The general conclusions derived from the present studies. Future scope of the work is highlighted.
Time series analysis, as described in previous chapters, can be used to study nonlinear systems and is an effective method from experimental point of view. We described certain applications of the technique to different systems like human brain, Asteroidal distribution and Saturn rings.

The analysis does not demand any equation of the state of the system and hence is an efficient tool to study complex nonlinear systems. As described at length, a pseudo phase space is constructed from the discrete time series using the method of delayed matrix. The matrix consists of m vectors in a \(d\)-dimensional phase space with \(m \geq d\). From the set of \(m\) vectors, a \(q^{th}\) order correlation integral in \(d\)-dimensional phase space, \(C_d^{q}(r)\) can be constructed. By varying \(q\) from \(\infty\) to \(\infty\), \(C_d^{q}(r)\) has been calculated from which generalized attractor dimension \(D_q\) and generalized Kolmogorov entropy \(K_q\) are evaluated. \(D_0, D_1\) and \(D_2\) are fractal dimension, information dimension and correlation or second order dimension respectively of the attractor which characterizes the nonlinear dynamics. \(K_2\) is known as second order Kolmogorov entropy.

For a homogeneous system, \(D_0 = D_1 = D_2\) and for a nonhomogeneous system, \(D_0 > D_1 > D_2\) (Schuster 1984). \(D_2\) is the lower bound of the Hausdorff dimension. Of all the generalized quantities \(D_q\) and \(K_q\), \(D_2\) and \(K_2\) are the most important parameters (Caputo and Atten 1987) which depend sensitively on the state of the nonlinear system. Moreover, these quantities can be derived easily from a time series and it is independent of the number of
data. Integer $D_2$ value corresponds to $K_2=0$ and will represent regular dynamics, while a noninteger value of $D_2$ corresponds to $K_2 > 0$ will quantify the degree of chaos. For a completely stochastic system $K_2 = \infty$ and $D_2$ is not defined. In this case the plot slope - dimension will have slope unity.

The programme we used to evaluate $D_2$ and $K_2$ is given in the appendix and was tested using a sinusoidal signal as the data. Result yielded the values $D_2 = 1$ and $K_2 = 0$.

It has been found that the present algorithm is more efficient than other methods like the box counting algorithm. The technique has inbuilt mechanism to remove noise, and it also differentiates between deterministic and nondeterministic components of the signal.

If the SNR is very low, we should search for alternate methods to remove noise from time series signals before applying the algorithm. In chapter 3 we described such a method to filter out noise from experimental data. This is an extension of the method described by Broomhead and King [1986]. The method has been demonstrated by filtering a noise - embedded sinusoidal signal. It has been found that the EEG is more or less free of noise and hence filtering technique is not necessary for time series analysis of EEG data.

In the thesis, various types of rhythms present in the electrical activity have been described along with the dynamical aspects of the neural system. A brief description of various types of time scales present in the neural system is also presented.

Our analyses show that $D_2$ and $K_2$ are sensitive to the state of the neural system, of which $K_2$ is more sensitive as compared to $D_2$. In comparison with other systems like certain astronomical systems, Raman attractor etc., the neural dynamics has attractor with high $D_2$ & $K_2$ values. This explains the complexity of the system.
All previous works (Babloyantz et al. 1985, 1986) deal with the analysis of only one of the channels of EEG and described the dynamics only in terms of $D_2$. We have analysed all the eight channels of EEG of a "clinically" normal brain during rest and mental activity. It was found that in all eight channels, $D_2$ is fractal and $K_2$ is positive. Analyses suggest the existence of a collection of strange attractors in the system and $K_2$ is very high compared to other systems. Left and right lobes are not symmetric with respect to the neural activity. In other words, brain has asymmetric activities in left and right lobes.

The study of information flow in the system in terms of $K_2$ variation shows that, the information flow is unidirectional when the person is immersed in some mental exercise. That is, the random firings are suppressed when the person is undergoing a mental exercise. Whereas, during the rest time, the random firings become prominent, and unidirectional nature of information flow disappears. The generalized dimension was also calculated for both mental activity and rest condition. From this the $f(\alpha)$ spectrum was deduced. The $f(\alpha)$ spectrum shows a well defined double peak during both at rest and mental activity, which suggests the existence of two attractors.

The present method of analysis gives the quantitative measures $D_2$ and $K_2$, which can be used to quantify chaos. Hence, $D_2$ and $K_2$ can be used as diagnostic tools since the degree of chaos depends on the state of the neural system under various pathological conditions like, epilepsy, migraine, tumour and psychotic.

We analysed the EEG of an epileptic (grand mal) patient during, before and after attack. The eighth channel (left parietooccipital region) shows a steady variation in $K_2$. It can be suggested that seizure is a defensive mechanism of the brain against any alteration of the neural dynamics.

The other epileptic case, which we have analysed is that
of a patient having both demylinated disease and epilepsy. In this case also we found that left parietooccipital region is more sensitive. The EEG in this case has been recorded while the patient had no seizure and the record indicated normal brain activity. However, our analysis shows the abnormality in the neural activity. This suggests the superiority of the present technique over the conventional method of EEG analysis.

The analysis of EEG of a patient having headache (EEG looks normal) revealed the fact that the left frontal region is the seat of abnormality. The $K_z$ shows periodic activity in the left frontal region as time evolves. In the case of Migraine also, frontal region is found to be sensitive, and $K_z$ shows large variation in this region as time evolves.

The analysis of epilepsy, tumour and psychotic cases at higher spatial resolution have also been carried out, by taking EEG at different electrode configurations. The pattern of $K_z$ distribution varies for different pathological conditions.

The $K_z$ value has been found to be low in the right parietooccipital region, where the tumour is growing, which is in the primary stage. Studies using both bipolar and monopolar techniques show the same behaviour.

The $K_z$ distribution in the psychotic case, shows a low $K_z$ value in the central region and high $K_z$ in the frontal and parietal regions, and thus creating certain segments in brain activity. The $K_z$ value is low in the left occipital region.

We have applied our analysis to two astronomical systems viz., asteroidal belt and saturn rings.

The studies by Wisdom [1983] showed that there exists an ordered and chaotic regions in the asteroidal belt and 3:1 resonance is the boundary between these two. Power spectrum analysis by Pratap [1977] showed that there are five dominant frequencies present in the system, and there is a change of
correlation from positive to negative at 3:1 resonance. This result agrees with our studies in Asteroidal belt.

We have analysed the density distribution of asteroids, and found that $D_2$ is 5 and embedding dimension is 15. The $K_2$ value is close to zero. This shows that the system is more or less completely ordered in the information sense and has very low stochasticity.

In the case of Saturn ring system, we have analysed the density distribution of matter (as a function of the distance from the planet Saturn), for a total span of $1.03 \, R_s$, i.e., from C ring (innermost ring) to the outer edge of A ring. We have calculated $D_2$ and $K_2$ for various ring and gap systems, as well as for the entire ring system.

For all rings and gaps $D_2$ is noninteger, and $K_2$ is positive but very low. For the Saturn ring system as a whole also, the $D_2$ is noninteger and $K_2$ is positive. That is, the system is chaotic both piecewise as well as, as a whole. The chaotic behaviour of the total system as a whole is a consequence of large number of interacting strange attractors, i.e., chaos - inducing chaos, whereas in Asteroidal belt it may be the case of chaos - inducing order.

**FUTURE SCOPE OF THE WORK**

The second order Kolmogorov entropy is identified as a sensitive parameter to characterize the neural system. By analysing the EEG of large number of patients with same type of abnormality or disease, we can standardize $K_2$, for each particular disease and can be used as a diagnostic tool. With this analysis, we can identify different stages of a disease. Also, we can develop the information flow pattern for "clinically" normal brain as well as for affected brain. The
change in the normal flow pattern can be used as a symbol of abnormality. The flow pattern also enables us to identify the mechanism of thought-processes in brain. If EEG is obtained with higher spatial resolution such as 32 channel or 64 channel, we can identify the localized effect of the brain, or detect which part of the brain is more involved in a particular activity. This will amount to certain type of imaging of the brain.

Simulation of complete signal from a part of the data is an interesting part of this analysis. By taking a part of the signal one can develop the whole signal by following the method by Parikh and Pratap [1990].

The neural system does not have an ideal equation. $D_2$ gives the number of independent parameters required to describe the state of the system. Thus by knowing $K_z$ and $D_2$, we can construct certain model equations of the system by the method suggested by Broomhead and King [1986].

We are now applying this analysis in Interplanetary scintillations (IPS), and also in Raman system. The spatial behaviour of stimulated Raman scattering in a medium can be studied by analysing the spatial variation of amplitude of stokes, antistokes and pump mode.

Developing the statistical mechanics of Neural system is a very exciting work, which will be carried out in the near future. Work in this direction has already been initiated by Parikh and Pratap [1984] and Pratap [1988]. This can also be carried out in the frame work of nonequilibrium statistical mechanics, as employed by Pratap and Sreekumar [1989] in the study of a many electron system.