Chapter 4

Inventory Models for Price-Sensitive Trapezoidal Demand under Biddable Two-Part Trade Credit
4.0 Introduction

In business world, the trade credit is considered to be a bridge between the suppliers and the buyers. This promotional tool encourages the buyer to purchase goods from the supplier without any investment. In inventory modeling, under two-part trade credit, it is assumed that the buyer either pays for all the procured items within a shorter allowable credit period and avails a cash discount or settles the account due for all the purchased items within a long allowable credit period at the regular price. In this chapter, we analyze the decision policy when buyer may pay any fraction of purchase cost within an allowable shorter credit period and receives a cash discount and then the remaining balance is paid within the long allowable credit period.

Two models are formulated under the following sections (viz.):

Model 4.1 Optimal retail price, replenishment time and payment scenario under biddable two-part trade credit for price-sensitive trapezoidal demand

Model 4.2 Optimal replenishment and payment policies for deteriorating inventory model with price-sensitive trapezoidal demand under biddable two-part trade credit

4.1 Optimal Retail Price, Replenishment Time and Payment Scenario under Biddable Two-Part Trade Credit for Price-Sensitive Trapezoidal Demand

In financial management, two-part trade credit is denoted as \( \beta/ M_1\text{net}M_2 \) under which the buyer receives a cash discount of \( \beta \) percentage if dues are paid
by $M_1$; otherwise the buyer can pay by $M_2$ at a regular purchase cost. In present business scenario, the buyer is considered to be a dominant player who has the choice to bid for favorable contract terms or/and flexible payments to reduce the overall purchasing cost. When the buyer has constraint of investment, the supplier can use the flexible payments to boost the demand from the buyer and increase cash-in-flow. This concept attracts to discuss a flexible-two part trade credit ($\beta / M_1$ net $M_2$) under which the buyer can pay any $\lambda$ percent of the total purchase cost within $M_1$ time and avail of the cash discount of $\beta$ percentage and then pay off the remaining account of $(1-\lambda)$ percent of the total purchase cost within allowable period $M_2$. The demand rate is considered to be price-sensitive trapezoidal. Buyer’s profit is maximized for the retail price, the cycle time and the payment policies. It is studied how the flexible two-part trade credit is advantageous compared to traditional one for the buyer.

**4.1.1 Assumptions and Notations**

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model.

**4.1.1.1 Assumptions**

1. A supply chain of single supplier and single buyer is considered.
2. Shortages are not allowed.
3. The market demand rate $R(=R(P,t))$ for the item is price-sensitive trapezoidal. So as per assumption A. 2, the functional form of the demand is
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\[
R(P,t) = \begin{cases} 
  f(t)P^{-\eta}, & 0 \leq t \leq u_1 \\
  R_0 P^{-\eta}, & u_1 \leq t \leq u_2 \\
  g(t)P^{-\eta}, & u_2 \leq t \leq T 
\end{cases}
\]

and hence demand function can be written as

\[
R(P,t) = \begin{cases} 
  R_1(P,t), & 0 \leq t \leq u_1 \\
  R_2(P,t), & u_1 \leq t \leq u_2 \\
  R_3(P,t), & u_2 \leq t \leq T 
\end{cases}
\]

where \( \eta > 1 \) is a price-elasticity mark-up.

4. The buyer can accumulate the revenue and earn interest from the beginning of the cycle until the end of the allowable trade credit offered by the supplier.

In \( (\beta/M_1 \text{ net } M_2) \) net credit scenario, the buyer can earn interest during \([0,M_2]\) at rate \(I_{bc}\).

**4.1.1.2 Notations**

\( \lambda \) : The percentage of the purchase cost that the buyer pays to the supplier at the time \( M_1 \)

\( 1 - \lambda \) : The fraction of the purchase cost that the buyer pays to the supplier at the time \( M_2 \)

\( \pi(\lambda,P,T) \) : The buyer’s profit ($ per unit time)

**4.1.2 Mathematical Model**

The inventory level in warehouse changes due to the price-sensitive trapezoidal demand in the warehouse. The rate of change of inventory at any instant of time \( t \) is governed by the differential equation
\[
\frac{d I(t)}{dt} = -R(P,t); 0 \leq t \leq T \quad \text{with the boundary condition } I(T) = 0.
\]

The solution of the differential equation is

\[
I(t) = \begin{cases} 
I_1(t), 0 \leq t \leq u_1 \\
I_2(t), u_1 \leq t \leq u_2 \\
I_3(t), u_2 \leq t \leq T
\end{cases}
\]

where

\[
I_1(t) = aP^{-\eta} \left( u_1 - t + \frac{1}{2} b_1 \left( u_1^2 - t^2 \right) \right) + aP^{-\eta} (1 + b_1 u_1) (u_2 - u_1)
\]

\[
+ \frac{aP^{-\eta}}{b_2} (1 + b_1 u_1) \left( 1 - e^{-b_2(T-u_2)} \right)
\]

\[
I_2(t) = aP^{-\eta} (1 + b_1 u_1)(u_2 - t) + \frac{aP^{-\eta}}{b_2} (1 + b_1 u_1) \left( 1 - e^{-b_2(T-u_2)} \right)
\]

\[
I_3(t) = \frac{aP^{-\eta}}{b_2} (1 + b_1 u_1) \left( e^{-b_2(t-u_2)} - e^{-b_2(T-u_2)} \right)
\]

Using \( I(0) = Q \), we get

\[
Q = I(0) = aP^{-\eta} \left( u_1 + \frac{1}{2} b_1 u_1^2 \right) + aP^{-\eta} (1 + b_1 u_1) (u_2 - u_1)
\]

\[
+ \frac{aP^{-\eta}}{b_2} (1 + b_1 u_1) \left( 1 - e^{-b_2(T-u_2)} \right)
\]

(4.1.1)

The total profit per unit time of the buyer comprises of the following components.

1. Gross revenue per unit time; \( GR = \frac{(P - C_b)Q}{T} \)

2. Ordering cost per unit time; \( OC_b = \frac{A_b}{T} \)

3. Holding cost excluding interest charges per unit time; \( HC_b = \frac{C_s I_b}{T} \int_0^T I(t) \, dt \)

4. Discount revenue per unit time; \( DR = \frac{\beta \lambda C_b Q}{T} \)
The interest earned and/or charged per unit time depends upon the lengths of $T, M_1$, and $M_2$. The three cases may arise: (i) $T \leq M_1$ (ii) $M_1 < T \leq M_2$ and (iii) $T > M_2$. Using assumption (4), we compute the interest charged/earned per unit time for the buyer as follows:

**Case 1: $T \leq M_1$ (Figure 4.1.1)**

![Diagram](image)

Figure 4.1.1 $T \leq M_1$

The buyer generates the revenue by selling all items before the due payment time. So, interest charges are zero. The buyer earns interest on the generated revenue during $[0,T]$, on full sales revenue during period $[T,M_1]$, and on $(1-\lambda)$-portion of full sales revenue during the interval $[M_1,M_2]$. Hence, the total interest earned per unit time; $IE_1$ is
\[
IE_1 = \frac{PI_{be}}{T} \left[ \int_{u_1}^{u_2} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_1}^{u_2} t \cdot R_3(P,t)dt \right] \\
+ \frac{PI_{be}(M_1-T)}{T} \left[ \int_{0}^{u_1} R_1(P,t)dt + \int_{u_1}^{u_2} R_2(P,t)dt + \int_{u_1}^{u_2} R_3(P,t)dt \right] \\
+ \frac{PI_{be}(1-\lambda)(M_2-M_1)}{T} \left[ \int_{0}^{u_1} R_1(P,t)dt + \int_{u_1}^{u_2} R_2(P,t)dt + \int_{u_1}^{u_2} R_3(P,t)dt \right]
\]

**Case 2: \( M_1 < T \leq M_2 \)**

Depending on values of \( M_1 \) and \( T \), the possible two sub-cases are (i) \( \lambda T < M_1 \) and (ii) \( \lambda T \geq M_1 \). In the section, the demand rate is changing at two time points viz. \( u_1 \) and \( u_2 \). Let us compute interest earned per unit time for \( \lambda T < M_1 \) by considering lengths of \( u_1 \) and \( u_2 \).

**Sub case 2.1: \( \lambda T < M_1 \) (Fig. 4.1.2)**

![Figure 4.1.2 \( \lambda T < M_1 < T \leq M_2 \)](image)
In this case, the buyer has generated more revenue than the $\lambda$-fraction of purchase cost at time $M_1$. Also $T \leq M_2$, the buyer has sold all the units before the payment is due. Hence, the interest charged $IC_{2.1}(\lambda T < M_1)$ for the buyer is zero. As shown in figure 4.1.2, the buyer accumulates interest (1) during $[0, M_1]$, the buyer earns interest on sales revenue; (2) at time $M_1$, the buyer pay $\lambda$ portion of purchase cost to the supplier and earns interest on $(1-\lambda)$-portion of purchase cost during $[M_1, M_2]$; and (3) after time $M_1$, the buyer earns interest on the sales revenue by selling the item until $T$ - time units, and on $(1-\lambda)$ fraction of full sales revenue during $[T, M_2]$. Hence, the total interest is

$$IE_{2.1}(\lambda T < M_1) = \frac{PI_{be}}{T} \left[ \int_0^{M_1} t \cdot R(P, t) \, dt + \frac{PI_{be}(M_2 - M_1)(1-\lambda)}{T} \int_0^T R(P, t) \, dt \right]$$

$$= \frac{PI_{be}}{T} \int_0^T t \cdot R(P, t) \, dt$$

$$\begin{align*}
IE_{2.1}(\lambda T < M_1) &= \begin{cases} 
IE_{2.1}(u_1 < u_2 < \lambda T < M_1) \\
IE_{2.1}(\lambda T < u_1 < u_2 < M_1) \\
IE_{2.1}(\lambda T < M_1 < u_1 < u_2) \\
IE_{2.1}(u_1 < \lambda T < u_2 < M_1) \\
IE_{2.1}(u_1 < \lambda T < M_1 < u_2) \\
IE_{2.1}(\lambda T < u_1 < M_1 < u_2)
\end{cases}
\end{align*}$$

where $IE_{2.1}(u_1 < u_2 < \lambda T < M_1) = \frac{PI_{be}}{T} \left[ \int_0^{u_1} t \cdot R_1(P, t) \, dt + \int_{u_1}^{u_2} t \cdot R_2(P, t) \, dt + \int_{u_2}^{M_1} t \cdot R_3(P, t) \, dt \right]$

$$+ \frac{PI_{be}(1-\lambda)RP(M_2 - M_1)}{T} = \frac{PI_{be}}{T} \int_0^{T-M_1} t \cdot R_3(P, t) \, dt$$

$IE_{2.1}(\lambda T < u_1 < u_2 < M_1) = IE_{2.1}(u_1 < u_2 < \lambda T < M_1)$
Sub case 2.2: $\lambda T \geq M_1$ (Fig. 4.1.3)

In this scenario, to avail of the discount in the unit purchase price, the buyer pays $\lambda$ - percentage of the purchase cost by time $M_1$. The buyer’s sales revenue is less than the $\lambda$ - percentage of the purchase cost because the offered credit
period $M_1$ is less than the time $\lambda T$. Hence, the buyer pays interest charges for the items in the stock during $[M_1, \lambda T]$ at an interest rate $I_{bc}$ per dollar per year.

Therefore, the interest charged per unit time is given by

$$IC_{22}(\lambda T \geq M_1) = \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I(t)dt$$

$$= \begin{cases} IC_{22}(u_1 < u_2 < M_1 \leq \lambda T) \\ IC_{22}(M_1 < u_1 < u_2 \leq \lambda T) \\ IC_{22}(M_1 \leq \lambda T < u_1 < u_2) \\ IC_{22}(u_1 < M_1 < u_2 \leq \lambda T) \\ IC_{22}(u_1 < M_1 \leq \lambda T < u_2) \\ IC_{22}(M_1 < u_1 \leq \lambda T < u_2) \end{cases}$$

where: $IC_{22}(u_1 < u_2 < M_1 \leq \lambda T) = \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I(t)dt$

$$IC_{22}(M_1 < u_1 < u_2 \leq \lambda T) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]$$

$$IC_{22}(M_1 \leq \lambda T < u_1 < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I_1(t)dt$$

$$IC_{22}(u_1 < M_1 < u_2 \leq \lambda T) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]$$

$$IC_{22}(u_1 < M_1 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I_2(t)dt$$

$$IC_{22}(M_1 < u_1 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{\lambda T} I_2(t)dt \right]$$

The buyer earns interest (1) during $[0,M_1]$ on the generated sales revenue; (2) during $[\lambda T,T]$, on the sales revenue by selling the items and on $(1-\lambda)$ fraction of
full sales revenue during the period $[T, M_2]$. Hence, the total interest earned per
unit time is

$$IE_{2,2}(\lambda T \geq M_1) = \frac{PI_{be}}{T} \int_0^M t \cdot R(P,t)dt + \frac{PI_{be}(1-\lambda)}{T} \int_{\lambda T}^T t \cdot R(P,t)dt$$

$$+ \frac{P(1-\lambda)}{T} I_{be} \left[ \int_0^T R(P,t)dt (M_2 - T) \right]$$

$$= \begin{cases} 
IE_{2,2}(u_1 < u_2 < M_1 \leq \lambda T) \\
IE_{2,2}(M_1 < u_1 < u_2 \leq \lambda T) \\
IE_{2,2}(M_1 \leq \lambda T < u_1 < u_2) \\
IE_{2,2}(u_1 < M_1 < u_2 \leq \lambda T) \\
IE_{2,2}(u_1 < M_1 \leq \lambda T < u_2) \\
IE_{2,2}(M_1 < u_1 \leq \lambda T < u_2)
\end{cases}$$

where $IE_{2,2}(u_1 < u_2 < M_1 \leq \lambda T) = \frac{PI_{be}}{T} \left[ \int_0^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_2}^{M_1} t \cdot R_3(P,t)dt \right]$
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\[
IE_{2.2}(M_1 < u_1 \leq \lambda T < u_2) = \frac{PI_{he}}{T} \int_0^{M_1} t \cdot R_1(P, t) dt + \frac{PI_{he}(1-\lambda)RP(M_2-T)}{T} + \frac{PI_{be}(1-\lambda)}{T} \left[ \int_{u_1}^{u_2} t \cdot R_2(P, t) dt + \int_{u_2}^{T} t \cdot R_3(P, t) dt \right]
\]

Case 3: \( T > M_2 \)

Depending on values of \( M_1, M_2 \) and \( \lambda T \), the possible three sub-cases are (i) \( \lambda T < M_1 \) (ii) \( M_1 \leq \lambda T < M_2 \) and (iii) \( \lambda T \geq M_2 \). The values of \( u_1 \) and \( u_2 \) will sub-branch each sub-cases as discussed below.

Sub-case 3.1: \( \lambda T < M_1 \) (Fig. 4.1.4)

In this scenario, since \( \lambda T < M_1 \), the buyer has sufficient amount in the account to pay the \( \lambda \) - fraction of purchase cost at time \( M_1 \). The buyer has unsold stock in the inventory at \( M_2 \). So the interest charges per unit time is
The buyer earns interest on (1) the generated revenue during \([0, M_1]\) (2) at time \(M_1\), the buyer pays \(\lambda\) percentage of the purchase cost and earns interest on \((1-\lambda)\) fraction of purchase cost during \([M_1, M_2]\) and (3) on the average sales revenue during \([T, M_2]\). Hence, the total interest earned per unit time is

\[
IC_{3,1}(\lambda T < M_1) = \frac{C_{b}I_{bc}}{T} \int_{M_2}^{T} I(t)dt = \begin{cases} 
IC_{3,1}(u_1 < u_2 < M_1 < M_2) \\
IC_{3,1}(M_1 < u_1 < u_2 < M_2) \\
IC_{3,1}(M_1 < M_2 < u_1 < u_2) \\
C_{3,1}(u_1 < M_1 < u_2 < M_2) \\
IC_{3,1}(u_1 < M_1 < M_2 < u_2) \\
IC_{3,1}(M_1 < u_1 < M_2 < u_2)
\end{cases}
\]

where \(IC_{3,1}(u_1 < u_2 < M_1 < M_2) = \frac{C_{b}I_{bc}}{T} \int_{M_2}^{T} I_3(t)dt\)

\(IC_{3,1}(M_1 < u_1 < u_2 < M_2) = IC_{3,1}(u_1 < u_2 < M_1 < M_2)\)

\(IC_{3,1}(M_1 < M_2 < u_1 < u_2) = \frac{C_{b}I_{bc}}{T} \left[ \int_{M_2}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{T} I_3(t)dt \right]\)

\(IC_{3,1}(u_1 < M_1 < u_2 < M_2) = IC_{3,1}(u_1 < u_2 < M_1 < M_2)\)

\(IC_{3,1}(u_1 < M_1 < M_2 < u_2) = \frac{C_{b}I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{T} I_3(t)dt \right]\)

\(IC_{3,1}(M_1 < u_1 < M_2 < u_2) = IC_{3,1}(u_1 < M_1 < M_2 < u_2)\)
\[
IE_{3.1}\left(\lambda T < M_1\right) = \frac{PI_{bc}}{T} \int_0^{\lambda T} t \cdot R(P,t)dt + \frac{PI_{bc}}{T} \int_0^{M_1} t \cdot R(P,t)dt\left(M_2 - M_1\right) + \frac{PI_{bc}}{T} \int_{M_1}^{M_2} t \cdot R(P,t)dt
\]

\[
\begin{aligned}
IE_{3.1}(u_1 < u_2 < M_1 < M_2) \\
IE_{3.1}(M_1 < u_1 < u_2 < M_2) \\
IE_{3.1}(M_1 < M_2 < u_1 < u_2) \\
IE_{3.1}(u_1 < M_1 < u_2 < M_2) \\
IE_{3.1}(u_1 < M_1 < M_2 < u_2) \\
IE_{3.1}(M_1 < u_1 < M_2 < u_2)
\end{aligned}
\]

where \(IE_{3.1}(u_1 < u_2 < M_1 < M_2) = \frac{PI_{bc}}{T} \left[\int_0^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_2}^{M_1} t \cdot R_3(P,t)dt\right] + \frac{PI_{bc}}{T} \int_{M_1}^{M_2} R_3(P,t)dt\]

\[
IE_{3.1}(M_1 < u_1 < u_2 < M_2) = \frac{PI_{bc}}{T} \int_0^{u_1} t \cdot R_1(P,t)dt + \frac{PI_{bc}}{T} \left[\int_0^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_2}^{M_1} t \cdot R_3(P,t)dt\right] + \frac{PI_{bc}(M_1 - \lambda T)}{T} \int_{M_1}^{M_2} R_3(P,t)dt
\]

\[
IE_{3.1}(M_1 < M_2 < u_1 < u_2) = \frac{PI_{bc}}{T} \int_0^{M_1} t \cdot R_1(P,t)dt + \frac{PI_{bc}(M_1 - \lambda T)}{T} \int_{M_1}^{M_2} R_3(P,t)dt
\]

\[
IE_{3.1}(u_1 < M_1 < u_2 < M_2) = \frac{PI_{bc}}{T} \left[\int_0^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt\right] + \frac{PI_{bc}}{T} \left[\int_0^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt + \int_{M_1}^{u_2} t \cdot R_3(P,t)dt\right] + \frac{PI_{bc}(M_1 - \lambda T)}{T} \int_{M_1}^{u_2} R_3(P,t)dt + \int_{u_2}^{M_2} R_3(P,t)dt
\]
\[
IE_{3.1}(u_1 < M_1 < M_2 < u_2) = \frac{PI_{be}}{T} \left[ \int_{0}^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt \right] + \frac{PI_{be}}{T} \int_{M_2}^{M_1} t \cdot R_2(P,t)dt
\]
\[
+ \frac{PI_{be}(M_1 - \lambda T)}{T} \int_{M_1}^{M_2} R_2(P,t)dt
\]
\[
IE_{3.1}(M_1 < u_1 < M_2 < u_2) = \frac{PI_{be}}{T} \int_{0}^{M_1} t \cdot R_1(P,t)dt + \frac{PI_{be}}{T} \left[ \int_{M_1}^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_2} t \cdot R_2(P,t)dt \right]
\]
\[
+ \frac{PI_{be}(M_1 - \lambda T)}{T} \left[ \int_{M_1}^{u_1} R_1(P,t)dt + \int_{u_1}^{M_2} R_2(P,t)dt \right]
\]

**Sub-case 3.2: \( M_1 \leq \lambda T < M_2 \) (Fig. 4.1.5)**

![Inventory Level Diagram](image)

Here, at time \( M_1 \), the buyer pays \( \lambda \) fraction of purchase cost and take advantage of the discount in the unit purchase price. Since \( M_1 \leq \lambda T \), the buyer pays the interest charges during \( [M_1, \lambda T] \) and also during \( [M_2, T] \). Hence, total interest paid per unit time by the buyer is
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\[ IC_{3,2} \left( M_1 \leq \lambda T < M_2 \right) = \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \int_{M_1}^{\lambda T} I(t)dt + \frac{C_b I_{bc}}{T} \int_{M_1}^{T} I(t)dt \]

\[
\begin{align*}
IC_{3,2} (u_1 < u_2 < M_1 \leq \lambda T < M_2) &= \\
IC_{3,2} (M_1 < u_1 < u_2 \leq \lambda T < M_2) &= \\
IC_{3,2} (M_1 \leq \lambda T < u_1 < u_2 < M_2) &= \\
IC_{3,2} (M_1 \leq \lambda T < M_2 < u_1 < u_2) &= \\
\end{align*}
\]

where \( IC_{3,2} (u_1 < u_2 < M_1 \leq \lambda T < M_2) \) is given by:

\[
\begin{align*}
IC_{3,2} (u_1 < u_2 < M_1 \leq \lambda T < M_2) &= \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \int_{M_1}^{\lambda T} I(t)dt + \frac{C_b I_{bc}}{T} \int_{M_1}^{T} I(t)dt \\
&+ \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\end{align*}
\]

\[
\begin{align*}
IC_{3,2} (M_1 < u_1 < u_2 \leq \lambda T < M_2) &= IC_{3,1} (M_1 < u_1 < u_2 < M_2) \\
&+ \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\end{align*}
\]

\[
\begin{align*}
IC_{3,2} (M_1 \leq \lambda T < u_1 < u_2 < M_2) &= IC_{3,1} (M_1 < M_2 < u_1 < u_2) + \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \int_{M_1}^{\lambda T} I_1(t)dt \\
&+ \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\end{align*}
\]

\[
\begin{align*}
IC_{3,2} (M_1 \leq \lambda T < M_2 < u_1 < u_2) &= IC_{3,1} (u_1 < M_1 < u_2 < M_2) + \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \int_{M_1}^{\lambda T} I_1(t)dt \\
&+ \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\end{align*}
\]

\[
\begin{align*}
IC_{3,2} (u_1 < M_1 < u_2 \leq \lambda T < M_2) &= IC_{3,1} (u_1 < M_1 < u_2 < M_2) \\
&+ \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\end{align*}
\]

\[
\begin{align*}
IC_{3,2} (u_1 < M_1 \leq \lambda T < u_2 < M_2) &= IC_{3,1} (u_1 < M_1 < u_2 < M_2) + \frac{C_b \left(1 - \beta\right) I_{bc}}{T} \int_{M_1}^{\lambda T} I_2(t)dt
\end{align*}
\]
I_{C,3,2}(u_1 < M_1 \leq \lambda T < M_2 < u_2) = \frac{C_b I_{bc}}{T} \int_{M_2}^{u_2} I_2(t)dt + \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I_2(t)dt

I_{C,3,2}(M_1 < u_1 \leq \lambda T < u_2 < M_2) = I_{C,3,1}(M_1 < u_1 < u_2 < M_2)

+ \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{\lambda T} I_1(t)dt \right]

I_{C,3,2}(M_1 < u_1 < \lambda T < M_2 < u_2) = \frac{C_b I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_2(t)dt \right]

+ \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_1(t)dt + \int_{u_1}^{\lambda T} I_1(t)dt \right]

I_{C,3,2}(M_1 \leq \lambda T < u_1 < M_2 < u_2) = \frac{C_b I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_2(t)dt \right]

+ \frac{C_b(1-\beta)I_{bc}}{T} \int_{M_1}^{\lambda T} I_1(t)dt

The interest earned by the buyer per unit time is

I_{E,3,2}(M_1 \leq \lambda T < M_2) = \frac{PI_{bc}}{T} \int_{0}^{M_1} t \cdot R(P,t)dt + \frac{PI_{bc}}{T} \int_{\lambda T}^{M_2} t \cdot R(P,t)dt

= \begin{cases} 
I_{E,3,2}(u_1 < u_2 < M_1 \leq \lambda T < M_2) \\
I_{E,3,2}(M_1 < u_1 < u_2 \leq \lambda T < M_2) \\
I_{E,3,2}(M_1 \leq \lambda T < u_1 < u_2 < M_2) \\
I_{E,3,2}(M_1 \leq \lambda T < M_2 < u_1 < u_2) \\
I_{E,3,2}(u_1 < M_1 < u_2 \leq \lambda T < M_2) \\
I_{E,3,2}(u_1 < M_1 \leq \lambda T < u_2 < M_2) \\
I_{E,3,2}(u_1 < M_1 \leq \lambda T < M_2 < u_2) \\
I_{E,3,2}(M_1 < u_1 \leq \lambda T < u_2 < M_2) \\
I_{E,3,2}(M_1 < u_1 \leq \lambda T < M_2 < u_2) \\
I_{E,3,2}(M_1 \leq \lambda T < u_1 < M_2 < u_2) \\
\end{cases}

where \( I_{E,3,2}(u_1 < u_2 < M_1 \leq \lambda T < M_2) \)

= \frac{PI_{bc}}{T} \left[ \int_{0}^{u_1} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_2}^{M_1} t \cdot R_3(P,t)dt \right] + \frac{PI_{bc}}{T} \int_{\lambda T}^{M_2} t \cdot R_3(P,t)dt
\[ IE_{3.2}(M_1 < u_1 < u_2 \leq \lambda T < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{M_2} t \cdot R_1(P,t)dt + \int_{\lambda T}^{M_2} t \cdot R_3(P,t)dt \right] \]

\[ IE_{3.2}(M_1 \leq \lambda T < u_1 < u_2 < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{M_2} t \cdot R_1(P,t)dt \right] + \frac{PI_{bc}}{T} \left[ \int_{u_1}^{u_2} t \cdot R_1(P,t)dt + \int_{u_1}^{u_2} t \cdot R_2(P,t)dt + \int_{u_1}^{u_2} t \cdot R_3(P,t)dt \right] \]

\[ IE_{3.2}(M_1 \leq \lambda T < M_2 < u_1 < u_2) = \frac{PI_{bc}}{T} \left[ \int_0^{M_2} t \cdot R_1(P,t)dt + \int_{\lambda T}^{M_2} t \cdot R_1(P,t)dt \right] \]

\[ IE_{3.2}(u_1 < M_1 < u_2 \leq \lambda T < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{u_2} t \cdot R_1(P,t)dt + \int_{\lambda T}^{u_2} t \cdot R_2(P,t)dt + \int_{\lambda T}^{u_2} t \cdot R_3(P,t)dt \right] \]

\[ IE_{3.2}(u_1 < M_1 \leq \lambda T < u_2 < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{u_2} t \cdot R_1(P,t)dt + \int_{\lambda T}^{u_2} t \cdot R_2(P,t)dt \right] \]

\[ IE_{3.2}(u_1 < \lambda T < u_2 < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{u_2} t \cdot R_1(P,t)dt + \int_{\lambda T}^{u_2} t \cdot R_2(P,t)dt \right] \]

\[ IE_{3.2}(M_1 < u_1 \leq \lambda T < u_2 < M_2) = \frac{PI_{bc}}{T} \left[ \int_0^{M_2} t \cdot R_1(P,t)dt \right] + \frac{PI_{bc}}{T} \left[ \int_{\lambda T}^{M_2} t \cdot R_2(P,t)dt + \int_{\lambda T}^{M_2} t \cdot R_3(P,t)dt \right] \]

\[ IE_{3.2}(M_1 < u_1 \leq \lambda T < M_2 < u_2) = \frac{PI_{bc}}{T} \left[ \int_0^{M_2} t \cdot R_1(P,t)dt \right] \]
\[ IE_{3.2}(M_1 \leq \lambda T < u_1 < M_2 < u_2) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t) \, dt \right] \]
\[ + \frac{PI_{bc}}{T} \left[ \int_{M_1}^{u_1} t \cdot R_1(P,t) \, dt + \int_{u_1}^{M_2} t \cdot R_2(P,t) \, dt \right] \]

**Sub-case 3.3: \( M_2 \leq \lambda T \)** (Fig. 4.1.6)

In this scenario, the interest earned per unit time by the buyer is given by

\[ IE_{3.3}(M_2 \leq \lambda T) = \frac{PI_{bc}}{T} \int_{0}^{M_1} t \cdot R(P,t) \, dt = \begin{cases} 
IE_{3.3}(u_1 < u_2 < M_1 < M_2 \leq \lambda T) \\
IE_{3.3}(M_1 < u_1 < u_2 < M_2 \leq \lambda T) \\
IE_{3.3}(M_1 < M_2 < u_1 < u_2 \leq \lambda T) \\
IE_{3.3}(M_1 < M_2 \leq \lambda T < u_1) \\
IE_{3.3}(u_1 < M_1 < u_2 < M_2 \leq \lambda T) \\
IE_{3.3}(u_1 < M_1 < M_2 < u_2 \leq \lambda T) \\
IE_{3.3}(u_1 < M_1 < M_2 \leq \lambda T < u_2) \\
IE_{3.3}(M_1 < M_2 < M_1 < u_2 \leq \lambda T) \\
IE_{3.3}(M_1 < u_1 < M_2 \leq \lambda T < u_2) \\
IE_{3.3}(M_1 < M_2 < u_1 \leq \lambda T < u_2) 
\end{cases} \]
where

\[
IE_{3,3}(u_1 < u_2 < M_1 < M_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{u_1}^{u_2} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt + \int_{u_2}^{M_1} t \cdot R_3(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < u_1 < u_2 < M_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < M_2 < u_1 < u_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < M_2 \leq \lambda T < u_1 < u_2) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]

\[
IE_{3,3}(u_1 < M_1 < u_2 < M_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{u_1}^{M_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt \right]
\]

\[
IE_{3,3}(u_1 < M_1 < M_2 < u_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{u_1}^{M_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt \right]
\]

\[
IE_{3,3}(u_1 < M_1 < M_2 \leq \lambda T < u_2) = \frac{PI_{bc}}{T} \left[ \int_{u_1}^{M_1} t \cdot R_1(P,t)dt + \int_{u_1}^{M_1} t \cdot R_2(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < u_1 < M_2 < u_2 \leq \lambda T) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < u_1 < M_2 \leq \lambda T < u_2) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]

\[
IE_{3,3}(M_1 < M_2 < u_1 \leq \lambda T < u_2) = \frac{PI_{bc}}{T} \left[ \int_{0}^{M_1} t \cdot R_1(P,t)dt \right]
\]
and the interest paid per unit time on the unsold items by the buyer is

\[
 IC_{3.3} (M_2 \leq \lambda T) = \frac{C_b (1 - \beta)}{T} \int_{M_1}^{T \lambda T} I(t) dt + \frac{C_b (1 - \lambda)}{T} \int_{M_2}^{T \lambda T} I(t) dt + \frac{C_b (1 - \lambda)}{T} \int_{\lambda T}^{T} I(t) dt
\]

\[
 = \begin{cases} 
 IC_{3.3} (u_1 < u_2 < M_1 < M_2 \leq \lambda T) \\
 IC_{3.3} (M_1 < u_1 < u_2 < M_2 \leq \lambda T) \\
 IC_{3.3} (M_1 < M_2 < u_1 < u_2 \leq \lambda T) \\
 IC_{3.3} (M_1 < M_2 \leq \lambda T < u_1 < u_2) \\
 IC_{3.3} (u_1 < M_1 < u_2 < M_2 \leq \lambda T) \\
 IC_{3.3} (u_1 < M_1 < M_2 < u_2 \leq \lambda T) \\
 IC_{3.3} (u_1 < M_1 < M_2 \leq \lambda T < u_2) \\
 IC_{3.3} (M_1 < u_1 < M_2 < u_2 \leq \lambda T) \\
 IC_{3.3} (M_1 < u_1 < M_2 \leq \lambda T < u_2) \\
 IC_{3.3} (M_1 < M_2 < u_1 \leq \lambda T < u_2) 
\end{cases}
\]

where \( IC_{3.3} (u_1 < u_2 < M_1 < M_2 \leq \lambda T) = \frac{C_b (1 - \beta)}{T} \int_{M_1}^{T \lambda T} I_3(t) dt \)

\[
 + \frac{C_b (1 - \lambda)}{T} \left[ \int_{M_2}^{T \lambda T} I_3(t) dt + \int_{\lambda T}^{T} I_3(t) dt \right]
\]

\[
 IC_{3.3} (M_1 < u_1 < u_2 < M_2 \leq \lambda T) = \frac{C_b (1 - \beta)}{T} \left[ \int_{M_1}^{u_1} I_1(t) dt + \int_{u_1}^{u_2} I_2(t) dt + \int_{u_2}^{T \lambda T} I_3(t) dt \right]
\]

\[
 + \frac{C_b (1 - \lambda)}{T} \left[ \int_{M_2}^{T \lambda T} I_3(t) dt \right]
\]

\[
 IC_{3.3} (M_1 < M_2 < u_1 < u_2 \leq \lambda T) = \frac{C_b (1 - \beta)}{T} \left[ \int_{M_1}^{u_1} I_1(t) dt + \int_{u_1}^{u_2} I_2(t) dt + \int_{u_2}^{T \lambda T} I_3(t) dt \right]
\]

\[
 + \frac{C_b (1 - \lambda)}{T} \left[ \int_{M_2}^{T \lambda T} I_3(t) dt \right]
\]

\[
 IC_{3.3} (M_1 < M_2 \leq \lambda T < u_1 < u_2) = \frac{C_b (1 - \beta)}{T} \left[ \int_{M_1}^{T \lambda T} I_3(t) dt \right]
\]

\[
 + \frac{C_b (1 - \lambda)}{T} \left[ \int_{M_2}^{T \lambda T} I_3(t) dt \right]
\]
Analysis of Inventory Policies in Supply Chain under Trapezoidal Demand

\[
IC_{3,3}(u_1 < M_1 < u_2 < M_2 \leq \lambda T) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(u_1 < M_1 < M_2 < u_2 \leq \lambda T) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(u_1 < M_1 < M_2 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(M_1 < u_1 < M_2 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(M_1 < u_1 < M_2 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(M_1 < M_2 < u_1 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]

\[
IC_{3,3}(M_1 < M_2 < u_1 \leq \lambda T < u_2) = \frac{C_b(1-\beta)I_{bc}}{T} \left[ \int_{M_1}^{u_1} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right] + \frac{C_b(1-\lambda)I_{bc}}{T} \left[ \int_{M_2}^{u_2} I_2(t)dt + \int_{u_2}^{\lambda T} I_3(t)dt \right]
\]
The total profit per unit time of the buyer is given by

\[
\pi(\lambda, P, T) = \begin{cases} 
\pi_1(\lambda, P, T) & T \leq M_1 \\
\pi_{2,1}(\lambda, P, T) & M_1 < T \leq M_2 \\
\pi_{2,2}(\lambda, P, T) & M_1 < T \leq M_2 \\
\pi_{3,1}(\lambda, P, T) & M_2 \leq T \\
\pi_{3,2}(\lambda, P, T) & M_2 \leq T \\
\pi_{3,3}(\lambda, P, T) & M_2 \leq T 
\end{cases}
\]

where \(\pi_1(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_1\) (4.1.2)

\(\pi_{2,1}(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_{2,1} - IC_{2,1}\) (4.1.3)

\(\pi_{2,2}(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_{2,2} - IC_{2,2}\) (4.1.4)

\(\pi_{3,1}(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_{3,1} - IC_{3,1}\) (4.1.5)

\(\pi_{3,2}(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_{3,2} - IC_{3,2}\) (4.1.6)

\(\pi_{3,3}(\lambda, P, T) = GR - OC_b - HC_b + DR + IE_{3,3} - IC_{3,3}\) (4.1.7)

With some algebraic computations, one can check that the objective functions defined in equations (4.1.2) – (4.1.7) are continuous functions of \(\lambda, P\) and \(T\).

Our objective is to determine the optimal selling price, the cycle time and the payment percentage that maximizes the annual profit \(\pi(\lambda, P, T)\) per unit time of the buyer for two-part trade credit policy.
4.1.3 Computational algorithm

We outline following steps to determine optimal policies for the buyer to maximize the total profit per unit time.

**Step 1:** Assign values to the inventory parameters.

**Step 2:** Solve $\frac{\partial \pi}{\partial \lambda} = 0$, $\frac{\partial \pi}{\partial P} = 0$ and $\frac{\partial \pi}{\partial T} = 0$ simultaneously. Check conditions given in case $i; i = 1, 2, 3$. Identify the time scenario and find out maximum profit.

**Step 3:** Knowing optimal policy $(\lambda, P, T)$, compute $Q$ (given in equation 4.1.1).

4.1.4 Numerical Examples

**Example 4.1.1:** Suppose $a = 30,000,000$ units, $\eta = 1.25$, $b_1 = 7\%$, $b_2 = 5\%$, $u_1 = 15$ days, $u_2 = 25$ days, $C_b = $10/unit, $A_b = $300/order, $I_b = 10\%/year$, $I_{bc} = 20\%/year$ and $I_{bc} = 15\%/year$. A credit term considered is ‘4/10 net 30’.

i.e. if buyer pays by $M_1 = 10$ days, the discount $\beta = 4\%$ in the purchase cost is offered. Otherwise, the payment is to be settled by $M_2 = 30$ days.

Following steps outlined in section 4.1.2, the buyer’s optimal fraction of payment $\lambda$ is 67% at $M_1$. This suggests 33% of the purchase cost should be paid at $M_2$.

With this payment policy, the buyer’s cycle time is $T = 24$ days and selling price $P = $48.77/unit. This obtains maximum profit of $9,07,739$.

From figure 4.1.7, it is observed that the buyer’s profit is convex with respect to the biddable payment to be made at earlier date to avail of the discount in the
unit purchase price. The concavity of the profit with respect to the retail price and the cycle time is shown in figure 4.1.8 and figure 4.1.9 respectively. The 3-D plots given in figures 4.1.10 - 4.1.12 suggest that the buyer’s profit is maximum for the proposed concept.

Figure 4.1.7 Convexity of profit w. r. t. biddable payment $\lambda$

Figure 4.1.8 Concavity of profit w. r. t. retail price $P$

Figure 4.1.9 Concavity of profit w. r. t. cycle time $T$

Figure 4.1.10 Concavity of profit w. r. t. biddable payment $\lambda$ and retail price $P$
biddable payment $\lambda$ and cycle time $T$

**Example 4.1.2**: Assuming all the parametric values of the model as given in example 4.1.1 except $u_1 = 15$ days and $u_2 = 45$ days gives that $\lambda = 1$ suggesting that the buyer should settle full account at $M_1$. The corresponding cycle time is 53 days and selling price is $48.65$. This results the maximum profit of $8,97,662$.

**Example 4.1.3**: When $\lambda = 0$ in example 4.1.1, the optimal selling price is $50.44$ and cycle time is 65 days which results profit of $8,93,187$.

From examples 4.1.1 and 4.1.3, when buyer takes advantage of fraction payment, the profit increases by $1.62\% \left(= \frac{907739 - 893187}{893187} \times 100\right)$.

Next, we carry out the sensitivity analysis by changing the inventory parameters given in example 4.1.1 as $-20\%, -10\%, +10\%$ and $+20\%$ to get the managerial insights for the decision maker. In figure 4.1.13, variations in the early payment
fraction $\lambda$ are studied. It is observed that $\lambda$ has highly positive impact due to changes in scale demand and interest paid by the buyer. Though scale demand is uncontrollable, the interest to be paid on unsold stock can be reduced by selling items in time. $\lambda$ is positively sensitive to the offered payment time and inventory carrying charge fraction. The ordering cost and the time point when demand starts decreasing exponentially are negatively related to $\lambda$. This suggests the buyer to save in ordering cost by larger order though the inventory carrying charged fraction will hinder for larger order. So, the buyer should find out the trade-off between these two parameters. The purchase cost of an item has negative impact on $\lambda$ which suggests that the decision maker should do smaller fraction of payment at earlier date for costlier items. From figure 4.1.14, we see that increase in purchase cost increases selling price while other inventory parameters decrease selling price. In figure 4.1.15, variations in the cycle time are depicted. The price elasticity, larger offered delay period and the time point when demand starts decreasing increases cycle time significantly. Increase in discount rate of units in purchase price decreases cycle time. This establishes that the offer of early payment by taking advantage of discount in unit purchase price is beneficial to the buyer. In figure 4.1.16, the profit variations are studied. The profit sharply increases with increase in discount in unit purchase price, scale demand, early payment time, the rate at which demand increases and the time in which demand increases and remains constant. Thus, the buyer is beneficial if the inventory policies are planned considering positive impact of these parameters.
Figure 4.1.13 Sensitivity analysis of inventory parameters on biddable payment

Figure 4.1.14 Sensitivity analysis of inventory parameters on retail price
Figure 4.1.15 Sensitivity analysis of inventory parameters on cycle time

Figure 4.1.16 Sensitivity analysis of inventory parameters on profit
4.2 Optimal Replenishment and Payment Policies for Deteriorating Inventory Model with Price-Sensitive Trapezoidal Demand Under Biddable Two-Part Trade Credit

The present inventory model is an extension of model discussed in section 4.1 with the assumption that inventory products get deteriorated at a constant rate.

4.2.1 Assumptions

The present section uses following additional assumption and notation with those in section 4.1.

1. The units in the inventory system deteriorate at a constant rate \( \theta \) \((0 < \theta < 1)\). The deteriorated units can neither be repaired nor replaced during the cycle time.

4.2.2 Mathematical Model

The inventory level in the warehouse changes due to price-sensitive trapezoidal demand and deterioration rate of units in the warehouse. The rate of change of the inventory at any instant of time \( t \) is governed by the differential equation

\[
\frac{d I(t)}{dt} = -R(P,t) - \theta I(t); 0 \leq t \leq T \quad \text{with the boundary condition} \quad I(T) = 0.
\]

The solution of the differential equation is

\[
I(t) = \begin{cases} 
I_1(t), & 0 \leq t \leq u_1 \\
I_2(t), & u_1 \leq t \leq u_2 \\
I_3(t), & u_2 \leq t \leq T 
\end{cases}
\]
Using \( I(0) = Q \), we get

\[
Q = \begin{cases}
  aP^{-\eta} \left[ -\frac{1}{\theta} + \frac{b_1}{\theta^2} + \frac{1 + b_1 u_1}{\theta} e^{\frac{\theta t}{1 - \theta}} - \frac{b_2}{\theta^2} e^{\frac{\theta t}{1 - \theta}} \right] \\
  + P^{-\eta} \left[ \frac{a(1 + b_1 u_1)}{\theta} e^{\frac{\theta t}{1 - \theta}} - e^{\frac{\theta t}{1 - \theta}} \right] \\
  + P^{-\eta} \left[ \frac{a(1 + b_1 u_1) e^{b_2 t}}{\theta - b_2} \right] - e^{b_2 t} \\
  \end{cases}
\]

Following the same procedure as discussed in section 4.1, three decision variables are computed that maximize the buyer’s annual profit \( \pi(\lambda, P, T) \) per unit time for biddable payment under two-part trade credit policy.

**4.2.3 Numerical Examples**

**Example 4.2.1:** Consider the same example from section 4.1 with \( \theta = 1\% \).

A credit term considered is ‘4/10 net 30’. Then the buyer’s optimal fraction of payment \( \lambda \) is 67\% at \( M_1 \). This suggests 33\% of the purchase cost should be paid at \( M_2 \). With this payment policy, the buyer’s cycle time is \( T = 24 \) days and selling price \( P = $48.77/\text{unit} \). This obtains maximum profit of $9,07,737.
The concavity of the profit with respect to the retail price and the cycle time is shown in figures 4.2.2 and 4.2.3 respectively. From figure 4.2.1, it is seen that the buyer’s profit is convex with respect to the biddable payment. The 3-D plots in figures 4.2.4-4.2.6 suggest that the buyer’s profit is maximum for the proposed concept.

Figure 4.2.1 Convexity of profit w. r. t. biddable payment $\lambda$

Figure 4.2.2 Concavity of profit w. r. t. retail price $P$
Example 4.2.2: Assuming all parametric values of the model as given in example 4.2.1 except $u_1 = 15$ days and $u_2 = 45$ days gives that $\lambda = 1$ suggesting
that the buyer should settle full account at $M_1$. The corresponding cycle time is 53 days and selling price is 48.65. This results the maximum profit of $8,97,652.

Next, we carry out the sensitivity analysis by changing inventory parameters given in example 4.2.1 as -20%, -10%, +10% and +20% to get managerial insights for the decision maker.

![Sensitivity analysis of inventory parameters on biddable payment](image1)

**Figure 4.2.7** Sensitivity analysis of inventory parameters on biddable payment

![Sensitivity analysis of inventory parameters on retail price](image2)

**Figure 4.2.8** Sensitivity analysis of inventory parameters on retail price
Figure 4.2.9 Sensitivity analysis of inventory parameters on cycle time

Figure 4.2.10 Sensitivity analysis of inventory parameters on profit
Conclusions

This chapter analyzes the buyer’s pricing, ordering and payment policies under flexible two-part trade credit scenario \( \left( \beta / M_1 \text{ net } M_2 \right) \) offered by a supplier when demand rate is price-sensitive trapezoidal. It is assumed that the supplier permits the buyer to pay any fraction of the purchase cost within time \( M_1 \) and the remaining account is to be settled by \( M_2 \); where \( M_1 < M_2 \). The buyer’s profit is maximized with respect to optimal selling price, ordering policy and payment option. The given analysis will help the supplier to allow the buyer to pay fraction of purchase cost at \( M_1 \) and rest within \( M_2 \). It suggests that the buyer should take advantage of time phase in which demand increases. It is established that the proposed flexible two-part trade credit is advantageous compared to traditional one for the buyer. By encouraging the buyer to pay early with the offer of discount in purchase price will reduce cash-out flow risk for the supplier. The second section takes care of the constant deterioration of the inventory products as well.