This chapter presents a Markov model for reliability analysis using different types of spares that replaces the faulty sensors in case failure occurs. The primary idea in this chapter is to address and analyze the reliability issues to devise a reliable and fault tolerant model for a sensor network system. We analyzed the model in terms of reliability and MTTF (Mean-Time-To-Failure). This research work focuses on the mechanism for providing an alternative of a redundant network by replacing the faulty sensor with the available spares.

This chapter has been organized into five sections. Related issues have been presented in Section 3.1. System model has been described in Section 3.2 to enhance the reliability of sensor network. Detailed analysis of the system is presented in Section 3.3 along with necessary assumptions and related terminology. Results and discussion have been shown in Section 3.4 followed by the summary of the chapter in Section 3.5.

3.1 ISSUES

WSN is a distributed system which possesses time varying wireless connectivity without any infrastructural backbone. Despite the constraints, like scarce energy and computational power of nodes, self configuration after deployment and unattended and harsh operation environment, WSN has been evolved as a tool to bridge the gap between physical world of sensors and the virtual world of information and services. Sensing the environment and sending the sensed data to a remote place via sink are the primary objectives of WSN. These objectives are to be achieved with greater reliability as some important decisions depend on the information received from WSN. As an application-driven network, WSN generally requires high data reliability to maintain detection and response capabilities.

There is an exciting new wave in sensor applications - wireless sensor networking - which enables sensors and actuators to be deployed independent of costs and physical constraints of wiring. Sensor networks do not rely on any hard-wired communication links; therefore, they can be deployed in places without infrastructure, and they can be used in medical assistance, surveillance, reconnaissance, disaster relief operations [10][15]. Increasing computing and wireless communication capabilities expand the role of sensor from mere information dissemination to more demanding tasks such as sensor fusion, classification,
etc. Fault tolerance and reliability performs exclusively vital role for embedded systems, such as obscured wireless sensors, which are deployed in some applications where it is difficult to access them physically.

In order to achieve a better reliability of the system, one solution is to improve the quality of spares; another one is to increase the number of spares. We consider in our model hot or stand-by spares, which means that they replace immediately the failed sensor (there is no gap in time between the moment the sensor has failed and the moment spares replace it).

When the spares substitute a module, then it has the same failure rate as the module.

3.2 SYSTEM MODEL

Consider a two sensor (of the same type) parallel-redundant system with single replacement facility of rate \( r \) as shown in Figure 3.1.

![Figure 3.1 Reliability Block Diagram](image)

Assuming failure rate of both sensors is \( \beta \). When both sensors have failed, the system is considered to have failed and no recovery is possible. Let the number of sensors in proper functioning represent the state of the system. The state space is \{0, 1, 2\} where 0 is the absorbing state while state 1 and 2 are transient states. Figure 3.2 shows the state diagram of the system under consideration.

![Figure 3.2 Finite Markov Chain for Two Sensor Parallel Redundant System](image)
3.3 ANALYSIS

Following assumptions are required to analyze the system model:

- **Fault Tolerance**: The fault tolerance is ability for a system to continue functioning properly even after failures in any part of the system have occurred. Fault tolerance in WSN [18] can be provided in three ways: 1. through hardware improvement and backup components, 2. through traffic management and 3. through redundant network design. WSN is transforming into a multi service medium leading to the convergence of voice, video and data communication. Each type of service has a particular constraint and it has to be satisfied for the communication to be effective. In [21] an interesting research regarding the fault tolerance aspects of a sensor network assumes that the nodes are either active or inactive with Bernoulli model. In case that one or more sensor fails, other sensors of a different type can substitute their work, such that the fault goes.

- **Reliability**: The probability that a component survives until sometime \( t \) is called the reliability \( R(t) \) of the component. Let \( X \) be the random variable representing the life time of a component then \( R(t) = P(X > t) = 1 - F(t) \); where \( F(t) \) is called the unreliability of the component.

The unreliability of a system is \( F(t) = 1 - R(t) \). Initially the system is functional at \( t=0 \) having \( R(0) = 1 \) and \( F(0) = 0 \). Eventually the system will fail at \( t=T \) having \( R(T) = 0 \) and \( F(T) = 1 \).

- **MTTF**: The expected life or the Mean-Time-To-Failure (MTTF) of the component is given by

\[
E[X] = \int_0^\infty f(t)dt = \int_0^\infty R'(t)dt \quad \text{where} \quad R'(t) = -f(t) \quad \text{and} \quad R(t) = P(X > t).
\]

\[
E[X] = \int_0^\infty R(t)dt \left[ R(t) \right]_0^\infty = \int_0^\infty R(t)dt
\]

Now, since \( R(t) \) approaches 0 faster than \( t \) approaches \( \infty \), we have \( E[X] = \int_0^\infty R(t)dt \).

- **Failure rate**: Failure rate, \( h(t) \), is the conditional probability that a component surviving to age \( t \) will fail in the interval \( (t, t + \Delta t) \).

\( H(t) = \frac{f(t)}{R(t)} = -\frac{R'(t)}{R(t)} \) and if component life time is exponentially distributed, then \( R(t) = e^{-\beta t} \) and \( h(t) = \beta e^{-\beta t} = \beta \).
For the finite Markov chain as shown in Figure 3.2, assume that the initial state of Markov chain is 2 when both sensors are functioning properly; i.e., \( P_2(0) = 1, P_k(0) = 0 \) for \( k = 0, 1 \). Then \( P_j(t) = p_{2j}(t) \) and system of differential equations for the system model can be given as:

\[
\frac{dP_2(t)}{dt} = -2\beta P_2(t) + rP_1(t)
\]

\[
\frac{dP_1(t)}{dt} = 2\beta P_2(t) - (\beta + r)P_1(t)
\]

\[
\frac{dP_0(t)}{dt} = \beta P_1(t)
\]

Taking Laplace transform, system can be reduced as under:

\[
sP_2(s) - 1 = -2\beta \bar{P}_2(s) + r\bar{P}_1(s)
\]

\[
s\bar{P}_1(s) = 2\beta \bar{P}_2(s) - (\beta + r)\bar{P}_1(s)
\]

\[
s\bar{P}_0(s) = \beta \bar{P}_1(s)
\]

Solving these equations for \( \bar{P}_0(s) \), we get:

\[
\therefore \bar{P}_0(s) = \frac{2\beta^2}{s^2 + (3\beta + r)s + 2\beta^2}
\]

After taking inverse Laplace, we can obtain \( P_0(t) \), the probability that no sensors are working at time \( t \geq 0 \). Let \( X \) be the time to failure of the system; then \( P_0(t) \) is the probability that system has failed at or before time \( t \). Thus the reliability of system at time \( t \) is:

\[ R(t) = 1 - P_0(t) \]

Laplace transform of failure density:

\[ f_x(t) = -\frac{dR}{dt} = \frac{dP_0(t)}{dt} \]

is then given by:

\[ L_x(s) = \bar{f}_x(s) = s\bar{P}_0(s) - P_0(0) = \frac{2\beta^2}{s^2 + (3\beta + r)s + 2\beta^2} \]

The denominator can be factored so that \( s^2 + (3\beta + r)s + 2\beta^2 = (s + \alpha_1)(s + \alpha_2) \), and above expression can be rearranged so that:

\[ L_x(s) = \frac{2\beta^2}{\alpha_1 - \alpha_2} \left( \frac{1}{s + \alpha_1} - \frac{1}{s + \alpha_2} \right) \]
Where: \( \alpha_i; \alpha_1 = \frac{(3\beta + r) \pm \sqrt{(3\beta^2 + 6\beta r + r^2)}}{2} \)

Taking the inverse transform, we get:

\[
f_X(t) = \frac{2\beta^2(e^{-\alpha_1t} - e^{-\alpha_2t})}{\alpha_1 - \alpha_2}
\]

Thus the MTTF (Mean-Time-To-Failure) of the system is given by:

\[
E[X] = \int_0^\infty x f_X(x)dx = \frac{2\beta^2}{\alpha_1 - \alpha_2} \left[ \int_0^\infty xe^{-\alpha_1x}dx - \int_0^\infty xe^{-\alpha_2x}dx \right]
\]

Re-calling that \( \int_0^\infty xe^{-ax}dx = \frac{1}{a^2} \), we get:

\[
E[X] = \frac{2\beta^2}{\alpha_1\alpha_2} \left[ \frac{1}{\alpha_2} - \frac{1}{\alpha_1} \right]
\]

\[
E[X] = \frac{2\beta^2 (\alpha_1 + \alpha_2)}{\alpha_1^2 \alpha_2^2} = \frac{3}{2\beta} + \frac{r}{2\beta^2}
\]

MTTF of the two sensor parallel-redundant system, in absence of a replacement facility (i.e. \( r=0 \)), is equal to \( E[X] = \frac{3}{2\beta} \). Therefore, the effect of a replacement facility is to increase the MTTF by \( \frac{r}{2\beta^2} \) or by a factor of \( \frac{r}{3\beta} \).

- **Modeling through Proposed System Model:** Example 1 describes the modeling of a redundant system where one type of spare can replace the faulty sensor of the same type.

**Example 1:** consider a system with two different types of sensors (X, Y) and two spares (SX, SY) that can replace only own type sensors in case fault occurs as shown in Figure 3.3.

![Reliability Block Diagram for Two Single Type Sensors](image)

**Figure 3.3** Reliability Block Diagram for Two Single Type Sensors
Assuming failure rates for all sensors are same, i.e., $\beta_X = \beta_Y = \beta_{SX} = \beta_{SY} = \beta$ and various states of the system has been represented as given in Table 3.1.

**Table 3.1** State Space for Redundant System with Single Type Spares

<table>
<thead>
<tr>
<th>State</th>
<th>Functional (Available) Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(X^2 Y^2 SX SY)$</td>
</tr>
<tr>
<td>1</td>
<td>$(X^2 Y^2 SY)$</td>
</tr>
<tr>
<td>2</td>
<td>$(X^2 Y^2 SX)$</td>
</tr>
<tr>
<td>3</td>
<td>$(X^2 Y^2)$</td>
</tr>
<tr>
<td>4</td>
<td>$(XY^2)$</td>
</tr>
<tr>
<td>5</td>
<td>$(XY)$</td>
</tr>
<tr>
<td>6</td>
<td>$(X^2 Y)$</td>
</tr>
<tr>
<td>7</td>
<td>$(X^2 Y SX)$</td>
</tr>
<tr>
<td>8</td>
<td>Failed</td>
</tr>
</tbody>
</table>

Markov model for the system described in Figure 3.3 can be shown as in Figure 3.4.

![Figure 3.4 Markov Model for a Redundant System for Single Type Spares](image-url)
Chapter 3: Reliability Analysis in Wireless Sensor Networks

Now the reliability and MTTF can be computed as:

1. For two single type spares:
   \[ R_s(t) = 9e^{-2\beta t} - 18e^{-3\beta t} - 6e^{-5\beta t} + e^{-6\beta t} \]
   \[ MTTF_2 = \frac{73}{60\beta} = \frac{1.217}{\beta} \]

2. For one single type spare:
   \[ R_s(t) = 6e^{-2\beta t} - 9e^{-3\beta t} - 5e^{-5\beta t} \]
   \[ MTTF_1 = \frac{21}{20\beta} = \frac{1.05}{\beta} \]

3. If we have no spare:
   \[ R_s(t) = 4e^{-2\beta t} - 4e^{-3\beta t} + 4e^{-4\beta t} \]
   \[ MTTF_0 = \frac{11}{12\beta} = \frac{0.917}{\beta} \]

MTTF for two single type spares is greater than that of one single type spare and no spare cases, i.e., \( MTTF_2 > MTTF_1 > MTTF_0 \). The comparison of these three cases has been shown in Figure 3.9.

3.4 RESULTS AND DISCUSSION

Figure 3.5, 3.6, 3.7 and 3.8 shows the mean time to failure, taking particular values for \( \beta \) as 0.01, 0.02, 0.03, 0.04, 0.05 and 0.06 for the number of failures per 10000 seconds. Figure 3.5 shows the comparison between the systems with replacement rate \( r=0 \) and \( r=\beta \). The MTTF for all values of \( r \) is significantly large in comparison to the MTTF of the system in absence of replacement facility. Figure 3.6 shows the comparison between the systems with replacement rate \( r=0 \) and \( r=0.001 \). The MTTF is still significantly large though the replacement rate is very small in comparison to the failure rate. Figure 3.7 shows the comparison between the systems with replacement rate \( r=0 \) and \( r=0.009 \). Figure 3.8 shows the comparison between the systems with replacement rate \( r=0.001 \) and \( r=0.009 \). There is significant improvement in MTTF when the replacement rate has increased to 0.009 from 0.001 with the same failure rate. In all the cases, if a spare sensor replaces a failed sensor then MTTF increases by a factor \( \frac{r}{3\beta} \).
Figure 3.5 MTTF vs. Failure Rate with $r=0$

Figure 3.6 MTTF vs. Failure Rate $r=0$ and $r=0.001$
Figure 3.7 MTTF vs. Failure Rate $r=0$ and $r=0.009$

Figure 3.8 MTTF vs. Failure Rate $r=0.001$ and $r=0.009$
3.5 SUMMARY

This chapter is a contributing effort to explore the reliability issues in WSNs. We presented the system reliability for the two cases: (1) without provision of standby spares, (2) with the provision of standby spares. The system lifetime is calculated and the suggestive values for the different $\beta$ are given. The comparison of these two models in terms of MTTF (Mean-Time-To-Failure) shows an increase in MTTF by a factor of $\tau/3\beta$ when standby spares are used. In the next chapter, a Markov model for reliable packet delivery in WSNs is presented.