in the interaction region and they are moving in and out of the interaction region. While atoms which are moving out of the interaction region most likely collide with the cell wall, and its quantum state created during the interaction with the laser fields is destroyed. To amend the situation, and prolong the atoms-field interaction time two methods are used i) anti-relaxation wall coating inside the cell, these coatings usually consist of paraffin waxes or silanes with long carbohydrate chains and ii) Addition of buffer gas to the vapor cell, like inert (He and Ne) or diatomic (N₂) gases. Buffer gas is able to prohibit atoms from diffusing rapidly out of the interaction zone by velocity changing collisions, as a result atoms spend a much longer time inside the interaction region, so that the quantum state is still preserved. Collisions with buffer gas atoms can cause dephasing as well result in changes in the velocity of a vapor atom. The effect of such collisions can be incorporated in the density-matrix formalism by including a term [31]

\[
\left(\rho_{ij}\right)_{\text{coll}} = -\gamma_{ph}(1 - \delta_{ij})\rho_{ij} - \Gamma_{ij}\rho_{ij} + \int W_{ij}(v' \rightarrow v)\rho_{ij}(v', t)d^3v'
\]  

(1.12)

Here \(\gamma_{ph}\) is the rate of collision-induced dephasing of optical coherences and \(\Gamma_{ij}\) is some average rate of change in velocity \(v\). For simplicity \(\Gamma_{ij}\) is assumed to be independent of \(v\) and related to the collisional kernel \(W_{ij}(v' \rightarrow v)\) by

\[
\Gamma_{ij} = \int W_{ij}(v \rightarrow v')d^3v'
\]  

(1.13)

The collision kernel in general is assumed to be of the form

\[
W_{ij}(v' \rightarrow v) = W_{ij}(v - \alpha v')
\]  

(1.14)

Where \(\alpha\) is a constant, \((1 > \alpha > 0)\). Physically the second term in Eq. (1.12) is the “out term” representing collisional shift of atoms with a velocity \(v\) to other velocity subclasses at some rate \(\Gamma_{ij}\) and the third term is the “in term” arising due to the collisional shift of atoms from other velocity subclasses into velocity subclass \(v\). In this thesis we however restrict our discussion to the case of most prevalent and experimentally relevant strong collision model in which collisions result in rapid thermalization of the velocity distribution of the system. The collision kernel of Eq(1.14) is assumed to be independent of the initial velocity and of the form
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\[ \lim_{\alpha \to 0} W_{ij}(v' \to v) = \Gamma_{ij} M(v) \]  

(1.15)

Where \( M(v) = \left( \frac{\sqrt{2 \ln 2/(\pi v^2)}}{v} \right)^3 \exp (-\ln 2 \frac{v^2}{v^2}) \) is the Maxwell velocity distribution of atoms with \( \bar{v} = \sqrt{\ln 2 \nu_{th}} \) and \( \nu_{th} = \sqrt{2k_B T/m_a} \) is the most probable thermal velocity at a temperature \( T \) of an atom of mass \( m_a \) and substituting Eq(1.15) in Eq (1.12) yields as

\[ \left( \rho_{ij} \right)_{col} = -\gamma_{ph} (1 - \delta_{ij}) \rho_{ij} - \Gamma_{ij} \rho_{ij} + \Gamma_{ij} M(v) \int \rho_{ij}(v',t)d^3v' \]  

(1.16)

1.6 Generalized density matrix equation:

The generalized density matrix equation can be written as using Eq(1.11) and (1.16) together as

\[ \left( \frac{\partial}{\partial t} + \bar{v} \cdot \nabla \right) \rho_{ij} = -\left( \frac{i}{h} \right) \sum_m \left\{ \rho_{im} v_{mj} + v_{mj} \rho_{im} \right\} + (\rho_{ij})_{relax} - \gamma_{ph} (1 - \delta_{ij}) \rho_{ij} - \Gamma_{ij} \rho_{ij} \]

\[ + \Gamma_{ij} M(v) \int \rho_{ij}(v',t)d^3v' \]  

(1.17)

Eq (1.17) describes dynamic equation of motion with inclusion of relaxation process & buffer gas collisions effect.
1.7 References:


P. R. S. Carvalho, L. E. E. de Araujo, and J. W. R. Tabosa, Phys. Rev. A 70, 063818 (2004); 

