ABSTRACT

By a graph $G = (V,E)$, we mean a finite undirected connected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak (2005).

In chapter 1, we collect some basic definitions and theorems on graphs which are needed for the subsequent chapters.

One of the fastest growing areas within graph theory is the study of domination. An excellent treatment of fundamentals of domination is given in the book by Haynes et al. (1998) and a survey of several advanced topics in given in the book edited by Haynes et al. (1998a). A set $S \subseteq V$ is said to be a dominating set of $G$ if every vertex in $V - S$ is adjacent to some vertex in $S$. A dominating set $S$ is called a minimal dominating set if no proper subset of $S$ is a dominating set of $G$. The domination number of $G$ is the minimum cardinality taken over all dominating sets in $G$ and is denoted by $\gamma(G)$.

Sampathkumar and Walikar (1979) introduced the concept of connected domination in graphs. A dominating set $S$ of $G$ is called a connected dominating set if the induced subgraph $\langle S \rangle$ is connected. The minimum cardinality of a connected dominating set of $G$ is called the connected domination number of $G$ and is denoted by $\gamma_c(G)$. 
Weichsel [See Haynes et al. (1998)] introduced the concept of perfect domination in graphs. A dominating set $S$ of $G$ is called a *perfect dominating set* if every vertex $v$ in $V - S$ is adjacent to exactly one vertex in $S$. The minimum cardinality of a perfect dominating set is called *perfect domination number* of $G$ and is denoted by $\gamma_p(G)$.

Arumugam and Sivagnanam (2010) introduced the concept of neighborhood connected domination. A dominating set $S$ of a connected graph $G$ is called a *neighborhood connected dominating set* (ncd-set) if the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a ncd-set of $G$ is called the *neighborhood connected domination number* of $G$ and is denoted by $\gamma_{nc}(G)$.

In chapter 2, we introduce the concept of neighborhood connected perfect domination and initiate a study of corresponding parameter.

A perfect dominating set $S$ of a graph $G$ is called the neighborhood connected perfect dominating set (ncpd-set) if the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a ncpd-set of $G$ is called the neighborhood connected perfect domination number of $G$ and is denoted by $\gamma_{ncp}(G)$. Several results on this parameter are presented in chapter 2.

Fink and Jacobson (1984) introduced the concept of $k$-domination in graphs. A dominating set $S$ of $G$ is called a *$k$-dominating set* if every vertex in $V - S$ is adjacent to at least $k$ vertices in $S$. The minimum cardinality of a $k$-dominating set is called *$k$-domination number* of $G$ and is denoted by $\gamma_k(G)$. Another
domination parameter, the k-tuple domination was introduced by Harary and Haynes (2000). A set $S$ is called a $k$-tuple dominating set of $G$ if for every vertex $v \in V(G)$, $|N[v] \cap S| \geq k$. The minimum cardinality of a $k$-tuple dominating set of $G$ is the $k$-tuple domination number $\gamma_{\times k}(G)$. The 2-tuple domination number $\gamma_{\times 2}(G)$ is called the double domination number and the 3-tuple domination number $\gamma_{\times 3}(G)$ is called the triple domination number.

In chapter 3, we initiate a study of neighborhood connected 2-dominating set and neighborhood connected 2-domination number of a graph.

A set $S \subseteq V$ is called the neighborhood connected 2-dominating set (nc2d-set) of a graph $G$ if every vertex in $V - S$ is adjacent to at least two vertices in $S$ and the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a nc2d-set of $G$ is called the neighborhood connected 2-domination number of $G$ and is denoted by $\gamma_{2nc}(G)$.

As an analogy to vertex domination, the concept of edge domination was introduced by Mitchell and Hedetniemi (1977). A set $X \subseteq E$ is said to be an edge dominating set if every edge in $E - X$ is adjacent to some edge in $X$. The edge domination number of $G$ is the cardinality of a smallest edge dominating set of $G$ and is denoted by $\gamma'(G)$. Arumugam and Velammal (2009) introduced the concept of connected edge domination of a connected graph. An edge dominating set $X$ of a connected graph $G$ is called a connected edge dominating set if the edge induced subgraph $\langle X \rangle$ is connected. The minimum cardinality of a connected edge
dominating set of $G$ is called connected edge domination number and is denoted by $\gamma'_c(G)$.

In chapter 4, we introduce the concept of neighborhood connected edge domination and initiate a study of corresponding parameter.

An edge dominating set $X$ of a connected graph $G$ is called the neighborhood connected edge dominating set (nced-set) if the edge induced subgraph $\langle N(X) \rangle$ of $G$ is connected. The minimum cardinality of a nced-set is called the neighborhood connected edge domination number (nced-number) and is denoted by $\gamma'_{nc}(G)$.

Telle and Proskurowski (1997) introduced restrained domination as a vertex partitioning problem.

A dominating set $S$ of a graph $G$ is called restrained dominating set if every vertex in $V - S$ is adjacent to a vertex in $V - S$. The minimum cardinality of a restrained dominating set of $G$ is the restrained domination number $\gamma_r(G)$.

The vertex connectivity number $\kappa(G)$ of a graph $G$ is the minimum number of vertices whose removal results in a disconnected or trivial graph.

In chapter 5, we find the upper bound for the sum $\gamma_r + \kappa$ and characterize the corresponding extremal graphs. Also we find the upper bound for sums $\gamma_{2nc} + \kappa$ and $\gamma_{ncp} + \kappa$ and characterize the corresponding extremal graphs.